Data Analysis & Decision Making - III
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Lecture 59

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you. And this is welcome to this DADM 3 course on the NPTEL MOOC series. And as you know this course is spread over 12 weeks. Total number of lecturers is 60 and total number of contact hours is 30 which means that each lecture is for half an hour and each week we have five lectures of half an hour each. And after 12 after each week of lectures of five number you basically taken assignment and after the 12 assignments you basically will be appearing for the final examination.

And my good name is Raghu Nandan Sengupta from IME department IIT Kanpur. And as you can see in the slide we are on the 59th lecture which is the last but one lecture and we are in the 12th week and the 50th lecture was basically but dealing with the concept that how the modelings would basically be done and if you remember the concept of modeling I had been mentioning time.

And again in the (previous) in the first few classes of one or two weeks that the methodology concept how you solve would be the main emphasis of this course but how you model how you consider the constraints would be an important point which will only come through, through lot of, lot of experience, lot of problem solving, lot of practical analysis.

And then in the last slide or last two slides we consider the concept of robustness and how robustness was important and how robustness and sensitive analysis are some concept which basically give you the same idea that what you want to basically propose or you want to find out. So, with this we will start the 59th lecture and basically discuss more about robustness part and again I will discuss some models, their proves, the data's, the pre-processing part and what are the answers and how we can analyze them.

So, considering robustness the salient points are so we can consider the constraints are robust that means the corresponding the fact that there would be a probability based on which we can say constraint one or two or three whatever it is, is greater than some some fixed value or changing value also with some level of reliability, some level of robustness. And those level of robustness or reliability I am using, going to use the word reliability and robustness interchangeably so please bear with me.

They would be given by the level given by beta 1 and this level of beta one and beta two would have same implication as we discussed in the diagrammatic form when we considered reliability, when we considered the PMA approach and the RIA approach where if you remember ((inaudible))(03:11)

Consider separately in one case the circle slowly started increasing till it was tangent to the feasible region and the size of that value of the radius was beta. In another case you kept beta fix when you basically shifted the feasible region more inside such that it was tangent to the circle. Why circle? Because we are considering that when you basically convert from the X space to the U space considering the univariate normal and all of them are orthogonal.

We will basically have circle in a two dimension case, in a higher dimension it will be a sphere and so on and so forth. And it will be a hypo-sphere corresponding to the higher dimensions. So, we so coming to the robustness part so the constraints can be robust.

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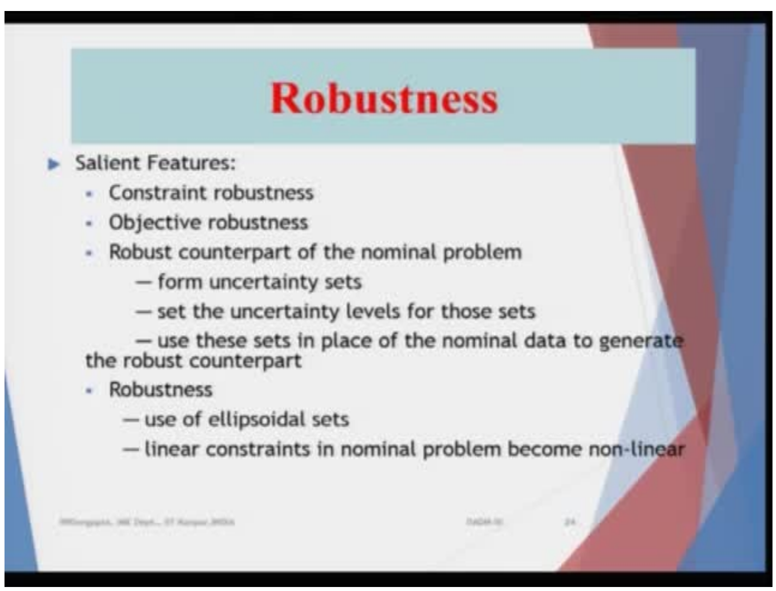

And we can consider the robustness by considering the concept of betas, you can also consider the objective functions to be robust. But in our discussion the problems we will be solving will only consider the concept of constraints to be robust and basically give the solutions accordingly. The robust counterpart for these nominal problems, so now here how we will solve or how we will basically propose the model would be obviously there is a deterministic part of the problem, we will convert the deterministic one in the probabilistic part in the sense the corresponding probability with the level of reliability of beta 1 to beta 2 or beta 3, beta 4 depending on number of constraints which we have would be considered in the constraints.

