Data Analysis and Decision Making-III
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Lecture-48

A very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe and a very warm welcome to all the students and the participants in the DADM III course which is Data Analysis and Decision Making – III under NPTEL mock series. And as you know, this course duration is total contact hours is 30 which when broken down into the number of lectures is 16 number and spread over 12 weeks, so if you see from the slide we are in the 48th lecture which is the third lecture in the tenth week.

We have already completed 9 weeks. Each week, we have 5 lectures of half an hour each and after each week we have an assignment, so considering that you are in the tenth week, by the time when you are attending this lecture, you have already completed 9 assignments in totality will consider and solve and attempt 12 assignments and then take the final examination.

So, if you remember, we were discussing in the last lecture, 47th one, when we started, actually going into details how branch and bound problem would be solved and we have taken and I mention, when we started the 47th lecture, we will take 2 problems, Problem 1 would have only two decision variables, the reason was very simple, we will be able to draw it and as you understand, as I proceeded in that problem, it was very easy for me to make you understand that how the search space changes, what is the feasibility and so on and so forth.

It is fine in a three dimension also, if we are able to draw it, we will be able to highlight it but I did not attempt, I only stuck to the two dimensional one. Now, in the problem, we have divided the whole region into 2 areas, in the right hand side area, the x1 and x2 values were like this, x1 was basically 4 and 5, x2 values was given as 2.8. So, for a value of x2 being 3, 4, 5, 6 we saw it was infeasible region and also by solution, we can find it as infeasible region.

And also in the diagram, I have also showed that the point 3 was outside the feasible region, so it is not plausible. Now, we will go into the other part where the x2 values would be less than 3 and obviously less than 3 considering, it is integer programming, it will be only values of 2, 1 and 0 or 0, 1, 2. So, this is the problem.

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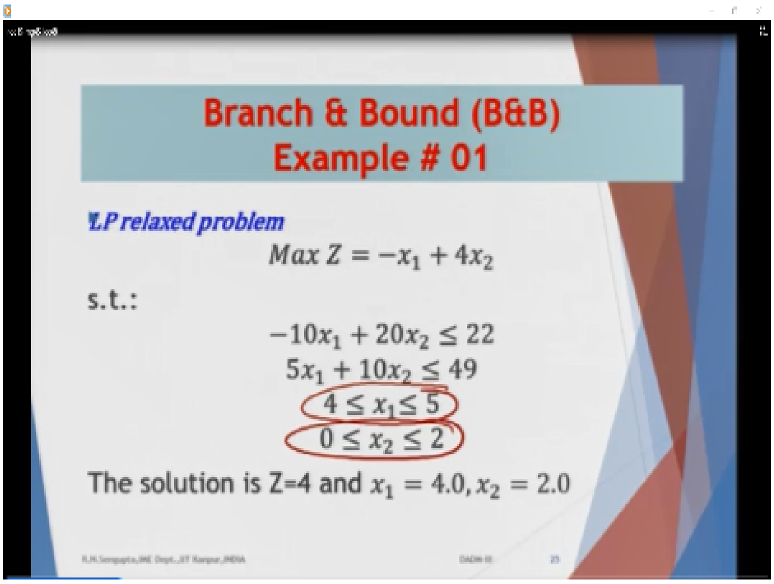

Again I am stating this LP relaxed problem formulation. I have purposely omitted writing integer program problem. So, it is like this, maximization of Z minus x1 plus 4x2 and the constraints remain the same minus 10x1 plus 20x2 is less than equal to 22. 5x1 plus 10x2 is less than equal to 49 is the second constraint. And actually what would have been the integer programming other constraints were, x1 is 4 or 5, x2 is 0, 1, 2 because we are in the other branch.

But now we basically formulate the LP relaxed formulation, the objective function maximization remains the same, the first constraint remains the same, the second constraint remains the same, the other decision variables corresponding constraints are now changing which are in the relaxed form linear programming. X1 is continuous between 4 and 5 any values, x2 is continuous between 0 and 2 any values. This is a simple linear programming problem. Use the tabular method, solve it, the linear programing concept using the simplex method and the solution comes now to be Z is equal to4, X1 is 4.0 and X2 is 2.0.

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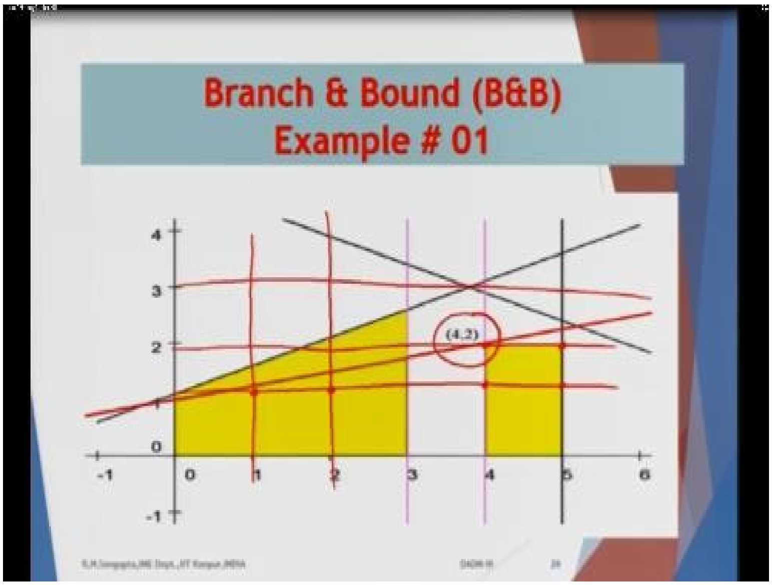

Now, here is the solution. So, when you check, when you solve the problem, the actual one, I will again draw it. So, this was; (())(05:07) we do not know. So, this is now ruled out because this is; let me check, ok, it can be, because 2 and 5 is there. So now again, the interesting part, this red line, which is the objective function has; you will be moving it in order to linear programming problem, what you do? You will shift it and find out the corner points.

So, the corner points technically for this are 1, 2, 3, 4 as it goes out for the maximization problem, the corner point is the solution. Here, actual in this whole square, rectangle, the integer points are x1 is 4, 5, x2 is 0, 1, 2 and technically if you solve it, for all these six points, the optimum feasible and best solution is 4, 2 and the actual z value is 4. So, this area, right hand side area, we have at least achieved. But we have not finished, we have to basically check the answer. On the left hand area, we have to consider the x2 values as I mentioned as 0, 1 and 2 which I have just mentioned.

