Data Analysis and Decision Making- III
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology Kanpur
Lecture 58

Welcome back my dear friends a very Good Morning, Good Afternoon, Good Evening to all of you wherever you are in this part of the globe and as you know this is the DADM 3 course on the NPTEL MOOC series the total course duration is for 12 weeks which when convert into number of contact hours is 30 and which when spread into the number of lectures is 60 which means that each lecture is for half an hour.

And after each week where in week we have 5 classes of half an hour each we have assignments and after the 12 assignments which is the end of the course we have the final examination and as you can see from the slide we are in the 12th week which is and we are in the 58th lecture that means we have 58, 59, 60 to go.

And my good name is Raghunandan Sengupta from IME department at IIT Kanpur. So, if you remember we have discussing about the concept of robustness of the concept of reliability and in the concept of reliability I did mention that they were reliability basically means that how reliable your answers are considering that the sets of information or parameters which are affecting or the data set which is a defecting your models are can be divided into 2 sets one is the uncertain set.

Uncertain set would also be utilized or used in the later part of the discussion, so they are uncertain parameters they are basically denoted by P and they are the deterministic or non-probabilistic parameters which are d and the weight decision variables are X. Now, we considered the concept of sequential optimization where the decoupled (())(02:04) was run where considering that there are 2 spaces X space and the U space, in the X space we have the actual optimization from model and considering is a deterministic one we can utilized any of the existing models which we have discussed.

There are other models also, different type of models also google programming and so on and so forth based on which we can solve the deterministic problem and get an answer. In case if they are not, we do not have any algorithm to solve we will try to basically utilized the concept of many heuristic methods, some of them we have discussed though briefly in DADM 2.

And based on the deterministic solution we end our discussion and give the results which was fine but in the case of reliability considered (())(02:56) the constraints are probabilistic with some probability beta 1, beta 2 or alpha 1, alpha 2 wherever which way ever way you want to discuss. We saw that in case if it was a normal distribution univariate or a multivariate one we can easily solve it considering for the univariate one we have the simple standard normal table the Z distribution and we can propose answers.

In case in the multivariate case also for a multivariate normal we can propose the answer and finish our task and we comfortable that yes considering normal distribution to be true and normal distribution can be considered for the reliability part for the constraints we get decent results.

And I also showed you by drawing diagrams which I did many times that as you have in the 2 dimensional case the corner points or the deterministic solution in case if they are reliable with certain distribution if it is normal with same variance, then obviously the reliable solution would be more inside the feasible region and the overall reliability the area would be a circle considering the variances for both the distribution univariate marginal distribution for x1 and x2 are normal.

In case if they are normal but variances are different then obviously will have one ellipse with the major axis or the minor axis being vertical along x1 direction or x2 direction whichever you want to basically denote. In case if it is higher dimension we will basically have a sphere on a ellipsoid and our life is also under control considering the symmetric distribution normality we can understand.

But in the case if the non-normal distribution holds then obviously we saw that the overall reliable space is so different looking and the overall shape and size is totally different then trying to find out conceptually we can understand that trying to find out the reliability point, reliability space is easy but trying to basically solve it is basically becomes difficult.

So, we then went into the concept of PMA, RIA and PMA method where we basically utilized the concept that either you minimize or maximize depending on how the problem need to be solved, the function g prime corresponding to some constraints where the distance measure in the U space.

Now, here what we also did was basically convert this X variables into the U variables and we into the U variables means the other univariate normal distribution and we use the concept

that the CDF or the distribution function on value are all between 0 1, 1 and 3 important properties hold, the 4 important properties hold and I did discuss that by writing it here also and we have discussed that in detail in DADM 1 also.

So, using Rosenblatt transformation we can transform the M into U space, find out the MPP using the RIA or the PMA method then map it back or do the inverse transformation of the X space and then basically continue. We also discussed that the starting point in the X space would basically be the mean value or the nominal value or the average values of the parameters P or X, based on that we do the first iteration, get the first answer, then map it into the U space do the PMA or RIA method find out the reliability point MPP point.

Then reverse do reverse transformation into the X space and then again do the optimization using the simple concept of the optimization technique we know and then again transform back in to the U space using the Rosenblatt transformation continue till we basically are able to reach the optimality that is the difference between X, FX T minus FXT minus 1 considering the 2 iteration values are less than some (())(07:10) which we have already pre-decided we stop our optimization.