And then this transformed form which is the probabilistic constraints would basically be solved using the robust counterpart. So, the first part with the deterministic models would not be written, we will immediately right the probabilistic counterpart of those deterministic model and then give an idea, initially we will give an idea how the problems can be solved and we will write their robust counterpart and then basically skip the simulation part because simulation would basically more of trying to basically build the codes in mat lab or whatever it is.

I will just give you the gist and then go into the data which is utilize how the pre-process data is analyzed and then how we basically utilize the preprocess data in the model formulation model solving and then give the results and try to analyze the results that is also very important. So, we can use the concept of uncertainty sets.

So, the uncertainty sets are would basically we mean set of values where the corresponding probabilities would be there probabilities would not be considered per se the probabilities or the chances would be there that each values in the uncertainty set would basically have a certain probability of that being true or that being picked up.

And we will basically have a set of the of the uncertainty level of the set, so each value which is appearing would basically have a probability which I mentioned and these uncertainty sets would basically be considered in our example we consider in such a way that we will basically consider a nominal value and there is nominal value would be the so-called expected value over and above, below which that actual value would be perturbed or changed and that perturbation and change would basically be considered as the level of reliability in the robust counterpart sense.

You will use this set in place of the nominal data. Nominal data would basically as I mentioned just few minutes back would be the average value and we will basically use this nominal data to generate the robust counterpart. Now, one of the main important difference between the reliability part or concept or the stochastic concept and the robust concept is, in the reliability part we would or we have considered the concept of some particular distribution per se. But in the robust counterpart we would not be considering but now later on you will see and obviously there will be a question from your part that we will be using some nominal values over and below which perturbation would be true.

So, these nominal values would be considering based on the fact that we have the information or we have the practical knowledge that the returns of the scripts, of the return of the portfolio would be extreme value distribution, so we will utilize the concept of expected value of the EVDs in order to basically find out what is the nominal value over and below which the perturbations will be true.

We will use the concept of ellipsoidal sets and the concept of linear constraints in the nominal problem may become non-linear. So, but obviously we will try to check that when it (())(08:01) non-linear the concept of convexity should be true or else the concept of optimization would not be able to be utilized in trying to solve the problem.

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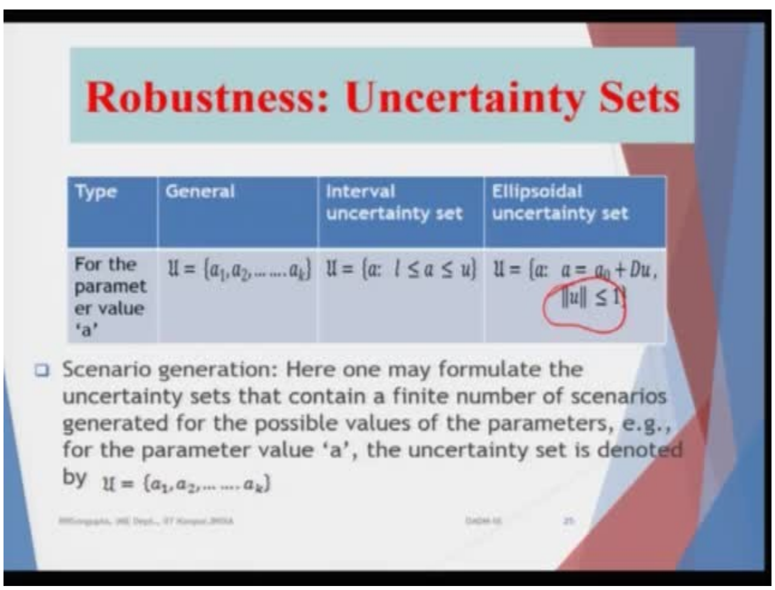

Now, in general robustness as I said in the uncertainty sets there are different ways to basically model the uncertainty sets. So, in general we consider the uncertainty sets given by U where the (())(08:26) values would be given from A1 to Ak and obviously each of them would be have some level of reliability or a probability. In the interval uncertainty sets we will consider the value of the fluctuation and the value of perturbations of the parameter A or it can be X, it can be W whatever it is.