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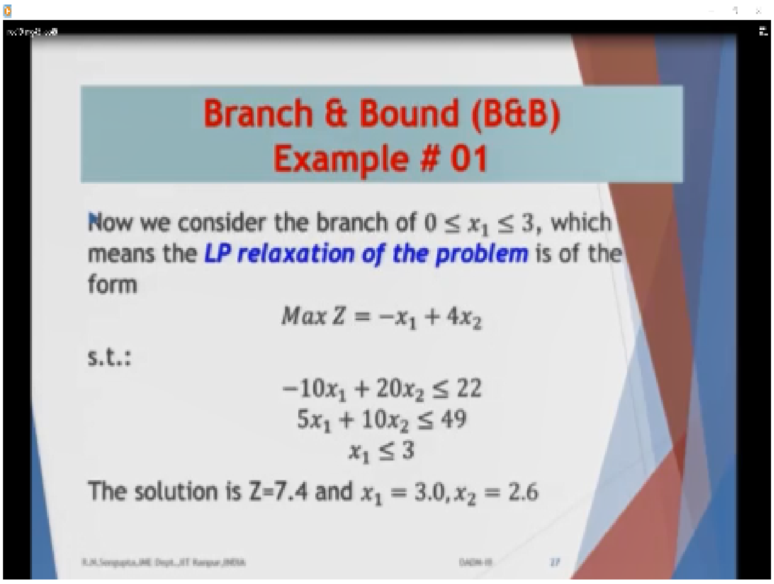

Now, we consider the branch of the values of x1, till now we have been all considering x1 is 4, 5, 4, 5 and that is why we have sub branch for x2. Now, once that sub branch x2 has given us an optimum solution, now we will go to the next primary branch which we have done where we are basically now going to take x1 values as 0, 1, 2, 3; 4 and 5 for x1 is already considered, we have finished that right-hand side. Right hand side means this one.

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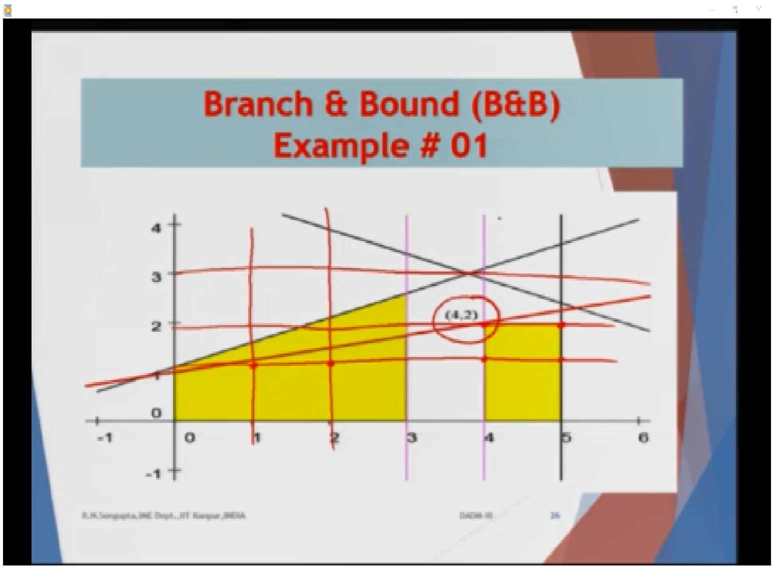

I am not going to highlight it but this is the whole area where we have been discussing for so long. Now, we are going to go out for the left-hand side, the other branch.

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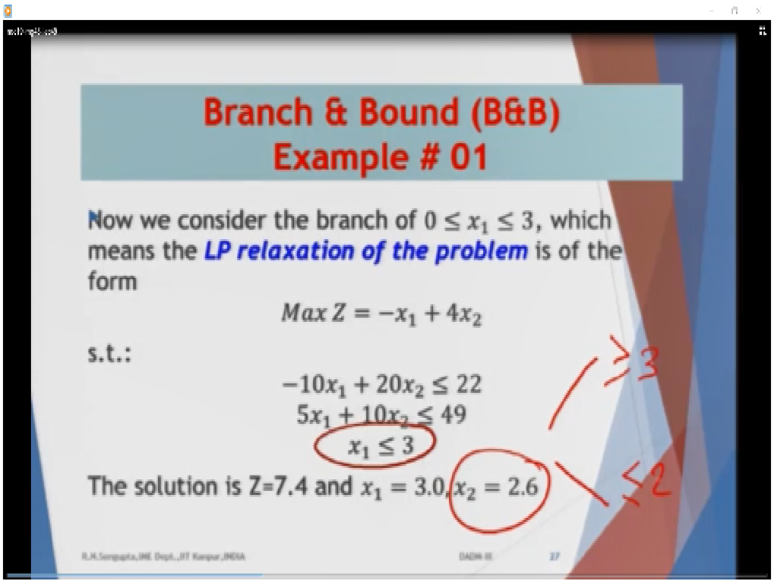

Now, we consider the branch of x1 between 0, 1, 2, 3. Now, we will basically solve it using the; first give the highlight of the integer programming which I have not done in order to save time. So, I will directly go to the linear programming relaxation and just highlight the fact where we have changed it, which I am doing for each and every step.

We consider the branch of x1 between 0 and 3 continuous, linear programming, remember which means the LP relaxation of the problem is of the form like this. It is a maximum problem, same objective, constraints, number 1 minus 10x1 plus 20x2 less than equal to 22 remain the same, 5x1 plus 10x2 less than equal to 49 which is the second constraint remains the same.

Actually the values of x1 in the integer programming would have been, it is only integers 0, 1, 2, 3 that we relax. This I will highlight. I am purposely using this darkish red color in order to highlight where the relaxation is happening. So, I relax and x1 values can take any values, non-integers also continuous between 0 and 3. Linear programming problem, tabular method, whatever you want to use and now the solution comes out to be like this, x1 is 3, x2 is 2.6 and Z value is 7.4.

