Data Analysis and Decision Making- 3
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Lecture 05

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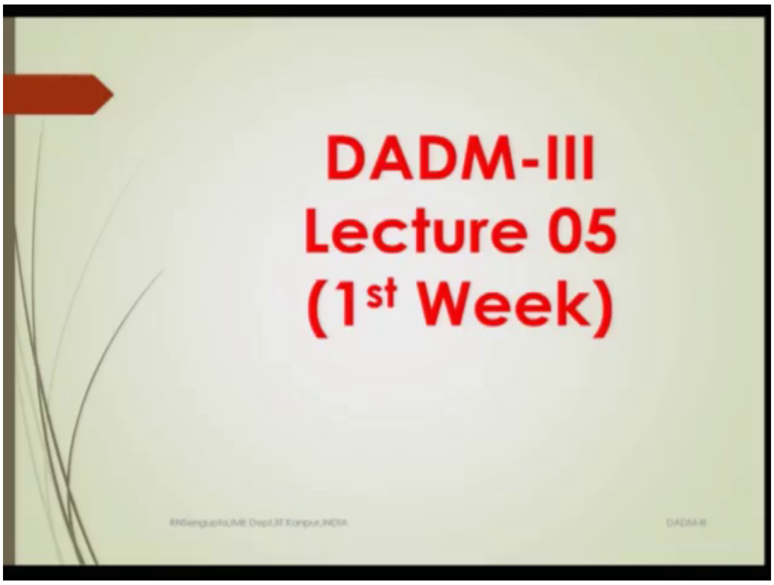

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you and this is the DADM-3 course which is Data Analysis and Decision Making 3 and you know this course duration is for 12 weeks which is 60 lectures and each week we have 5 lectures for duration, each being for half an hour.

So the total duration if we consider 60 lectures basically the total number of hours contact hours is 30 and after each 30 hours and after each week, after 5 lectures we have an assignment, so there will be 12 assignments and after the end of the course there would be question answer or final examination. So if you remember we are discussing about utility and I did mention in some conceptual framework that why utility was important. So, we will continue that and try to wrap up as soon as possible of the utility function and then go into the optimization.

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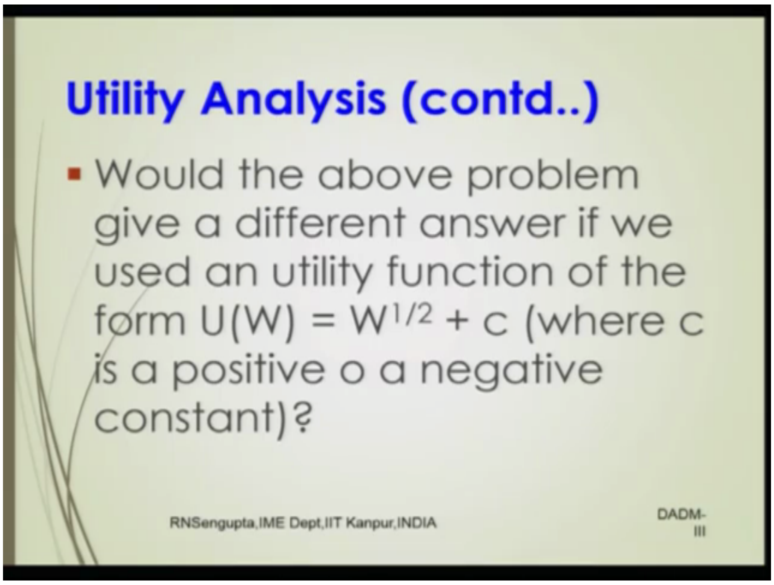

Now, if I consider the utility which I mentioned, I had mentioned the generally I considered the quadratic utility function, the polynomial utility function, the linear utility function. So in the above problem when there was a government bond treasury bills along with the option where decisions were three options with probabilities of 0.2, 0.4, 0.4 and the values not the utilities, the values were given in lakhs.

So the question would be what would be the above problem B? Would it give a different answer? If we use a utility function which is U W is equal to W to the power half plus 2, so half was basically the utility function, now you are basically adding a constant here. So if you basically add a constant here and so all the utilities function would be just increased by that values and when you multiplying the corresponding probabilities those probabilities also would basically give you the values as 1, but probably some of the probabilities are 1.

So, that C value would be just an addition for all the utility, so the ranking would not change, provided C is positive so it is negative there would be cases that if you have to come basically compare whether negative values are true or 0 is true, so obviously if the all are 0 then they become at the same level. If negative values are to be considered then obviously the ranking would be maintained.

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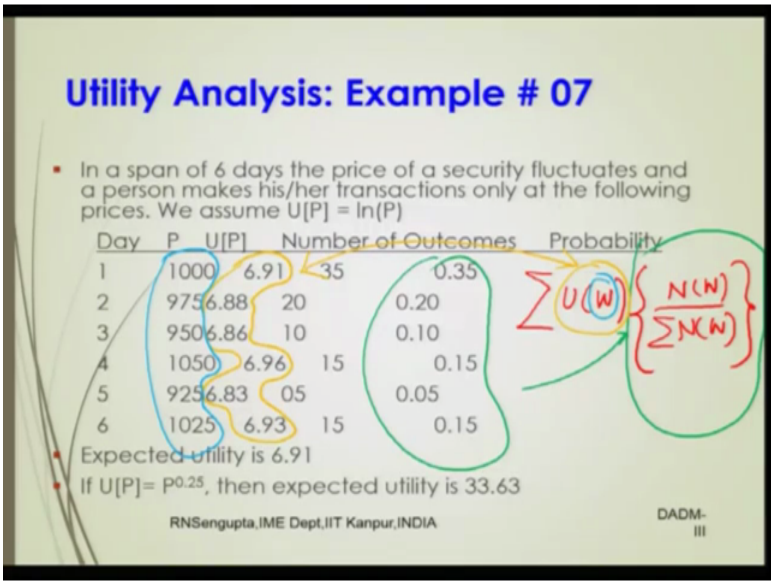

So consider in a span of 8 days prices of a security fluctuates and a person makes his, her transactions accordingly, so we consider the utility function to be logarithmic. So for day 1, 2, 3, 4, 5, 6 the prices are 1000, 975, 950, 1050,(1050), 925 and 1025. Now obviously when you when these values are given we are considering them as W some wealth, it they may have different implications also.