The parameter A would basically fluctuate between a minimum value L and a maximum value of U for each and every A there would be a L1, L2, L3 till Lk considering there are values A1 to Ak corresponding to A1 to Ak the upper amount would be given by a small u1 to small uk. And the ellipsoidal uncertainty sets would be considered based on the fact that will consider in a very simple sense, if it is two dimension 1 it will basically be a circle and the center of the circle would be the nominal value over and below which the perturbation set would be, so if it is the level of variances I am bringing the concept of variances why I will come to that later on.

If we consider the level of variances of the perturbations for both of these variables are to be of equal value then it will obviously it will be a circle in the case, if the variances are different in that case we will basically have ellipsoid with a major and minor axis in different direction depending on where the variance is high or where the perturbation is high.

Perturbation's concept is like a atom which is fluctuating, higher the level of energy more the fluctuation is and lower the energy less the fluctuation is. So, if you are trying to find out the common area it will be an ellipse or ellipsoid depending on which direction the perturbation or the moment or the vibration is high. Now, we will consider this ellipsoidal uncertainty set will be considered based on the fact that we are going to consider the L2 norm.

Now, this concept of L2 norm I have discussed that in DADM 2 when we were discussing the concept of electro and TOPSIS methods. So, this L1 norm, L2 norm till L infinity norm are the norms based on which we measure. And L2 norm is basically the concept of in very simply we use in the Cartesian coordinate where in order to find out the distances between the two points we basically find out X1 minus X2 whole square plus Y1 minus Y2 whole square and then find out the square root of that considering two-dimensional one, in three dimensional one it will basically be calculated accordingly.

So, we will consider and this norm of L2 norm has also be utilized in the case of reliability if you remember in the PMA and the RIA method the constraint of the objective function depending upon which problems formulation you are doing whether the PMA or the RIA we consider this norm of distance which was given by the distance norm which is here, which I have just circled.

So, that the distance norm is basically less than equal to 1 because we are considering a unit circle.

In the scenario generation method here one may formulate the uncertainty sets and obviously the general concept which will use would be the interval set or the ellipsoidal sets whatever all the other methods we consider. So, we will formulate the uncertainty sets that contain a finite number of scenarios as we generate them and they are being they are generated from a possible value of the parameter is and the parameter values 'a' that is the uncertainty set would be denoted between by a set u and the level of realize values for the uncertainty set.

So, the a value would basically take any other values between A1 to Ak depending upon the level of probability or the level of reliability which is true for those. So, realization of A1, A2, A3 would depend on the level of reliability which is there or you want to assign to those uncertainty sets, uncertainty sets. Now, these are just for your information need not be too much bothered into going into this books or the papers provided, if you are interested to go into the research it may be helpful.