Now, in the robustness case which I did mention few times that the concept of robustness is basically something to do with the sensitive analysis, sensitive analysis means the initial part or linear programming where we did where we considered the constraints either the variables which were multiplied by X1, X2, X3 all the decision variables or the variables on the right hand side that the values of B are basically change by 1 unit based on that what are basically the amount of slack or the surplus which are left and what is the utilization based on which we can find out the optimum solution and how does the corner point move. Considering that the corner points are the best solution in the amongst the feasible points that the optimum feasible points.

So, robustness would basically be a upper type of that concept which will be utilizing. So, the lecture 58 and 59 and also the major part of the 60th lecture would be something do with the robustness. What I will basically mention is basically the concept of robustness, what are the different type of robust sets which you consider, what is the concept how we basically model it, I will give you few models from the finance point of view, give the robust counterpart.

Initial problems obviously would be probabilistic get the give their robust counterpart, try to basically discuss what is the data set we are using and why we are using the data set the pre-

processing part so on and so forth. So, obviously if it would have a huge amount of conceptual finance concept which will use, which we have discussed in DADM 1 briefly and one of the courses which I taught in NPTEL was the quantitative finance, where we did discuss something to do with the concept of that how the financial data would be utilized when you are trying to do the portfolio optimization.

So, these all problems would be in the from the realm or the areas of portfolio optimization. Before I start of robust optimization there would be few other problems which we could not cover in the 57th lecture, so I will briefly discuss them and basically then move into the area of robust optimization.

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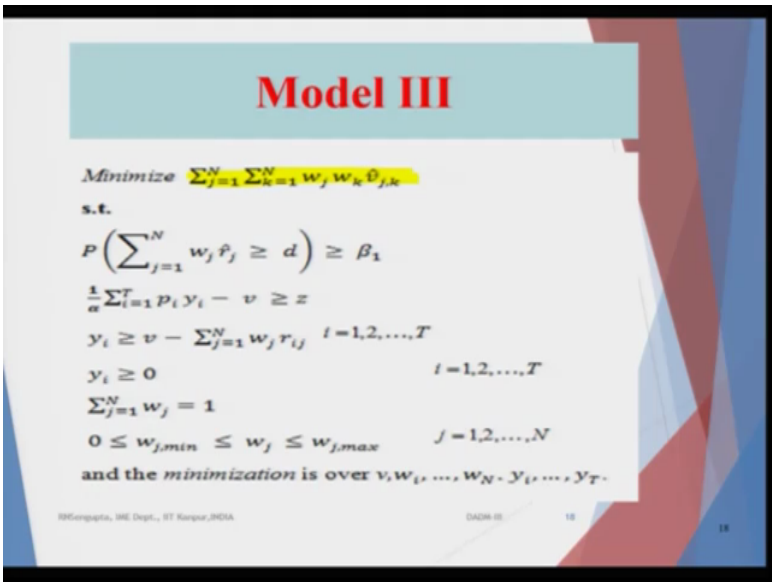

So, in the concept of basically model 3 what we have is basically you want to minimize some sort of risk, so risk basically for a portfolio considering that it is a symmetric distribution will always consider the covariances, covariance variance matrix and if the weights are W1, W2 to Wn considering there are n assets or n financial scripts in the portfolios we will try to basically minimize the variance.

So, here is the objective function. So, we minimize the objective function, these are the weights. So, I am not going to highlight it more so W's are the weights and Vjk considering j changes from 1 to N and k changes from 1 to N are the variance or the covariance values between the j th and the k th stock.

So, obviously we will try to utilized the estimate of that, the first constraints would basically be consider that the probability, so here the probability comes obviously they can be this problems which are discussing can be convert it solved using both the reliability part as well as the robust counterpart, robust optimization concept also.

The first part is that if I leave the probability so generally inside the bracket it means the returns which is Wi's into ri's or Wj's into rj's, j is equal to 1 to n is greater than some fixed value d, d is basically some stipulated value we have kept for yourself for the portfolio overall investment returns which you want to ensure and this value of d would basically depend on what is the market condition, what is a level of risk and all things considered together.