So the moment you have those W and (or) which is the price you have the utility function which is given as logarithmic, we convert them and find out the utilities, utilities are given as 6.91, 6.88, 6.86, 6.96, 6.83, 6.93 so that gives the utility which is W, U W. So W which is P is given then you find out the U W based on the logarithmic one. Now when I come to the outcomes so obviously outcomes technically means consider this scenario, this price of thousand has occurred 35 number of times in a total span of 100 days.

So if this is the case we will consider that outcomes giving as some information of the probability of occurrences which is N W divided by the summation of N W, N W is the total number of sums of the total occurrence, total occurrences in different numbers. So the outcomes are given by 35 individually 35, 20, 10, 15, 5, 15 and if I divide by the corresponding total number which is 100 the corresponding probabilities are 0.35, 0.2, 0.1, 0.15, 0.05 and 0.1.

So if I find out the expected value again I use the same formula it is summation of U W into N W, N W is a numbers, summation of N W. So these probabilities are, so this is given if I want to find out the utility I have already mentioned that but still I have mark, so these you please I am sorry are given, so this is the utility, so I am basically mapped them and if I want to find out W, W values are guess, this is the W or should not be green I will use a different colour wait, so this is the value of W, so this is the value of W.

So the green-green basically gives you the probability the this not light orange, saffron sort of colour would give you the utility and the blue one gives you the W value. So the value comes out to be 33.63 which is the expected utility on the decision.

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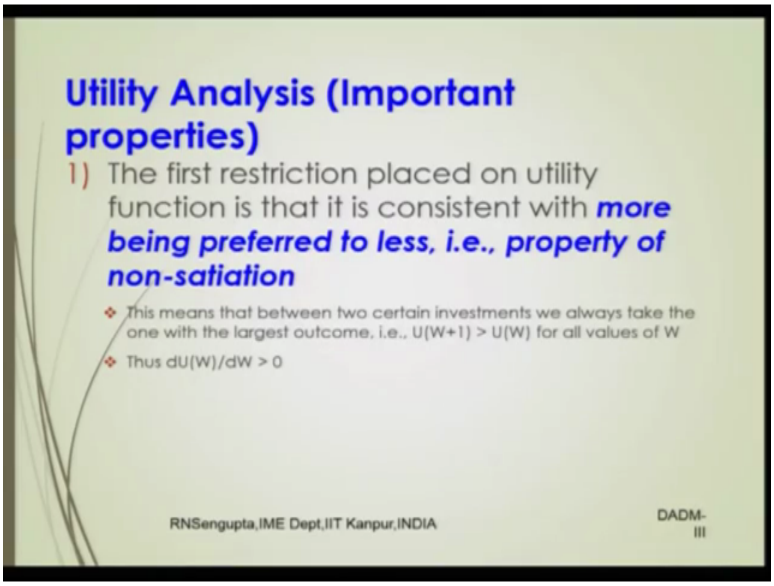

So there are some properties of a utility function because I want to maximize and if you remember in the optimization problem (I also) I always have mentioning they are trying to find out the optimum one whether you want to maximize or minimize. The first restriction placed on utility function in that it consists it is consistent with more being preferred to less that is the property or non-satiation that means more I give you more you want.

So I give you 10 rupees, 20 rupees, 30 rupees, 40 rupees so these are on the wealth fund, so if when I convert into a utility, your utility function keeps increasing. Now the question may be asked from your point for point of view whether this increase is always happening on increased strength? So, I would say no because you will try to basically differentiate these three categories, I will come to that in a very simple way.

So the first restriction placed on the utility function is that it is consistent with more being prefer to less there is the property non-satiation. This means that difference between the two certain investments we have we will always take the one with the largest outcome that means the utility based on the fact it is from the output which is W plus 1 would be greater than the utility which we are getting from W that means the first derivative of the utility function would always be greater than 0 convert me, so that is theoretically, practically it may not be true now.

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Now coming to the second property, so utility analysis important properties would say, that if you consider the investment on the decision maker perception of the concept of risk then we have the concept of the property of risk can be summarized as property of risk aversion, property of risk neutrality and property of risk seeking that means if I want to avoid risk that means, which means what? I would be getting a (man) main worth which is always increasing but my liking for that will start decreasing. That means it is increasing at a decreasing rate.

When I consider the concept of neutrality I will show you that with the graphs, so when I am showing the concept of risk neutrality. So risk neutrality would basically mean that as my value increases, the utility increases obviously as per the first property but is increasing at a steady rate, fixed rate and if it is risk seeking again the property of the first one that means more I get more I want that is the first utility the derivative is positive and this second derivative is also positive that means of the utility function with respect to W will also be positive because it will mean that I want more and more at an increasing rate.

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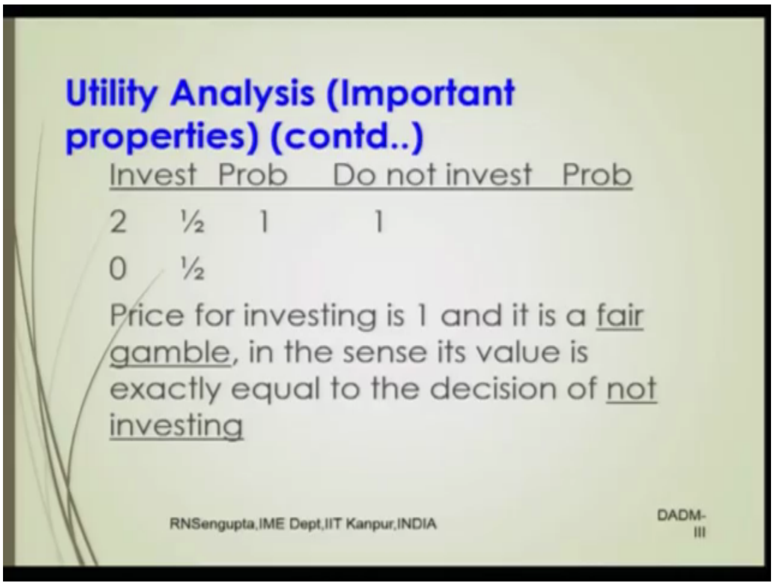

So let us consider a simple example, so consider I want to play a game with you, so in one sense we have a coin which is the (prob) unbiased coin probabilities is half and half and a head and a tail and on the other hand you have a bias coin consider is a coin, you must somebody of you must have seen the movie Sholay, so it is the coin which was used by in Sholay which is both are the same face, so considering the probability is 1.