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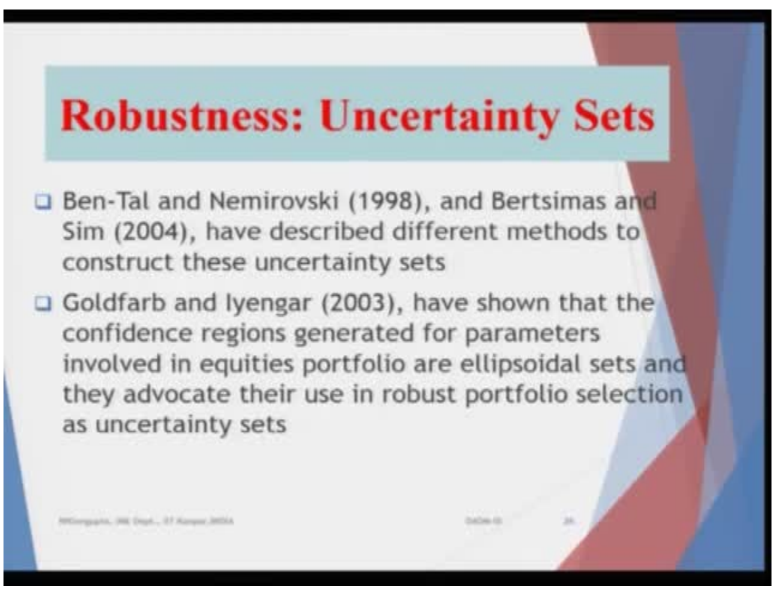

So, Ben-Tal and Nemirovski, they are considered very well-known figure and Bertsimas and Sim are also considered stalwarts in trying to basically model the concept of robustness using the uncertainty level sets or ellipsoidal sets concepts. So, their paper and their book came out in the years 1998 and after that in 2004 also there are seminal work by them. And they have described the different methods to construct this uncertainty sets.

Now, the second point is important for our discussion here Goldfarb and Iyenger have shown that the confidence region generated for parameters involved in equities portfolio in trying to basically model the portfolio optimization. We consider the concept of ellipsoidal sets, so the reason why we are mentioning that second point is that we will be discussing many of the problems on the portfolio optimization point of view that is why we want to give you the reference.

That is why we are going to consider the concept of ellipsoidal sets in order to model the portfolio optimization problem. So, let me continue reading it, so they have shown that the confidence regions generated for parameters involved in equities portfolios are ellipsoidal sets and the advocate that they are used in robust portfolio selections as uncertainty sets are the best way how we can solve them.

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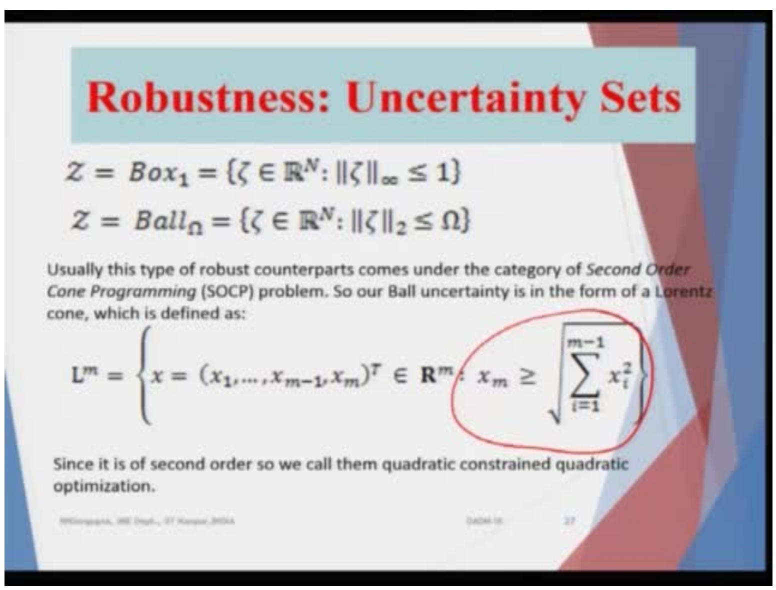

Now, in case of our solving our methodology of solving would be considered this box and ball plot. In the box plot we consider the L infinity norm and in the ball plot we consider the L2 norm and we will consider the transformation being done in such a way we will consider the standard normal deviate this concept of U set as we did in the reliability sense that transforming from X space to U space would also be utilized here.

So, usually this type of robust counterparts, once we basically have the probability convert using the concept of box and ball into the robust counterpart find out the commonality or the intersection between them and then basically solve this problem accordingly. Usually this type of robust counterparts comes under the category of Second Order Cone Programming problem. So, our ball uncertainty is in the form of a Lorentz Cone. So, Lorentz cone is considered ice cream.