So, we will just pause here, in the sense x1 is 3 which is fine, as per the norm it is an integer, we are happy but are we happy with X2? X2 is 2.6. So, again technically, we should branch and find out X2 for values of greater than equal to 3 and X2 for values less than equal to 2. Now that means, we would basically have from here a branch going out where it is greater than or equal to 3 and less than equal to 2. And obviously X1 remains as 3, so you will keep checking it for a combination of 3 and 3, 3 and 4 and so on and so forth.

And another case we will check where combination of 3 and 2. So, the first value I am talking about is X1,3 and 2, 3 and 1, 3 and 0 and so on and so forth. But after we complete it, again we will go back to the right-hand side area.

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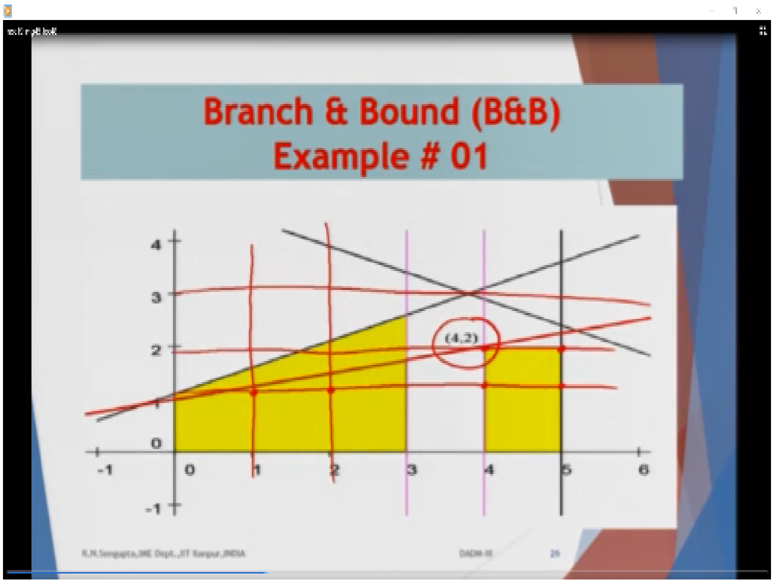

We have already solved it, stopped there because we have already obtained a good solution. I am not saying it is an optimum best solution, it is a good solution in the sense that I have obtained X1 as integer, X2 as integer and the value obviously for Z. Here considering the objective function is the value which is also an integer.

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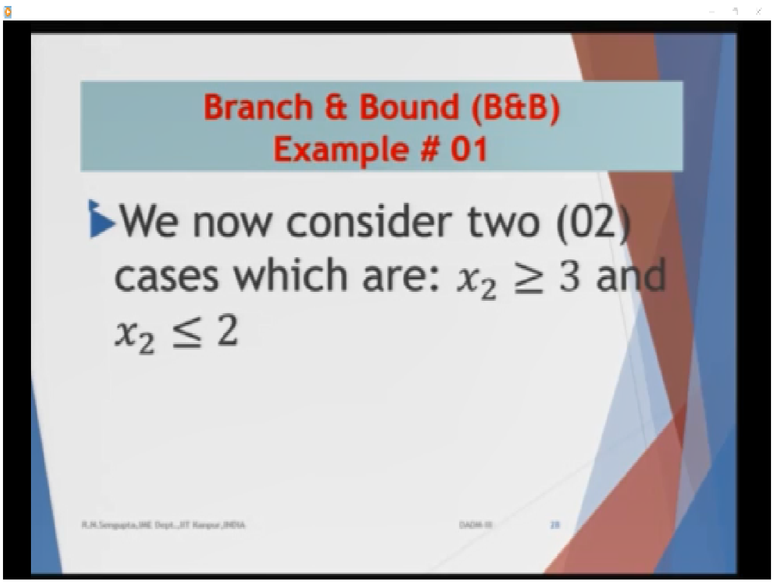

Now, as I just mentioned, we consider two cases where X2 is greater than and equal to 3, 3, 4, 5, 6 till infinity because X2 can be any values or integers and the other set is X2 is less than equal to 2 this is 0, 1, 2. And for this we will consider the combinations, for these two branches along with the values of X1.

So, this branch is going so called, I have not been able to draw it. I will come into the second problem. We are considering the left branch or the left part of the diagram. Right branch, we have already solved it and stopped there. We will compare it once we finish.

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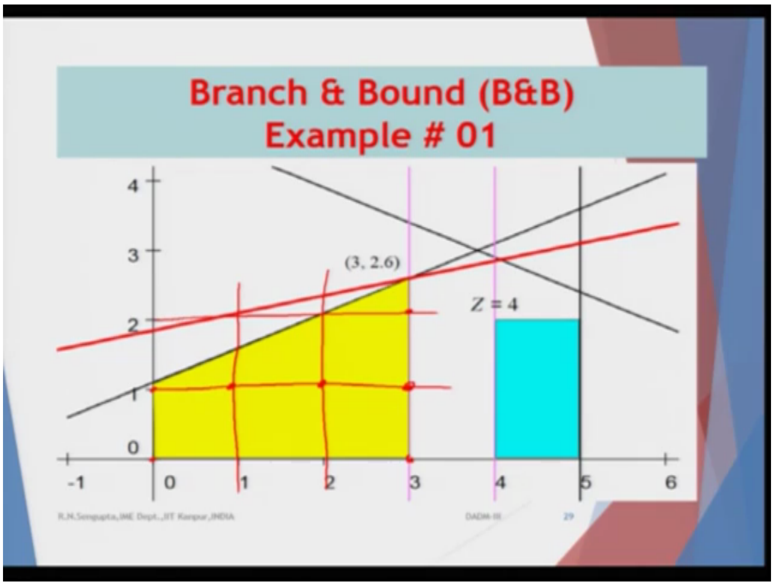

Now, see, there is a change of the color. This blue region, we have already solved, found out the value of X1 and X2 which are integers, Z value who is coming out to be integer also. That is a different question. And once we solve it, so we will compare, this yellow region we are going to solve with the blue one which have already been solved.

So, now in this, again I am drawing but which are actually feasible or non-feasible, I am going to highlight. Technically, it looks plausible but I do not think it is. If I drew it very clearly and neatly then obviously this would have been very clear. So now I basically concentrate on the left-hand side.