Now, (in seen) in the first case when it is an unbiased actual coin probability half and half we will consider that if you invest some value, some value your outcomes would basically be the value will be getting will be 2 with the probability of half and in another case when it is tail, so first case it is head it is with probability half with outcome is 2, for the second case probability is half the outcome is 0.

Now if I tell you to find out the expected value you will immediately find out and tell me it is 2 into half plus 0 into half which is 1 which is good, the expected values this 1. Now if I tell you about that bias coin where the probability is 1, you will again say and then the outcome is 1 you will say that for the probability 1 as it is 1 so also the expected value is 1. Now if I place this one scenarios in front of you and these two values 2 or 1 which is our notion values, consider they are the outcomes which are coming in Rupees, some value you can if have them.

So if I ask all of you which decision which you will take? So by and I am sure everybody will say that we will try to take the probability 1 where the probabilities are half and half, so that means you want to take that value because you want to take a risk and you (win) want to win a value of 2 and not 0, but the probabilities are same remember half and half. Now let me increase that 2, to say for example 2000 or let me increase the value 2 so when I am using the increasing the value 2 to 2000, 1 also increases by 1000 so 1 to 1000.

So in this third scenario consider that 2 (in) it becomes now 2 lakhs, 1 becomes 1 lakh but in all the cases whichever example I am giving the expected value for both the scenarios would be the same. So in the long run it would never mean that one is better than other but if I keep asking you the question in this first case when it was 2 and 1 you are attempted to take the decision of the gamble but as the stakes its stakes increases the value increases, people may slowly change the decision from the risky one, they may become neutral and in some of the cases they may basically take the case which is the deterministic one.

Which means depending on the scenarios people will change their decisions accordingly. So here let me continue price for investing is 1 and it is a fair gamble, in the sense its values exactly equal to the decision of not investing whatever it is that so the values which are giving, so not investing means that means you just play the game and get the same output but the values, expected values are same for both of the case what utility change.

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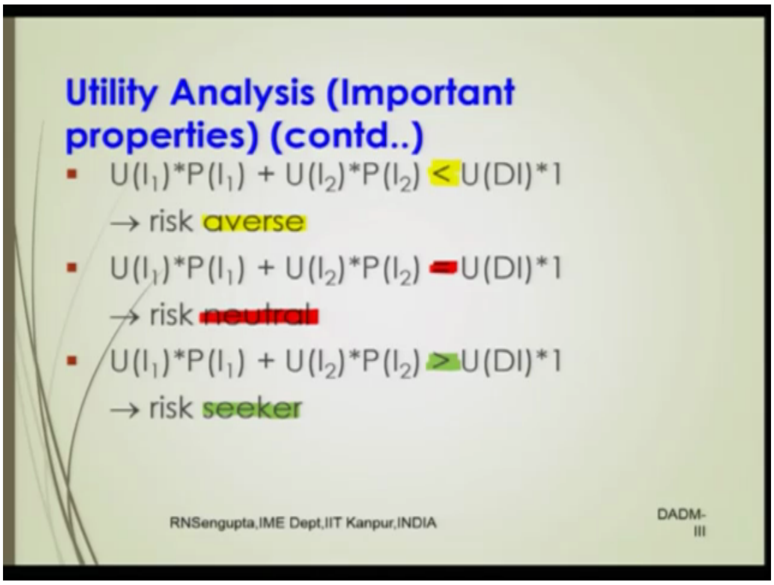

Now if I am considering the concept of the, so the human being in the case when you are going to take the gamble when you are indifferent between the gamble and the sure event and when you are going to take the sure event, obviously your characteristics are changing so those characteristics would be when you take the gamble your risk seeker when you basically are indifferent, you are a risk neutral person and when you take the deterministic one you are basically a risk averse person.

So the form, so the work, the risk averse person I will use the highlighter now, so for a risk averse person that means I want to avoid that means the less than equal to sign would hold true with respect to the deterministic one that means I will be more pull towards the (distant) deterministic scenario. If I am a risk neutral person obviously I am indifferent so obviously in that case it is an equality sign with base to the gamble and the certain event.

And when I am basically a risk seeker that means I want to run after risk, so it will be greater than sign, in the sense that I will take that gamble such that the returns to me are expected well again I am not going to consider, I am not saying they remain the same in all those cases, I would basically take the gamble because I think in the long run it will give me more benefits.

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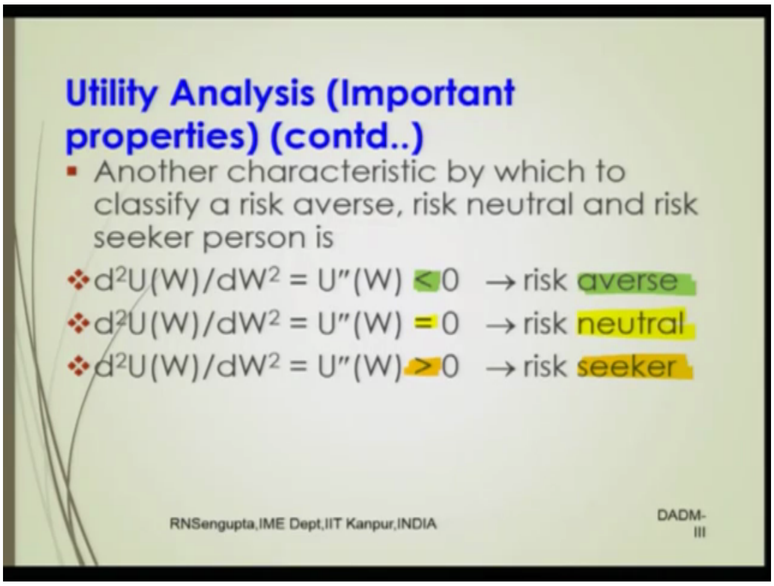

Another characteristics by which to classify a risk averse, risk neutral and risk seeking person is by considering the second derivative. If you remember I did mention the second derivative, the first derivative is always greater than 0, the second derivative will be U double prime by U of or U double prime which is dt U dW 2, U mean basically being the utility function and from there you can find out whether the person is risk averse, risk neutral or risk seeking person.

So in the case, if the second derivative is which is less than 0 the person is risk averse, so that means because the (second) which means that they are increasing but increasing at decreasing rates hence I am risk averse. So if I consider the equality sign that means I am indifferent so it is increasing but increasing on a steady rate, straight line and if I consider the greater than sign that means the second derivative is greater than 0 that means I am considering a risk seeking person that means I want to take the risk.