So, if you have a cone or set of an ice cream or the softy cone we consider, so they would basically be of the Second Order Cone Programming and the order the solution would be the quadratic in order to basically give us an idea that why we are using the concept of quadratic programming is that because remember the concept of variance would be coming time and again and variance is basically coming from the fact that we are considering the second moment. So, the first moment was basically related to the mean value, second moment is basically related to variances.

So, our ball uncertainty would be in the form of the Lorentz Cone. And then Lorentz Cone would be given if you remember. So, we will basically the mean part for the Lorentz Cone to be true would be that the (square) some of the squares of all the I's, I is equal to 1 to M minus 1 if I basically add them and find out the square root that should be less than equal to the m^{th} one Xm.

Xm is basically the dimension, not the dimension part it is basically the measure which we are doing in the m^{th} dimension. So, we will basically have the concept utilize here. Now, this has something to do if you remember for a triangle to be true that the sum of the two sides would be always be greater than the length of the third side or else the triangle would not be formed. So, this is some concept which you try to utilize in the Lorentz Cone considering the L2 norm in a quadratic programming concept.

So, since it is a second order problem we will call them the quadratic constraint quadratic optimization. And this quadratic optimization we utilize based on the fact that it will be a matrix queue, if it is in a higher dimension and the property if you remember which I mentioned the positive definite, positive semi-definite all this would basically hold true such that we are able to solve the problems accordingly utilizing the robust counterpart.

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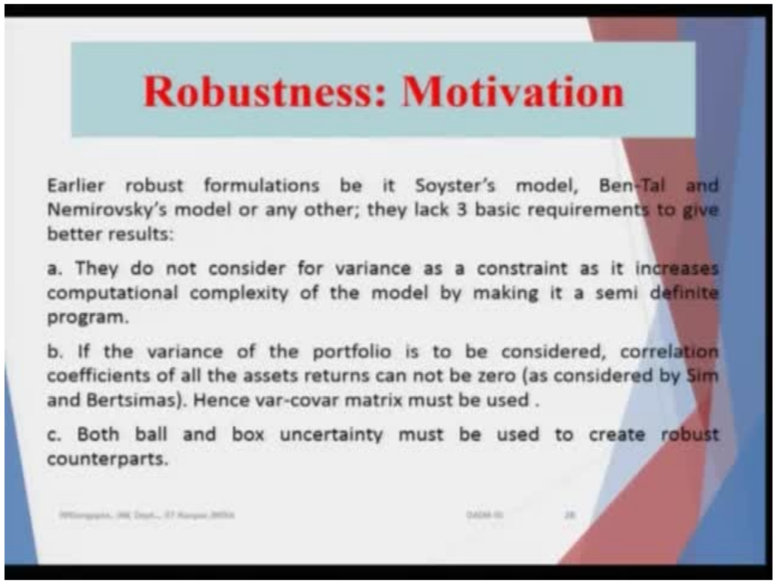

Earlier robust formulations has been solved by the Soyster's model, Ben-Tal and Nemirovsky's model but they lack three basic requirements to give a better results. So, that means what are the requirements which are lacking? They do not consider for variance as a constraint as it increases computational complexity of the model by making it a semi-definite programming. Because the more complexity you want to bring obviously the answers will be more near to practicality but it will basically increase the solution methodology it will make much more difficult for us to solve.

The variances of portfolio but also remember that we have to basically consider the concept of variance of the portfolio to be incorporated in a model because we are trying to basically balance between the return which is the first moment and the risk which is the second moment. The variance of the portfolio is to be considered and computed correlation and coefficients of all the assets cannot be 0 as considered by Bertsimas in his work. Hence variance covariance matrix must be utilized which is basically the principal diagonal, obviously you know but I will still repeat.

The principle diagonal are the variances of the first two itself the 2,2 element is the variance of the second to itself and so on and so forth. And of the diagonal element are basically the co-variances. Both box and ball uncertainty must be utilized to create the robust counterpart and we will take basically the intersection of that with the common area based on that we will solve and try to basically formulate the probability constraints into a robust counterpart and then solve them.