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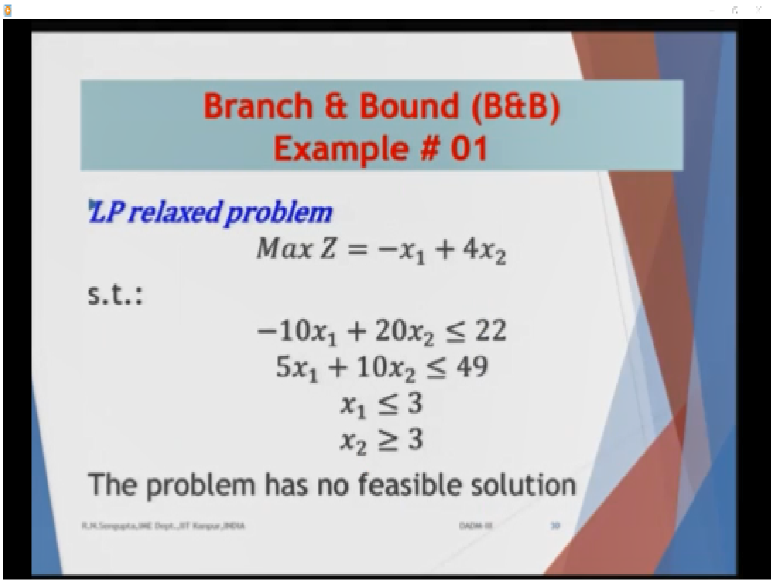

So, when I solve this problem, listen to me carefully, we are considering for the area of X2 greater than 3.

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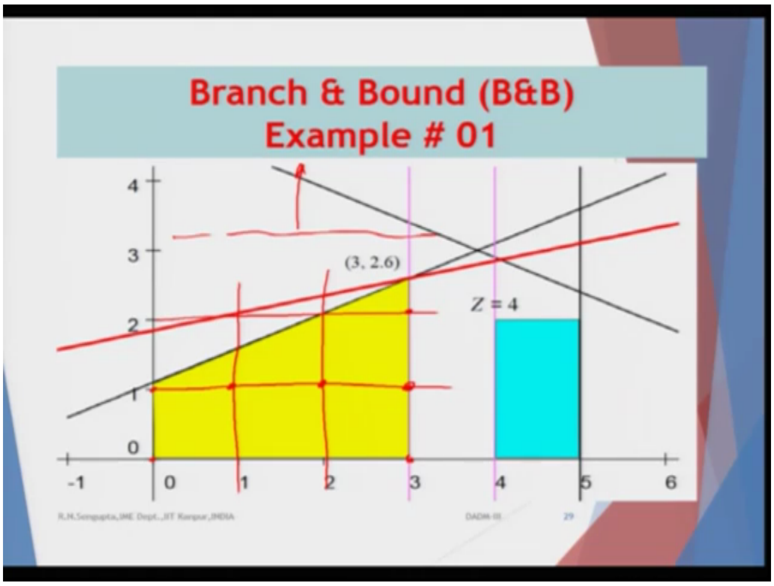

X2 greater than 3 is the portion above. So, obviously I will come to the diagram is immediately sees, you do not have to solve, immediately it is obvious that there is no feasible region here. X2 greater than 3 would be the region above. So, that is not feasible. So, obviously it is infeasible region and you do not have any solution. But still I will basically go through the same steps in order to make you understand.

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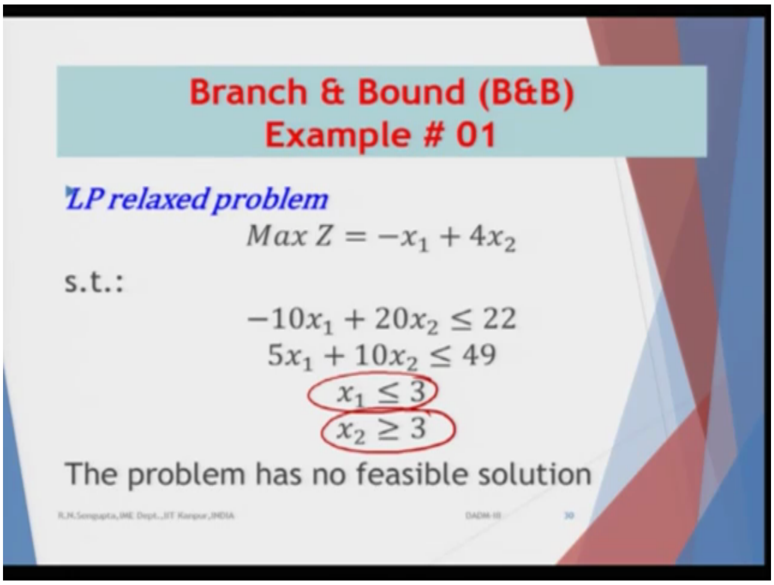

So, the initial problem is like this, I will read it and listen to me carefully. I am first reading the non relaxed form of the integer form. Maximization of Z, minus X1 plus 4X2, which is same, as same is as per the original problem. The constraint minus 10x1 plus 20X2 is less than equal to 22 which is same, as per the original problem. The second constraint, 5X1 plus 10X2 less than equal to 49 is same as the original problem.

Now, the integer part for X1 values were 0, 1, 2, 3. X2 values were 3, 4, 5, 6 till infinity. That we relax. And once we relax, we have the LP relaxed problem and the relaxed concepts are like this.

X1 can take any value, so, objective function remain the same, first constraint remains the same, second constraint remains the same. X1 values are from 0 to 3, all (())(13:47) continuous. We have basically changed the integers to the continuous one. And X2 values are 3 to infinity. And obviously as I showed you in the diagram just few minutes back, there is no feasible region so obviously we will not proceed. So, there is a dead end. In that arm we do not proceed further.