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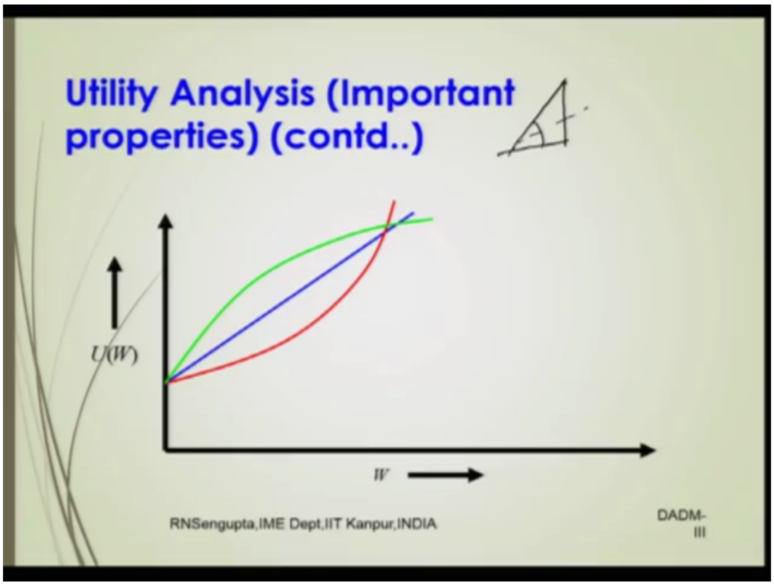

So utility analysis if I consider from the diagrammatic point of view, the pick I will take the so in the case if it is (in) so this the first derivative is for the green one is always increasing but if I consider the slope, slope is basically going down that means it is positive because this is less than is between 0 and 90 it is positive but the slope is slowly now I trying to decrease such that it is second derivative is decreasing, so the graph so this basically decreases, so it is the second derivative decreasing.

For the blue line, the so technically it would mean that if I am on the green line I am trying to avoid the risk that means I am a risk avoider, for the blue line again the first derivative is increasing and the second derivative is also fixed because if the first derivative is increasing but it is fixed that means the second derivative basically becomes 0 that means we cannot say whether the 1, 2 person wants to take the risk or avoid the risk so he or she is at risk neutral person.

Now if I consider the red line, so the red line is going like this, so the first derivative so in between 0 and 90 degrees the first derivative is obviously 0 but the tan on the that angle second derivative of that angle basically starts increasing that means which means that I am willing to take the risk which means I am a risk seeker.

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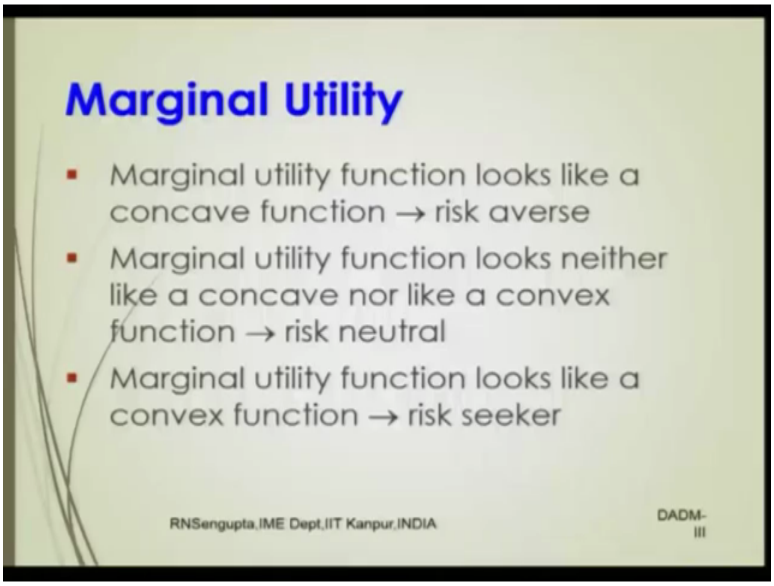

Now I have considered the marginal utility, marginal to mean the first derivative concept. So marginal utility functions looks like a convex function which means I am a risk averse person, marginal utility function looks neither like a convex nor a concave function which is a risk neutral person and marginal utility function looks like a convex function risk seeker, so that means it is the convex function that I am a risk seeker means I am want to a lot risk so it will be increasing at an increasing rate and for the first case is increasing and decreasing rate and for the (third) second case it is increasing at a constant rate.

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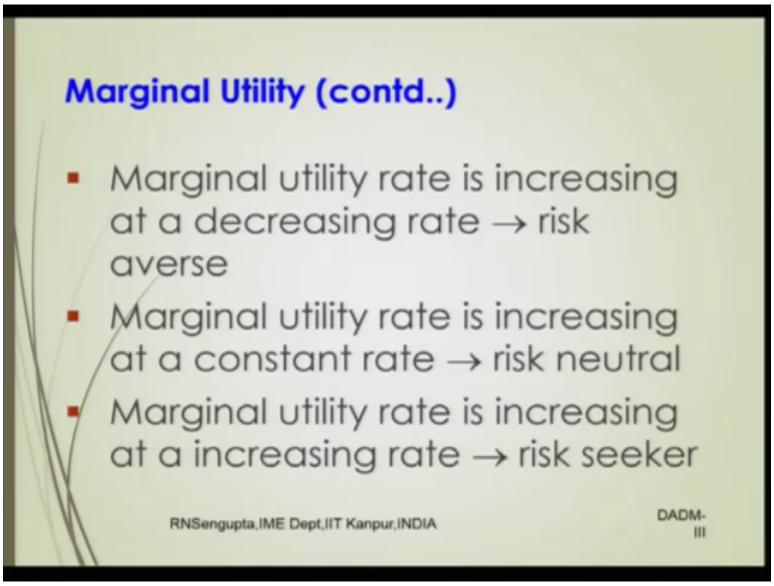

So if I consider increasing that is what I said marginal utility is increasing at and at a decreasing rate which is risk averse, marginal utility rate is increasing at a constant rate less neutral, marginal utility rate is increasing at an increasing rate which is risk seeking.