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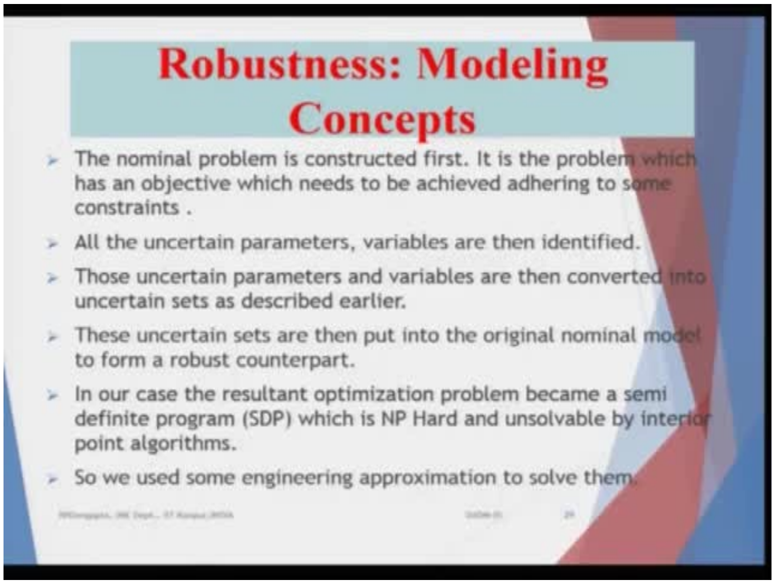

First we will basically have the nominal problem to be constructed, so remember the nominal problem would be based on the fact that we will take the expected value or the mean value of them and then basically try to model the problems according to the perturbations which we will have for over and below the nominal value. So, it is a problem which has an objective function which needs to be achieved adhering to some constraints.

So, those constraints in the case, if they are deterministic we will solve them using the determinist method if they are not then obviously we will have the probabilistic counterpart. All the uncertain parameters, variables are then identified. So, you have to basically first identify the variables and then model them at the constraints. Those uncertain parameters and the variables are then converted into uncertain sets as described above.

So, first so what we have skipped here I did mention that. We will basically have the probabilistic constraint, convert this probabilistic constraint in the robust counterpart and then proceed. And using the concept of uncertainty sets for each and every of this robust or this perturbations values for each and every variable which we are going to consider. These uncertain sets are then put into the original nominal model to form a robust counterpart as I just mentioned.

In our case the resultant optimization problem becomes a semi definite programming considering that we are trying to going to consider variances which are NP Hard problems and unsolvable by basically interior point algorithms. So, obviously you have to use either heuristic methods in order to get near optimum solutions. So, we will basically utilize some concept of very simple engineering in order to solve them. Now, we will consider one by one four models and also give their where as necessary we will give their robust counterpart.

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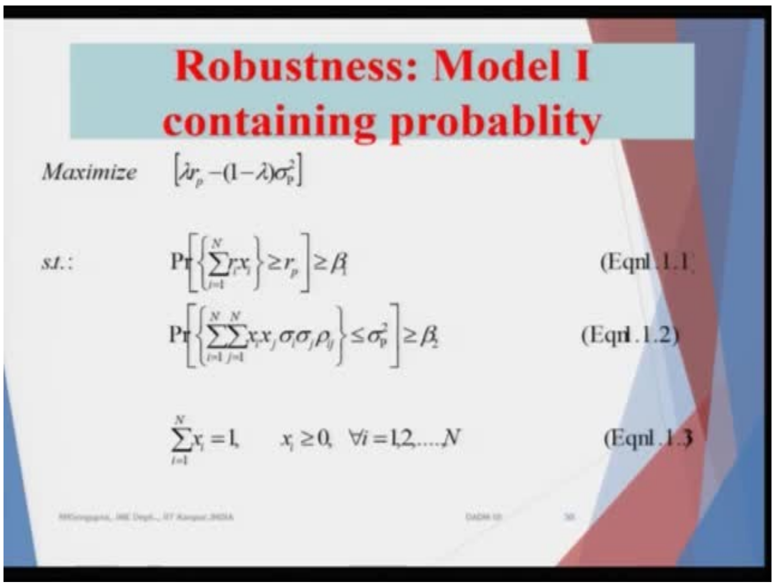