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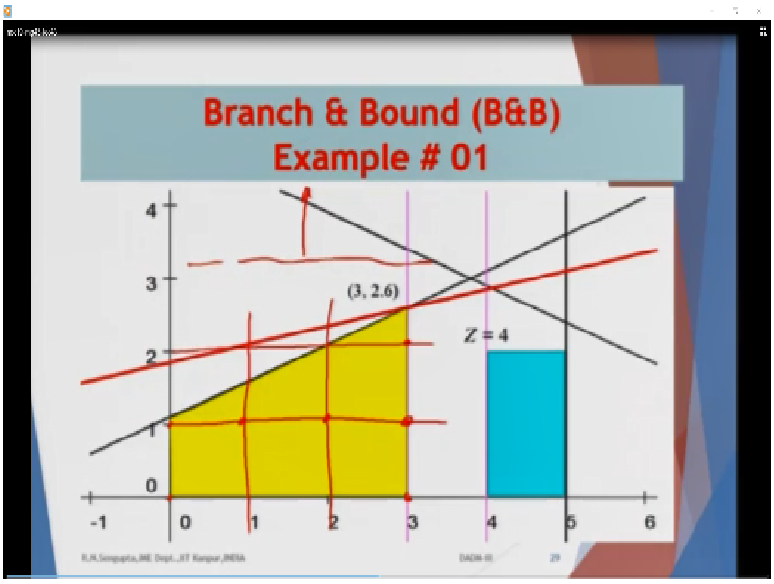

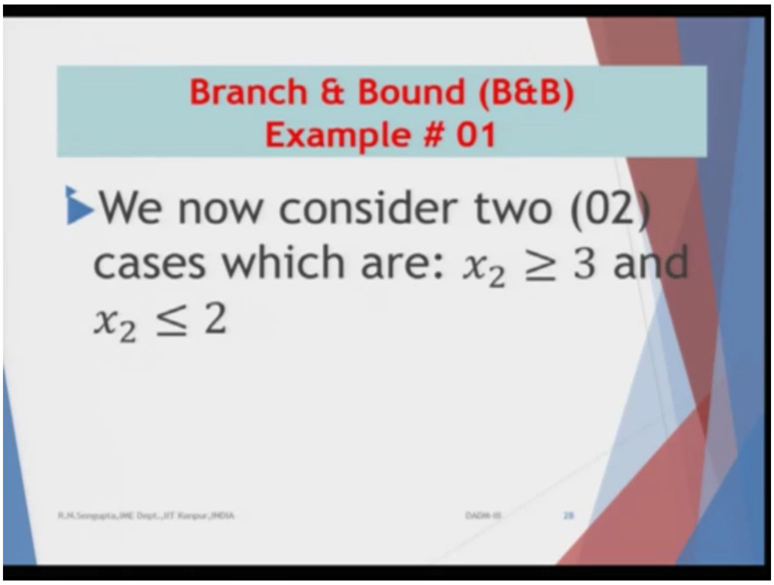

So, this portion we have eliminated. Now, we are going to consider the region here, below, that means for X2 less than the value which have already; so that was would be less than equal to 2.

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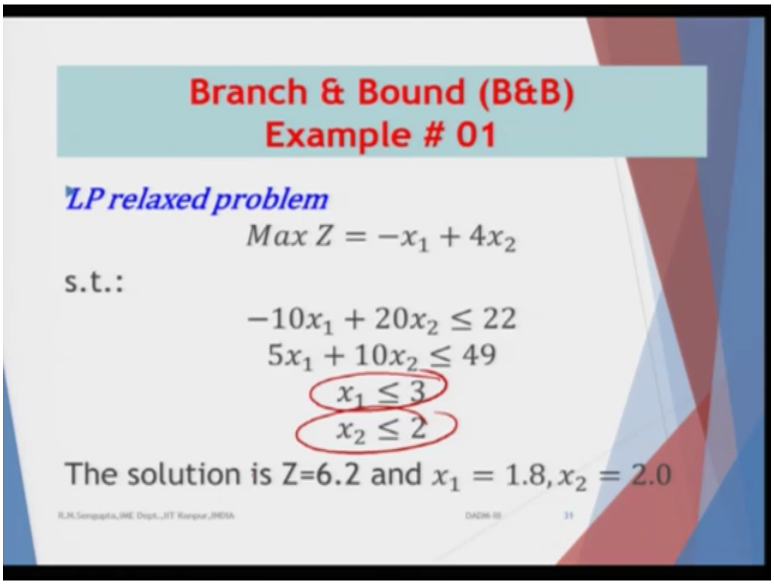

So, here it is. I will not highlight it. I will come to it later. So, X2 is less than 2. So, actually the integer programming is like this in the other branch. So, one branch has stopped because infeasible, the other branch, the contemporary the other branch.

Maximization of Z minus X1 plus 4X2 and the constraints are minus 10X1 plus 20X2 is less than equal to 22 is the first constraint. The second constraint is 5X1 plus 10X2 is less than equal to 49 and the relaxed version for the X1 variables is X1 is 0 to 3, both inclusive continuous which is right and the other branch which we have going to consider is X2 values are less than equal to 2, so the relaxed versions are 0 to 2, inclusive, everything continuous.

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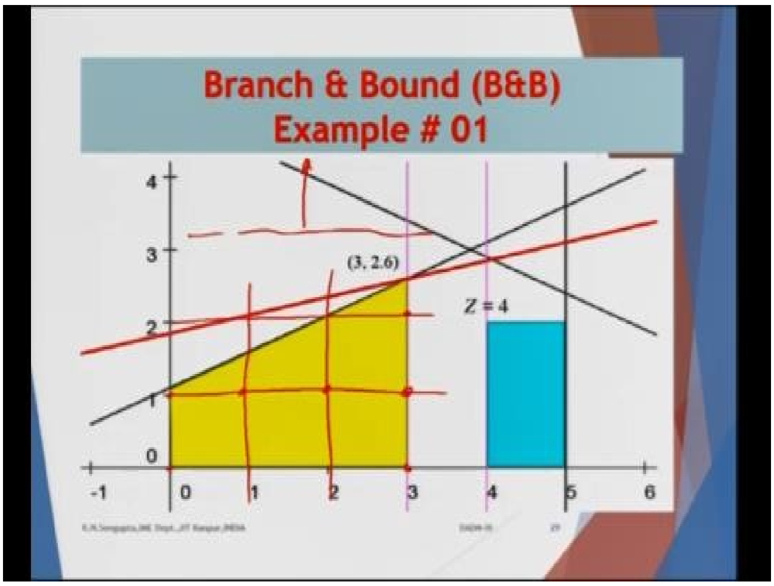

So this is actually, if you see the diagram is plausible because it will come inside the yellow region which is fine. So, the red line we will see would move inside and then it can be here, here, here, whichever it is and then we will basically solve it. So, it will move in the direction, parallel to it.