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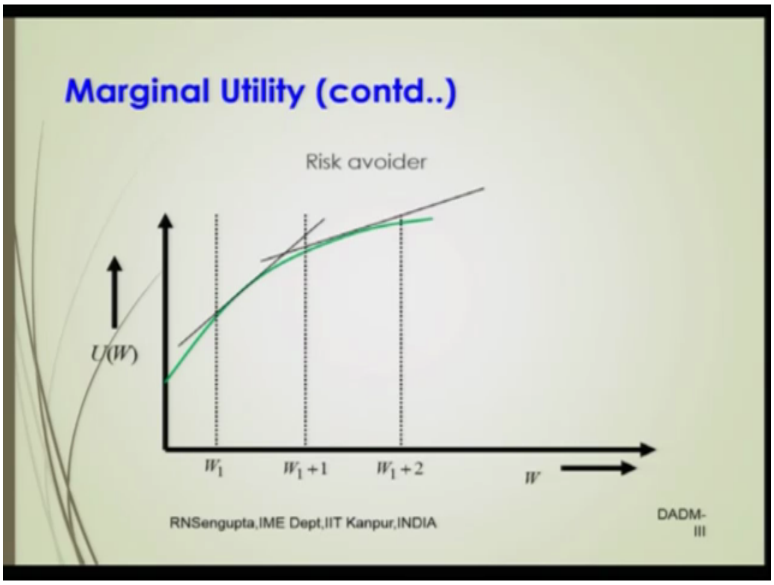

So these are the graphs, the green one so it is sliding going down so tan of this angle is in decreasing second derivative is basically decreasing it means (d 2 y) d 2 U dW 2 is now negative, so it is a risk avoider.

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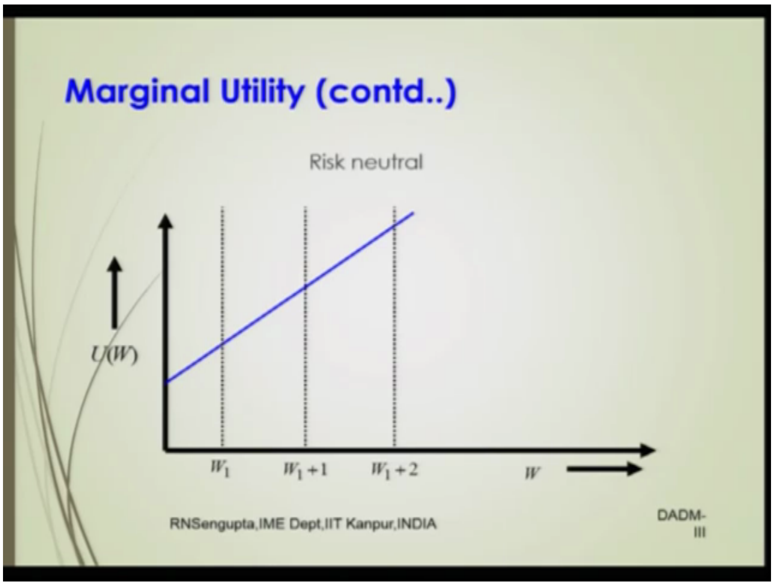

Consider a the blue line, so d Y d X is positive, d 2 Y, d X 2 is 0 so in that case it is a basically a risk in different person whatever happens you will take both the decisions with equal values because he is trying to basically he or she trying to find value the expected value and find it out.

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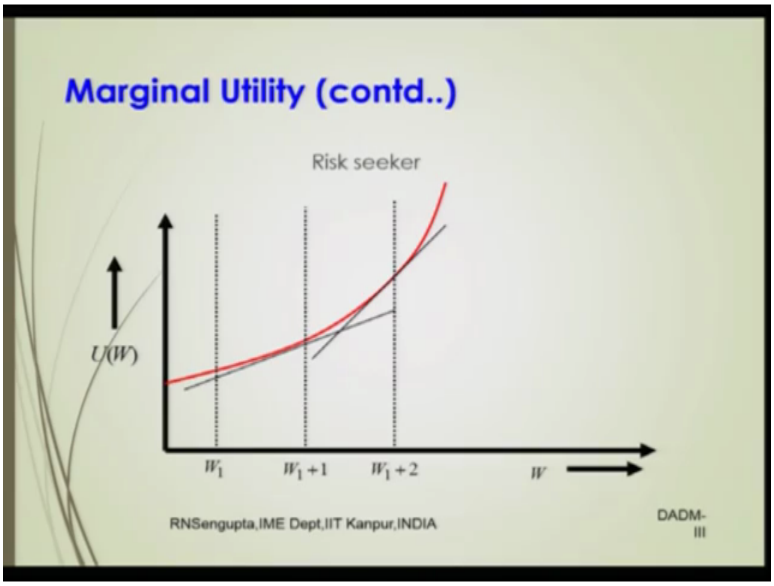

In case is a risk seeker, so the line is increasing, so in that case the first derivative is obviously positive, second derivative would be as the tan on that angle is increasing so obviously the second derivative which is d 2 U dW 2 is positive and the person is a risk seeker person.

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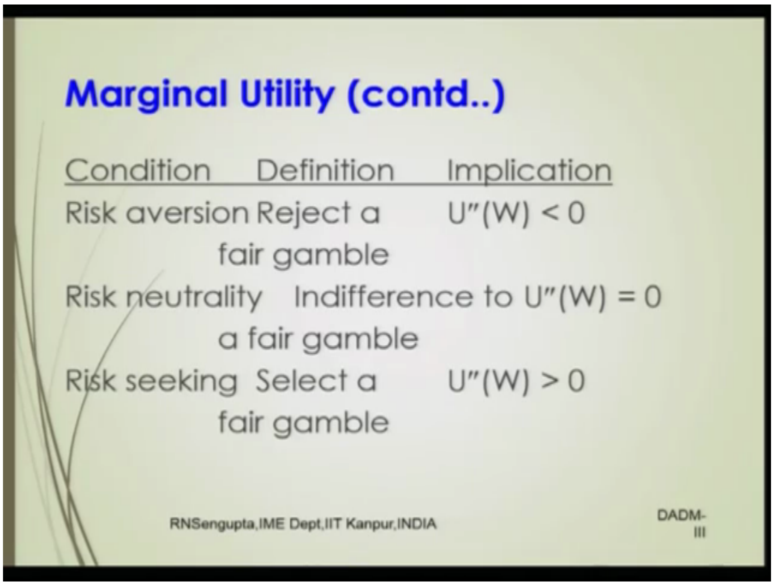

Now consider the marginal rates, so if now and that implications. So if I am a risk averse person which means I will reject a fair gamble and the second derivative as I have been telling about the green line would be the second derivative would be less than 0. If I am risk neutral person, I will be indifferent to the fair gamble which means that my second derivative will be 0. If I am a risk seeking you will select a fair gamble such that this second derivative would be greater than 0.