So, in the robustness part in model one which consist containing probability. So, the first problem is like this. We have the return on the portfolio rp, we have the variance of the portfolio Sigma Square b and we want to basically formulate the problem in such a way that we want to basically maximize a convex combination now the return under risk. So, if Lambda is high it will basically put more weightage, if you are concentrating on the objective function if lambda is high we will put more weightage is to the return, if lambda is low we will put more weightage on the variances.

Now, why we are maximizing? We are maximizing and trying to basically pull up the expected value and maximizing a negative value which basically, we are trying to basically pull down that value which is the variances. The second and the third constraints are very simple, still give me some time to repeat it. So, if you considering inside the bracket what you have is basically the return on the portfolio is greater than are some value of rp which we have set for ourselves.

The second constraint again if we only the part which is inside the bracket it means the double summation of Xi, Xj, Sigma I, Sigma J and rho IJ basically means we have the variance co-variances of each and every portfolio or script that is less than equal to Sigma Square P which we have set for ourselves for the problem based on the level of confidence which the investor has.

And the last constraint is basically related to the fact that the sum of the weights of X size. X size are the weights is equal to 1. Interestingly we have considered the values of X I is greater than zero. So, here obviously we have we have not written it but it should be that the values of XI's are between 0 and 1. Now, this would basically be the when I have not mentioned probability this is basically the simple deterministic part.

The moment we bring in equation 1.1 and 1.2 as written in the slide, the concept of probability then we have basically converting this deterministic problem into the probabilistic sense. Now, and obviously on the right-hand side we have the probabilities of returns being greater than RP is greater than equal to beta 1 where beta 1 is the level of probability corresponding to the first constraint.

And in the second constraint the probability of the co-variance of the portfolio being less than equal to Sigma Square P with some set value depending on the market conditions that overall probability is greater than equal to beta 2 it means basically that the level of confidence which we are putting for that second constraint is greater than equal to some value of beta 2 or the beta 2 value which you have set for ourselves.

What we will do is that what I mention is that, convert the first constraint using the nominal values and the different concept of ellipsoidal set into a robust counterpart then convert the second constraint again depending on the level of robustness and the nominal values into the second robust counterpart. Now, what we consider as the nominal value in the first case and the second case? In the first case we will consider the mean values of each and every stocks arise which are there.

So, we will basically collect the data, find out the expected value and basically consider the nominal values or the mean values for each and every stock. And in the second constraint we will consider the variances and find out the average variances based on which we will try to basically find out, consider them as nominal values and based on that we basically find out the counterpart for the second constraint. And obviously as I mentioned we are not going to consider any level of robustness in the objective function.

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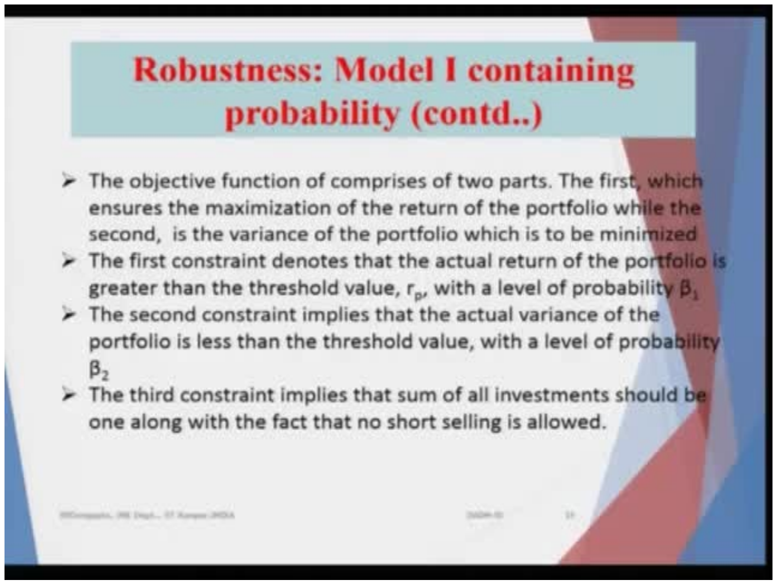