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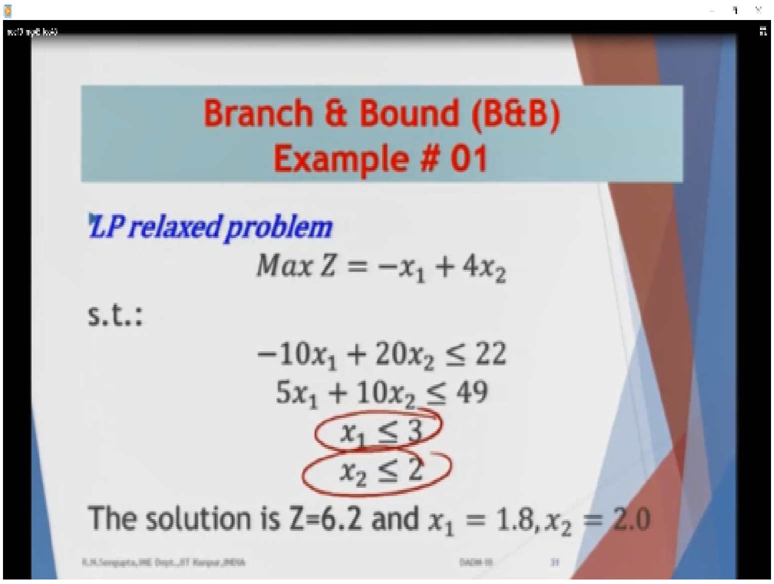

Once you solve X2 comes out to be 2, X1 comes out to be 1.8 and Z value comes out to be 6.2. So remember, we are solving the linear programming that is why the value of X1 is 1.8. So technically, what you would have done, logic says that we will now go for the other branching, lower branching, in the sense X1 would be greater than equal to 2 and X1 is less than equal to 1.

So, one set of less than equal to 1 would be, we will only do a search for X1 for 0 and 1 and other values we will do for the search for X1 is greater than equal to 2. But already we have put a constraint that X1 cannot exceed 3, so only the search is on the right-hand side would be X1 is 2 and 3. And other side, other branch, would be X1 is 0 and 1 and X2 is 2 already.

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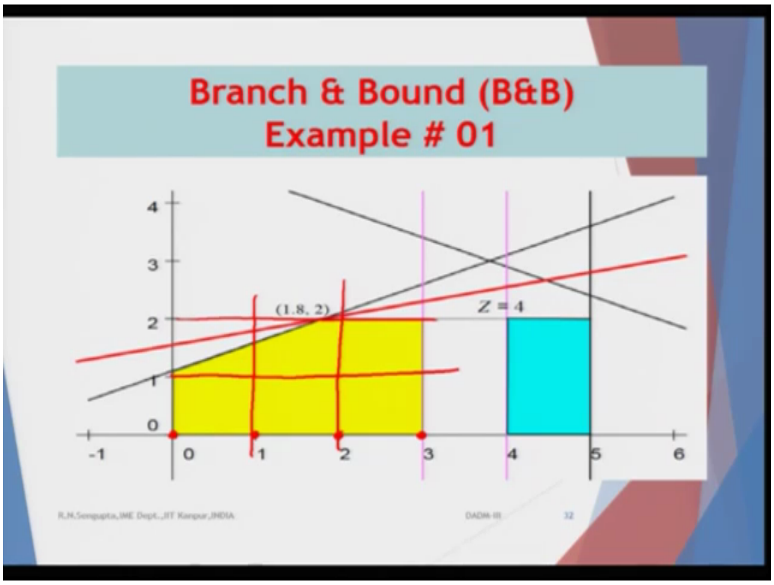

Now, this is the graph. So, this X1 is divided, 1 and less. I will do the search here and here for X1. Other part I will do on the search only here and here. And X2 is already 2, so my overall feasible region actually will shrink.

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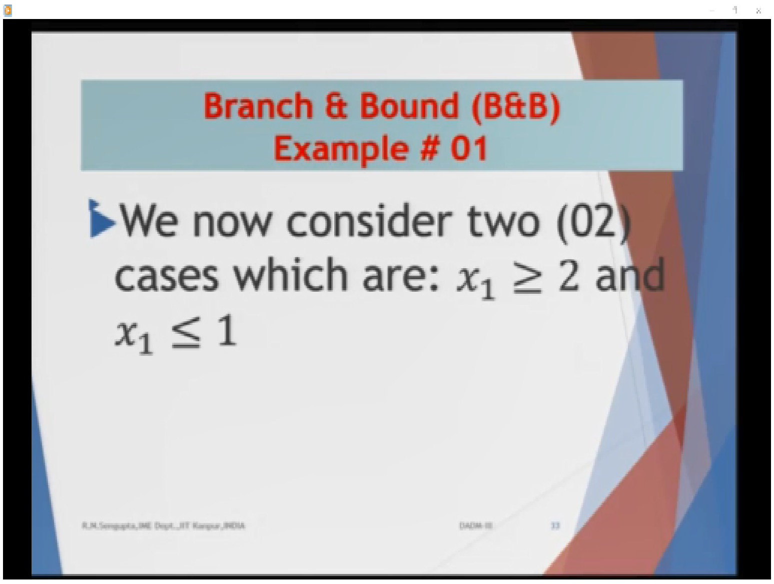

So, now we consider two cases which is X1 is 2 but it will be only 2 and 3 and another value is X1 is less than equal to 1. So, X1 is greater than equal to 2, sorry, X1 is greater than equal to 2 that is .2 and 3 and X1 is less than equal to 1 which is basically 0 and 1. So, we have already branched.