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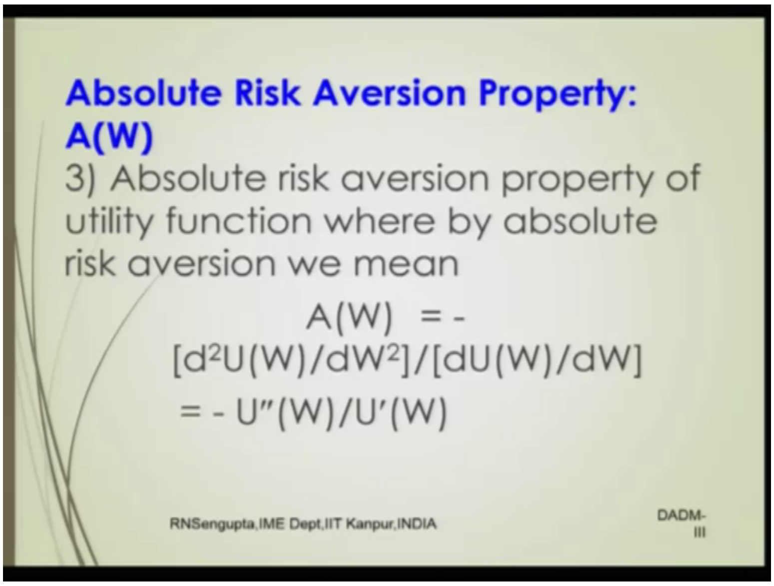

So now we will basically consider, now this would be becoming important when we consider a few simple problems from the point of the optimization for finance. So absolute risk aversion property or for the utility function would basically mean where this absolute risk aversion would be given by the value of A, which A is an absolute risk aversion property and that will be given by minus U double prime by U prime.

So, now consider U prime is always positive which is the denominator, in the numerator you have U double prime, so U double prime will be positive, negative or equal to 0 depending on whether you are risk person, risk seeking person, risk indifferent person, risk avoiding person. Such that combined with U double primes property and the minus negative sign you will basically have the property given on to A which is absolute risk aversion property and A prime.

Now, I would not go into the details, so generally we can prove the concept of absolute risk aversion property and generally relative risk aversion property will come to later by using a simple fair gamble with the value such that you are risk neutral and then expanding it using the Taylor series expansion and solving the problem. So, this is they are more from the theoretical point of view but we would not consider it as it is but I will just read it out.

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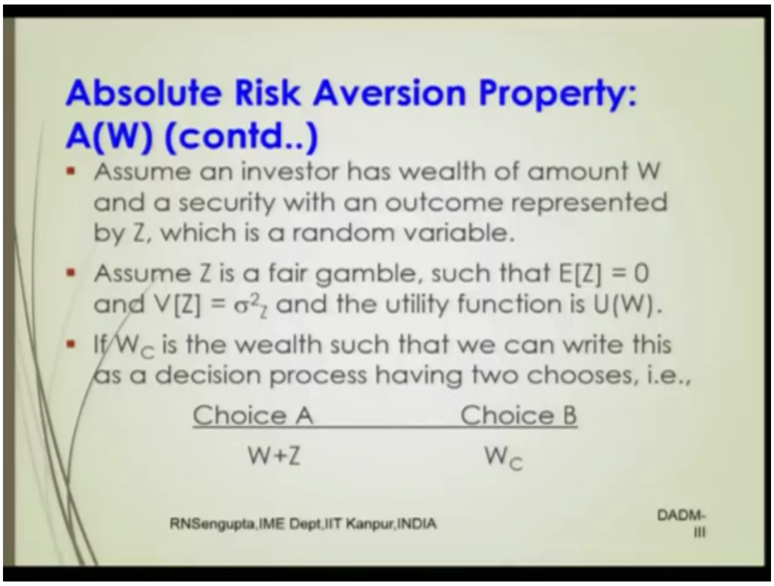

Assuming an investor has wealth of amount W and a security with an outcome presented by represented by Z which is a random variable, so that outcome will change. So assume Z is a fair gamble, fair mean gamble that means the gamble based on the expected value for the gamble and the expected value of the sure event would be seem. Now we are considering the expected value of the gamble is such that it is equal to 0, so obviously there are fluctuations positive and negative what is 0 and the variance is given by sigma square suffix Z and the utility function is say for example U W.

Now if we if we choose W C or certain amount of wealth so what we are trying to basically (value) balances is W C was on the right hand side and the fair gamble along with that money is on the left hand side, we want to basically balance them. Based on the fact that as the random fluctuations are that there the in the overall run the average value basically comes out to be the same.

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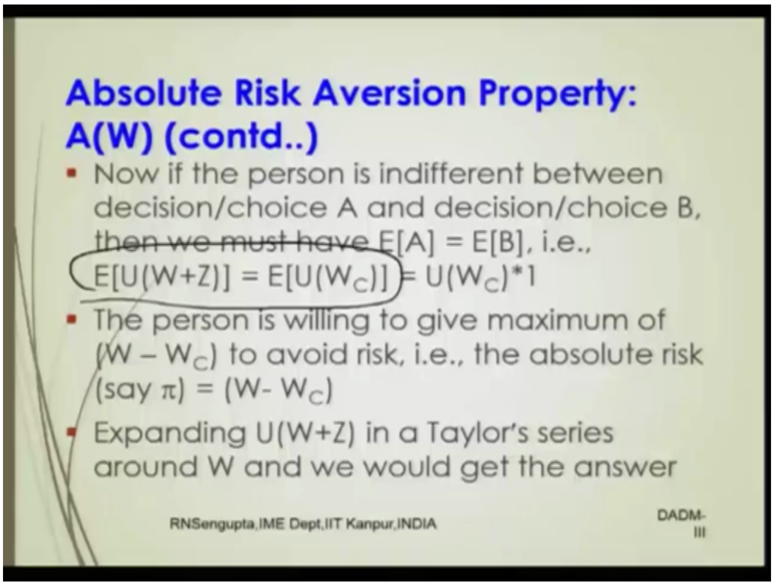

Now consider, considering this absolute risk aversion property, now if the person is indifferent between decision and choices A and choices B, so you will basically balance both the left hand side and the right hand side which would be given by this equation, which means the person is willing to give maximum some amount which will be basically with the difference between the W C on the right hand side and the value which you have along for the case when Z is there.