So, obviously it will depend on the investor and investor into investor the value of d definitely would change as the beta values would also change. So, in the initial case if there is no probability you would basically have the summation of the return of each and every stock when combine together, there is the return of the portfolio is greater than d and if I want to basically put a level of reliability it would be that probability of that summation being greater than equal to d is greater than equal to beta 1.

So, beta 1 would basically be the level of confidence or level of reliability the person would have on his or her investment portfolios based on which he or she is trying to basically achieve asset or certain return. The next one would be that in case if you have the concept of risk change.

Now, what is the concept of risk change? Here I will basically discuss some ideas of finance even though that is not directly related to this course of DADM 3, but I will still discuss that and I remember that I have mention all these points in quantitative finance course which was there in the earlier part of the NPTEL MOOC series or NPTEL series and that lecture, total lecture was for about 20 hours.

So, as this DADM 3 is for 30 that was for 20 hours. Now, generally the concept of diversification when we mean in portfolio optimization it means that more you basically put your eggs in different baskets, so if you put all your eggs in the 1 basket or all your assets or amount of money in very few number of assets so obviously any change for fluctuation, negative fluctuations in the returns of these assets would basically have a devastating effect on your total investment.

So, what you try to do is that you try to basically invest small proportions of your total money in different type of assets then combine them and form of portfolios such that you one to ensure that the return of the portfolio almost tries to mimic the market or beat the market such that you are always one step ahead than the market value of returns.

So, obviously in the market is falling, year fall is also there in the return of the portfolio fall is also there but it does not fall that dramatically as the return in the market, on in case if it is increasing the market returns is increasing you will try to basically beat the market and get some extra amount of return with respect to the market.

Now, generally in that case it was proved, initially it was proposed and proved by Markowitz that the risk and the return, return means the expected values and the risk being the variance, covariance matrix value of the portfolio was the best measures to find out what is the average return and what is the average risk for that particular portfolio.

Now, later on we use the concept of VAR which is value at risk, we use the concept of conditional value at risk and it was found that if the returns of the individual stocks are not normal, so normality was one of the main assumptions of the symmetric distribution the set of important assumptions, if those fails then the concept of utilizing the standard deviation or the variance or the value at risk, do not give the actual risk of the portfolio because in that case we will be utilizing the concept of extreme value distribution.

Those extreme value distribution of type 1, type 2 whatever it is and so some of them are the Gumbel distribution and generalized Pareto distributions all these things can be utilized but in that case the variance or the risk of the portfolio would not be given by the concept of variance or value at risk, in that case we generally try to use the concept of conditional value at risk which means that the conditional value at risk gives the value of the risk for the portfolio in the best possible manner both in the case when the symmetric distribution and this extreme value distributions are utilized to find out what are the returns of the individuals assets of the financial scripts.

So, based on that if you consider then the model corresponding to constraint 2 and 3 would be based on that fact that can utilized the conditional value at risk and provided the conditional value at risk is greater than or less than some stipulated value which you have for yourself. And obviously we would always have that the some of the weights for the for all the investment which we are doing in the N number of financial assets is sums up to 1, point 1.

And point number 2 the investment which we are doing proportion wise Wi's for each and every stock or financial scripts are between Wi min and Wi max and in the case when there is no short selling the values of Wi mean can be at minimum can be 0 and value of Wi max maximum can be 1 which means Wi would be bounded between 0 and 1. But in case if short selling is then obviously the first point related to the summation of the weights is equal to 1 will hold true but the case of where Wi's are between 0 and 1 would not hold true I am going to come to that to later on.