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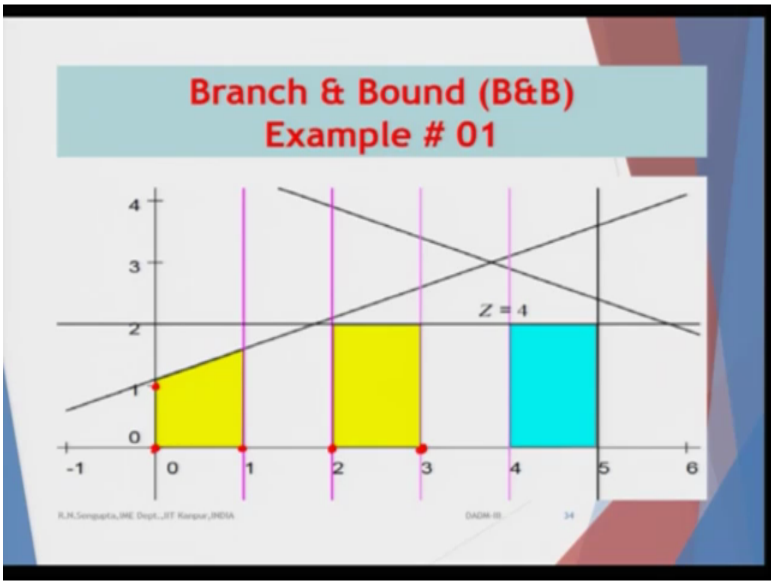

Now, this is the diagram. So, now your search would be, I am not going to now draw the vertical. So, this can be 1, this can be 1, this can be 1. In this region, this can be 1, this can be 1 and this blue portion already solved. So, the actual search considering the integers are now in these highlighted red points.

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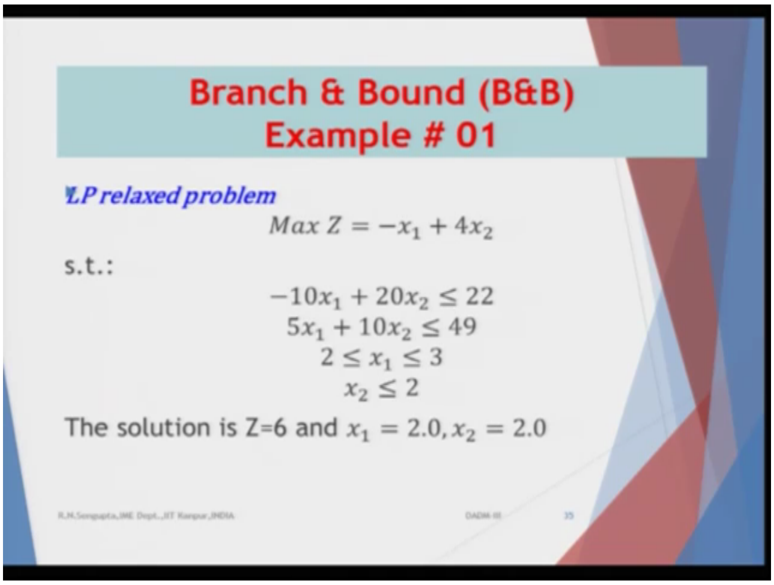

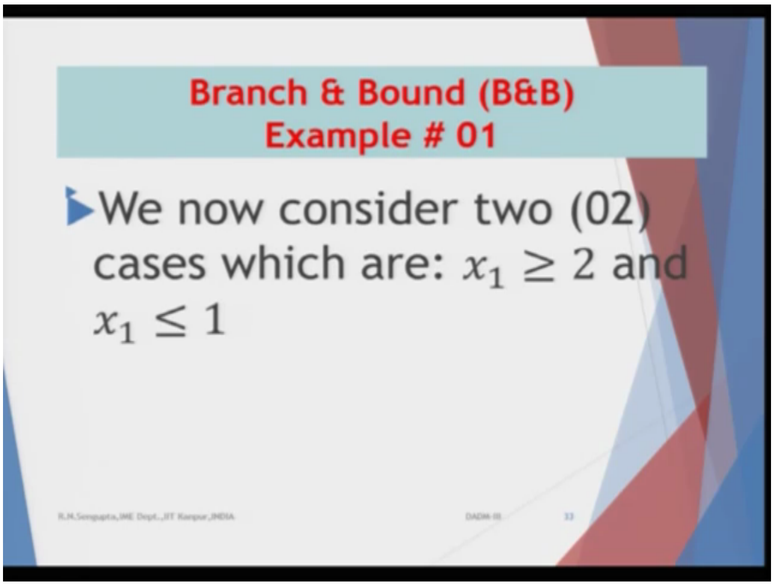

Now, we have again this problem formulation and this problem formulation is being done in two regions. As I said, for values of X2, 0 and 1. So, 0 and 1 is basically in this region, so it will be greater than 2 and less than equal to 1.

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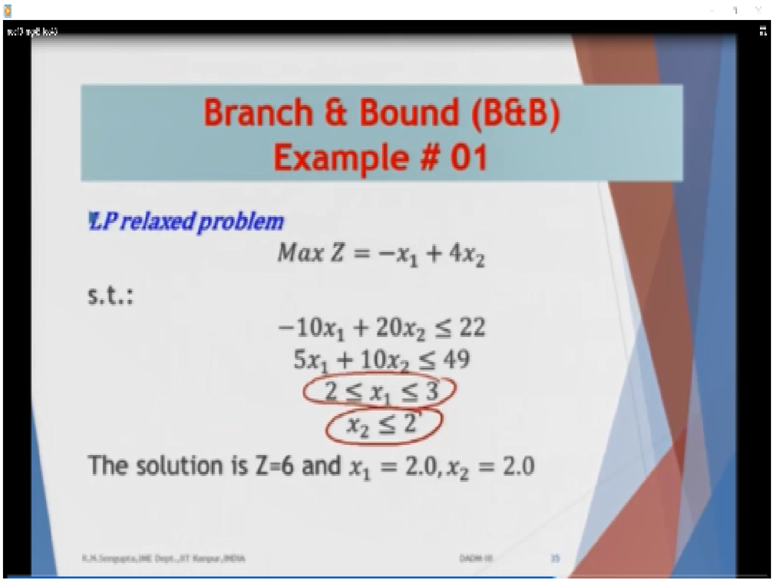

This is what I am doing. I just wanted to repeat that. So, the actual integer programming is Maximization of Z is minus X1 plus 4x2. The (())(19:31) constraints are minus 10x1 plus 20x2 is less than equal to 22. The second constraint 5x1 plus 10x2 is less than equal to 49 and X1 is between 2 and 3 which we have already decided. Why it is less than 3? Because that is already pre-defined in the upper branch and in this branch we have considered that it is greater than 2.