So in the first scenario on the left hand side you have W and Z which is fair gamble on the right hand side you have W C only, so the differences would basically give the balancing act. So if I basically expand U W plus Z using a Taylor series expansion you can find out the absolute risk aversion property.

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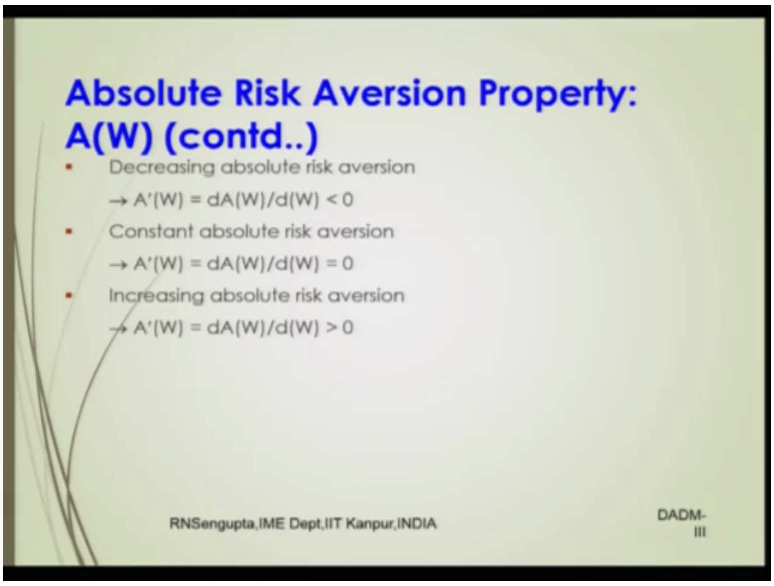

Now, if I consider the absolute risk aversion properties in general. So if I find out the derivative of A which is dA d W, so in the 3 scenarios when you have a risk avoiding person, risk seeking person and risk neutral person, the answers would become answers means the concept was coming out to be A prime is less than 0 means decreasing absolute risk aversion property that means my aversion for risk in the absolute sense will start decreasing. If I am basically A prime is 0 and that means I am a constant absolute risk aversion property and if A prime is greater than 0 using I would have basically have or the person would have increasing absolute risk aversion property.

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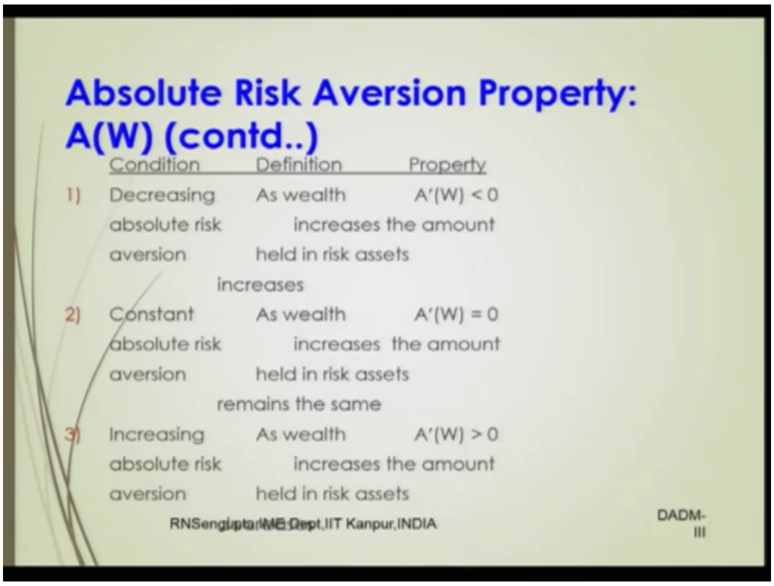

So decreasing absolute risk aversion property which would mean our wealth increases the amount of held, held in the risk asset basically increases. Now it mentions about the quantum nothing to do with the percentage time wait so that percentage would come later on. So if I would decreasing absolute risk aversion property so obviously we know that A prime would be less than 0 but it will also mean conceptually as wealth increases the amount held in risk asset also increases.

Similarly, whenever I go to the second one which is constant absolute risk aversion property where A prime is 0, it would mean that has wealth increases the amount held in risk asset remains the same. And in the third scenario, if it is increasing absolute risk aversion property which is A prime is greater than 0 so which means that as wealth increases the amount held in risk asset also increases. So it will basically find out using the properties of A and A prime and provided that would obviously come from the property of U double Prime.

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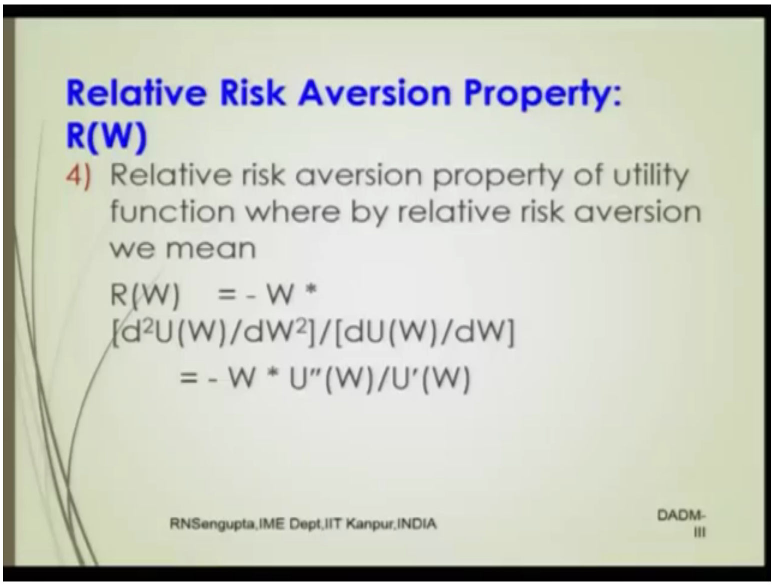

Now consider the relative risk aversion properties. So you have considered the absolute risk aversion property we will consider the relative risk aversion property. Now in that case the value of A which you have we will just multiplied it by W, now let us come to the details. So it is now minus W into U double prime divided by u prime, so u prime we already know its positive has to be positive. Now U double prime would be greater than 0, equal to 0, less than 0 depending on the property of risk aversion.