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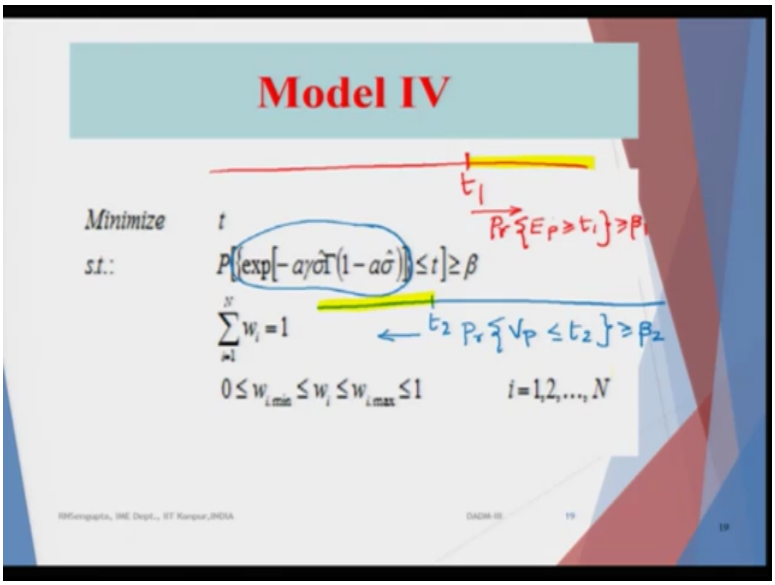

Now, this problem formulation is as something to do with robust optimization but the concept which I have written there on the slide has something to the concept of safety first principle which we considered in the utility concept, so safety first principle there were 3 rules.

So, one of the rules can be utilized here, where we have you want to minimize some value of t, so t is some fixed value and for our case we will consider that it can be change so you want to minimize it and bring it as low as possible and what we want is there are 2 ways how we can handle it, I am not going to the formula, I am basically stressing more on the concept how you can utilize.

So, consider that if you have a portfolio and the return of the portfolio is there and also the risk of the portfolio is there, so what you want to do is that you want to bound both the return and the risk by 2 separate values considering t1 and t2, where t1 would be the value over and which the return of the portfolio should definitely be.

So, there is the probability that the return of the portfolio is greater than t1 with some probability beta 1, where beta 1 again will depend on the investors concept of risk or concept of utility and in the other case if you want to bring the t2 also corresponding to the fact that the probability of the risk being less than equal to t2 is greater than equal to beta 2.

And what you will try to do is that in the first case you will try to maximize t1 as far as possible that means you if you consider the number line for the returns you will try to push t1 on to the right from my consideration such that the return of the portfolio will be greater than t1 with some probability beta 1 and in the case when you are basically measuring the risk or the variance whatever the or the conditional value at risk or the value at risk will ensure with some probability beta 2 that the value of the risk of the portfolio would be less than t2 and you will try to basically push t2 as low as possible.

So, if I draw the number line for both; number line means the line based on which I just discuss it will look like this, I will first use red color in order to highlight the concept of the return. So, here is t1 and you want your return of the portfolio to be greater hence on to the right hand side and you will try to basically push t1 on to the right and the probability would be probability of the return of the portfolio so is the expected value of the.

So, portfolio I will use E suffix P is greater than equal to t1 and that would be with a probability beta 1 and when I do it for the concept of the variance let me use a different color, what I will ensure that this is t2 some t2, so on the number line 1 which is red in color I am only measuring the returns, on number line 2 which is blue in color I am only measuring the risk.

So, they may be return can be say for example to the value of 10 to the power minus 2 while risk can be in the value of 10 to the power minus 6 whatever it is, considering that we are considering the returns based on the fact. The return can be either considering the concept of small r or capital R, which we have already discussed time and again when we are doing the concept of the utilities also.

So, here risk would be less than t2 or this is true what will ensure is probability this V is variance the suffix means for the portfolio I am not writing the total formula is less than equal to t2 that is greater than equal to beta 2, so we will try to basically ensure that and if I consider both of them separately they will be single objective function optimization problem based on some constraints and if I combine them it becomes a multi objective problem.

Now the formula, the constraints which is given here with respect to the exponential part is a something to some of the proves which we have done considering the hyper, HARA utility functions, I am going to come to that in the robust counterpart area and in the case if you want to consider the symmetric distribution obviously this part which I have circle would basically be the corresponding variance which I can formulate using the normal distribution or the non-normal distribution as the case may be.

The second (constraint) as so here the concept of written as one of the constraint based on the fact that t1 and beta 1 are not there so if they are included obviously it would be become not no too complicated but that will take case care or both the return and the risk. Now, the second constraint here in the model is basically the sum of the weights is equal to 1.