The objective function here comprises, I will just read it, I have already mentioned that but still am reading it. The objective function of this model comprises of two parts, the first which ensures that the maximization of the return of the portfolio whereas the second is the variances of the portfolio which is to be minimized. So, as I mentioned maximization of lambda into RP minus of 1 minus lambda into Sigma Square P basically means that I am trying to basically increase the first moment of the return of the portfolio and decrease the second moment which is the variance of the portfolio.

The first constraint denote as I said that the actual return of the portfolio is greater than the threshold value RP with the level of probability or reliability beta 1. The second constraint implies that the actual variance of the portfolio is less than the threshold value with a level of probability of beta 2 and this beta 1 and beta 2, I am again repeating depends on the investor.

The third constraint implies that the sum of all the investments should be one along with that fact that no short selling is allowed here, but if you remember I mentioned the word these are very interesting in the sense, if they are between 0 and X it means that we are not short selling in case if some of them are negative then obviously some of them would be positive these weights of X.

Which means I am trying to basically utilize some stock which I do not owe, sell it and give it in the market to someone utilize that money from selling that, that stock or scripts which I do not own and basically utilize that money in trying to basically jack up the total amount of investment which I have for those stocks which are already with me and I think they will give such higher returns.

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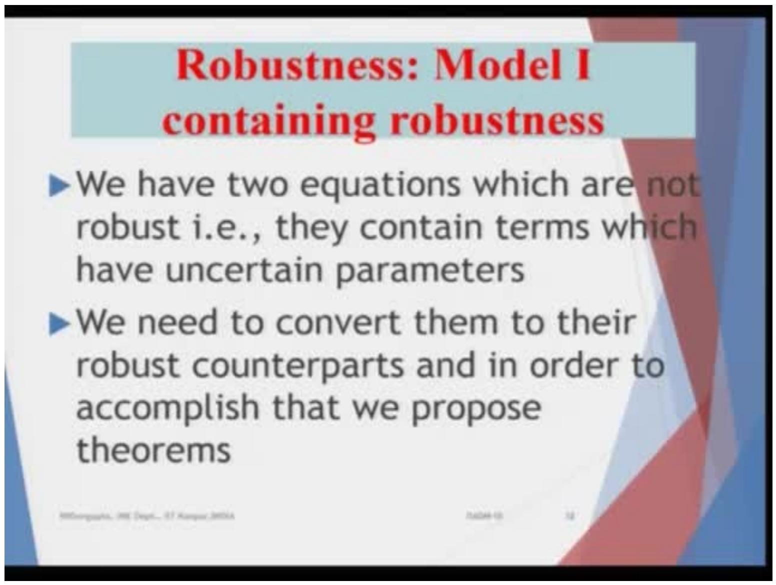

So, in this model we have two equations which are not robust and they contain terms which are not robust and they contain terms which have uncertain parameters but we will try to basically ignore them accordingly. We need to convert them that these two constraints to the robust counterparts in order to accomplish that we propose the particular theorem. And the fact is that whenever you are trying to basically solve the concept of robustness they would be either less than type and greater than type depending on the returns being greater than RP and the risk been less than equal to Sigma P.

So, what we will try to do is that we will try to basically convert them using the plus and minus some concept of perturbation which is there and convert them into the robust counterpart and that we will basically see in the in the later part as we solve the problems accordingly. With this part I will basically end the 59th lecture and basically continue giving a discussion on the robust counterpart in the last lecture which is the 60th one. Have a nice day and thank you very much.