So if I like the risk, do not like the risk, I am indifferent to risk so and my overall decision will be based on the fact that what is the value of A and A prime and that would basically have an effect on what is the value of R and R Prime.

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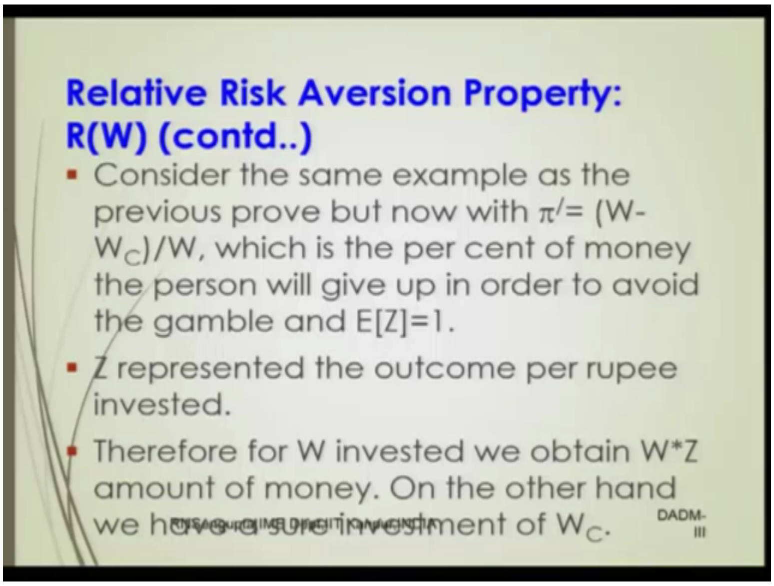

So consider the same example, ok now we want to basically prove the this concept of absolute risk aversion property. Now, we will basically go (in the app) in the relative risk aversion property we will go into the relative sense. So the main difference between the absolute risk and the conceptually how we analyse it, in absolute risk will (turn find) try to find out in absolute sense the increase and decrease while in the relatively sense it will be the proportions of the probabilities.

So we will basically and find out the probability based on the amount of investment which I have been done which is W. See here also the expected value there the expected value was basically 0, we will consider in the gamble the expected value is 1 and therefore for different values of W invested we obtain the values of W into Z and do the again the simple concept of Taylor series expansion and find.

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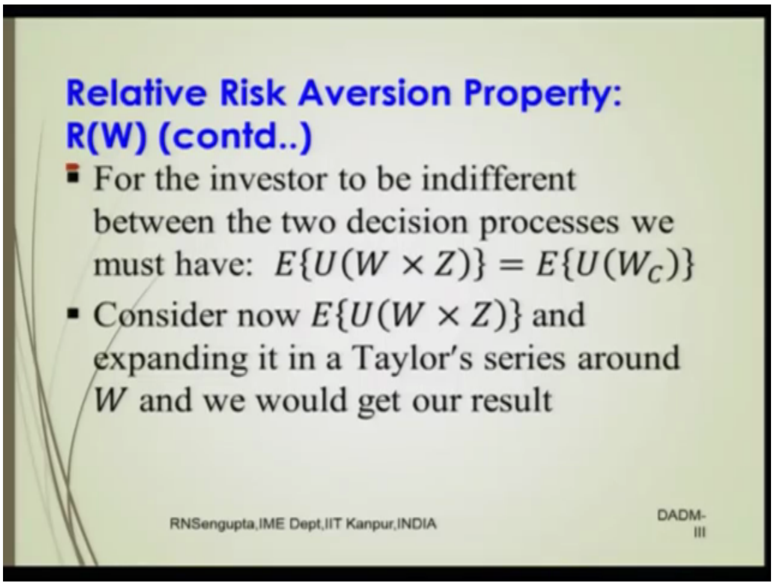

Now relative risk aversion property, so for the investor to be indifferent between the two decisions we must have the utility which I am trying to find out using W and Z and remember we are trying to find out in the relative sense and the utility we are trying to find out for the W C which is my actually investment, so actually investment means is that on the left hand side there is certainty and a uncertainty factor. So again use the Taylor series expansion you can get the values.

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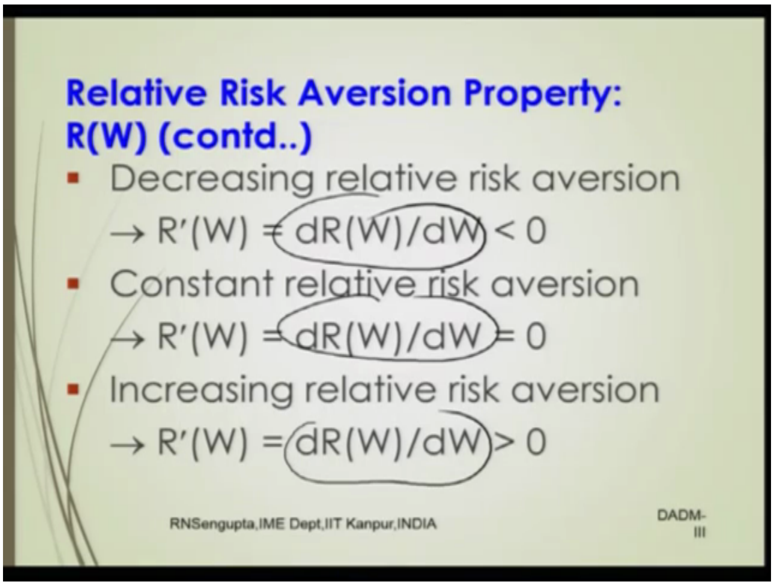

Now, you may be think okay let me complete one important point. So here the decreasing relative risk aversion property again will consider R prime P less than 0 it is decreasing, decreasing my it will be decreasing relatively, so that is why it is in a relative risk aversion property. Constant would be basically be the (constant risk relative) constant relative risk aversion property and increasing would basically mean when it is greater than 0 that means R prime, so this is R prime, this is R prime, this is R prime.

So, that is greater than less than 0, equal to 0 and greater than 0, so that will (me) also give me the concept of relative sense also you have the in the absolute sense and we will try to combine them to find out the properties of the utility functions and try to be trace them later. With this I will end this fifth lecture, have a nice day and thank you very much.