And the third constraint is also the fact that the weights of the investment for each and every I th financial script or the stock is bounded between Wi min and Wi max. Again I said Wi min minimum value can be 0, Wi max maximum value can be 1 provided no short selling is there in case if short selling is there, this bounds of 0 and 1 would change.

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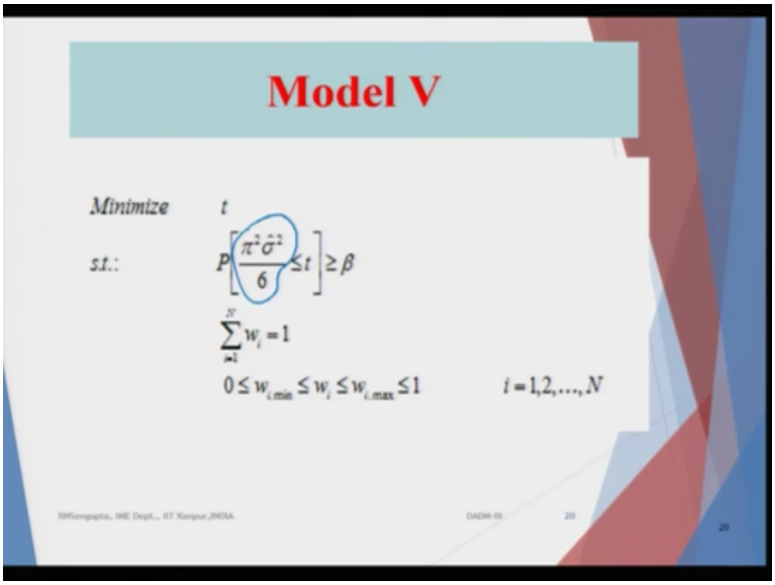

In case again I am continuing the problem formulation again is exactly the same, here this pi square sigma hat square by 6 is basically the variance of the EVDs or extreme value distribution for a portfolio and then sigma hat square would be calculated based on the portfolio which we have formulating considering N number of stocks where each stocks has EVD returns and how we combine them we can utilize the concept of Copula theory also to

find out what is the inter relationship between the stocks and hence we can combine them to find out the portfolio.

So, again the concept remains the same probability being less than t would basically mean for the variance probability of some part being greater than t would be corresponding to the returns and on the right hand side the value of beta 1, beta 2 would denote the level of confidence I have on these less than type or greater than type constraints. The second constraint which is given here in the slide also I am not repeating here is the some of the weights is equal to 1 and the third last constraints is basically Wi's are between min and max.

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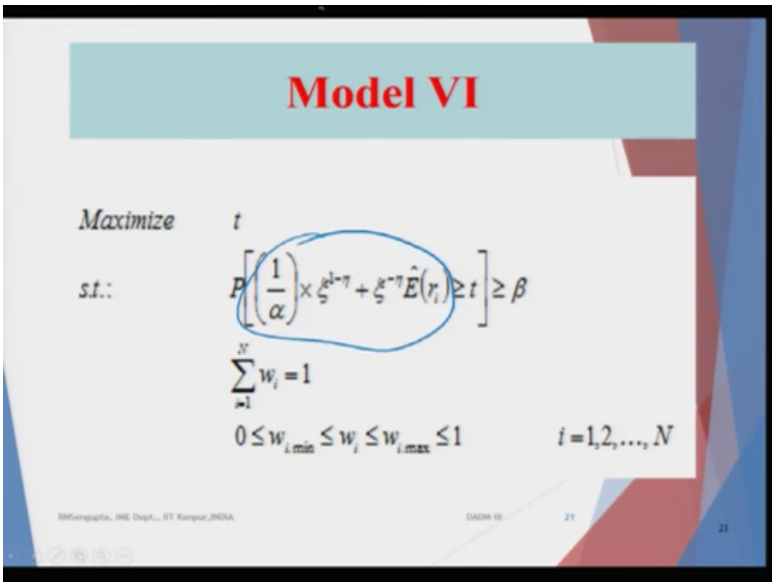

Here if you see for the first time I am writing a problem formulation again I am trying to maximize the t. In the last model which was model number 5 I minimize t that means I am trying to push is as far as possible to the left as I have drawn in the number line. So, in this case I am trying to basically maximize t and the maximization t happens because I want some probability based on the fact that the return of the EVDs or the portfolio considering the extreme value distributions is true for each and every stock we combine them using the Copula theory or the correlation coefficient whatever it is.

Then the return of the portfolio is greater than equal to t and on the right hand side again we have the probability beta 1 or beta 2 whichever way you denote. The second constraint is again the same thing sum of the weights is equal to 1 and the last constraints are the weights of each and every stock is bounded between 0 and 1.

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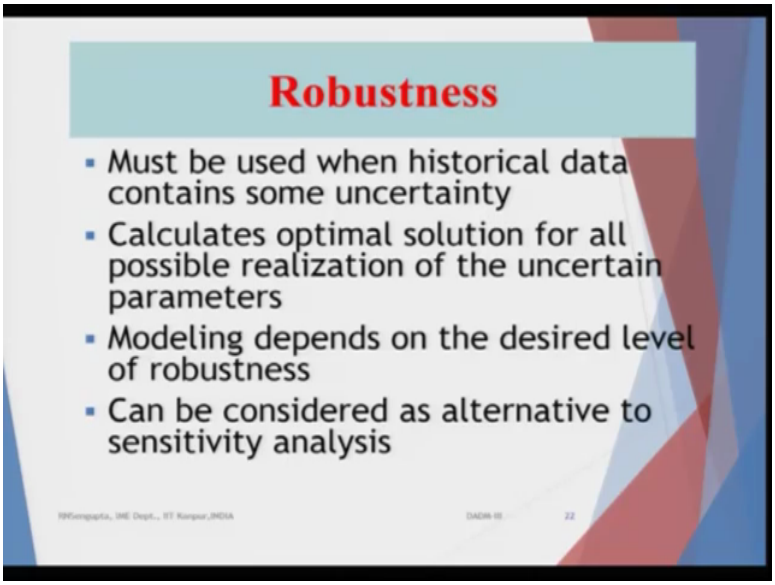

We will solve few of the problems in the robustness part please have patients. Now, when we are basically trying to consider the robustness or the concept of robustness we should consider or analyze that why are we trying to basically use the concept of robustness, is it necessary? Or could we have done or solved a problem utilizing no robustness concept?

So, the answer is that historically or actually in practical sense all the data which we get from the external source based on which we model or based on which we try to find out what is the return distribution in the case of a portfolio or what is the speed of a car or what is basically the tensile strength of a material we are going to build all these things are random they are fluctuating and they are uncertainty.

So, we should use the concept of robustness when the historical data contains some uncertainty, some uncertainty may be due to errors, (())(27:27) noise meet due to the collection of the way we collect the data may be due to the fluctuation or the background, how they supply the data for us and based background means it is a like we are basically measuring the temperature or the humidity or the wind speed.

So, obviously they would change accordingly depending on natural causes and we will consider the these variables or the inputs are non-deterministic, now why I am mention the word of humidity, temperature, wind speed is that, say for example you want to model the car and at what speed it can travel, so if it is travelling in an area where the wind speed is very high so that has to be considered or we have want to consider where the temperature fluctuation on the humidity are very high such that we can model the car how the (())(28:20)

how the doors would be what is the tensile strength of the door, what is the crash resistance speed of the car.

So, all these things when we consider, I am giving a very simple example, so when we do that obviously we have to consider these are the parameters, so they are very simple what I mention it can be say for example what is may be what is the traffic congestion, it can be what is the salt level in the air because that would basically have an effect on the wear and tear of the tier.

So, the wear and tear of the car. So, all these things would basically matter. So, we want to basically find out an optimum solution for all the possible realization the uncertain parameters, so we want to basically have a gamut of all the possible solutions of parameters and then basically consider though which has feasible and find on the optimum solution from them.

We will be modeling these ideas of robustness which will depend on the desired level of robustness you want to ensure, so are we very very strict on the level of reliability? Do you want the results to be to be absolutely certain or we want to basically give some relaxation. Consider some levels of beta high or low that would depend on the model formulation and as I mentioned robustness can be considered as an alternative to sensitive analysis.

So, we will see that how it can be considered in our models. So, with this I will end this 58th lecture and considered the concept of robustness in more details in the 59th and the 60th lecture, have a nice day and thank you very much.