So, obviously in the integer programming concept the x1 values can only be 2 and 3 and X2, we have already considered, it is basically less than 2. So two values, 2 it was the case so it will be less than equal to 2. Now, in LP relaxed formulation, you have X1 is continuous between 2 and 3 and X2 is continuous for all values from 0 to 2. When I basically solve this problem, the solution comes out to be this, X1 2, X2 2, Z is 6.

Now, we are happy, happy in the sense we have reached a value where integer values are 2 and 2 for X1 and X2 and Z value is obviously as per the objective function is 6. So, now we will basically reached a position where the integers but now we will compare this value of Z with the right-hand portion of the solution which we have already obtained, if you remember that the Z value was coming out to be 4 and X1 was 4, X2 was 2, that blue region.

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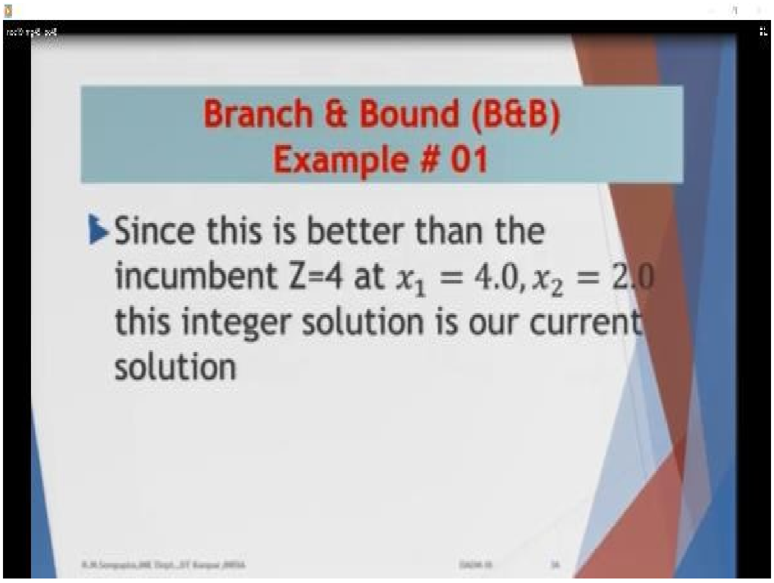

Since this is better than the incumbent which is already there, the contender, it was standing there, waiting to be compared with the result. So, it was basically X1 was 4, X2 was 2 and Z was 4. Since this is better than already incumbent 1 which was as I mentioned so this integer solution is our current solution based on which we will solve.

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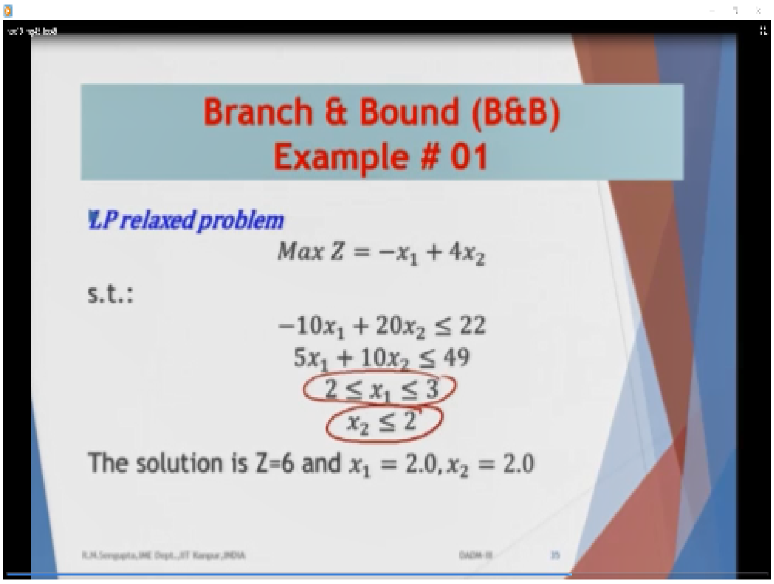

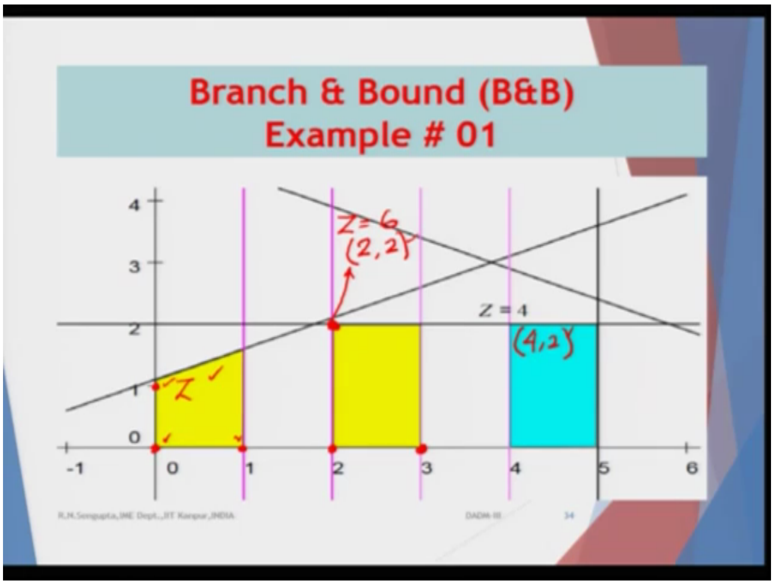

Now, remember one thing, when you solve this and get the value 6, so it is in the region of X1 2 and 3, so it is here. This is 2 and 3. So, it was Z was 6 and the values for X1 was 2. Here Z was 4 and the value was 4 and 2.

Why has it decreased? Because the Z graph line is like this. It will shift. So, obviously this area should be now blue. So, we will compare this blue 1 with blue 2. But what about here. We can; so at this point 0, 0 obviously is just I am enumerating and highlighting that we can solve also.

So, technically the problem would have been, again the same maximization problem, based on the same 2 constraints and the LP relaxed formulations whatever you solve but before that the integers would have been that X1 is only 0 and 1 and X2 is only 0. So, X2 was less than equal to 2 here, so it will be this so it is 0 and 1.

So, we will get a value here, here and here. So the obviously the objective function will give us some Z value, we will try to compare this, this and this but 6 is the highest value and we will basically report that value and get a value of 2 and 2 for the integers.

So, what we have done is that like this. This will become more clear in the second example. I will draw this diagram later.

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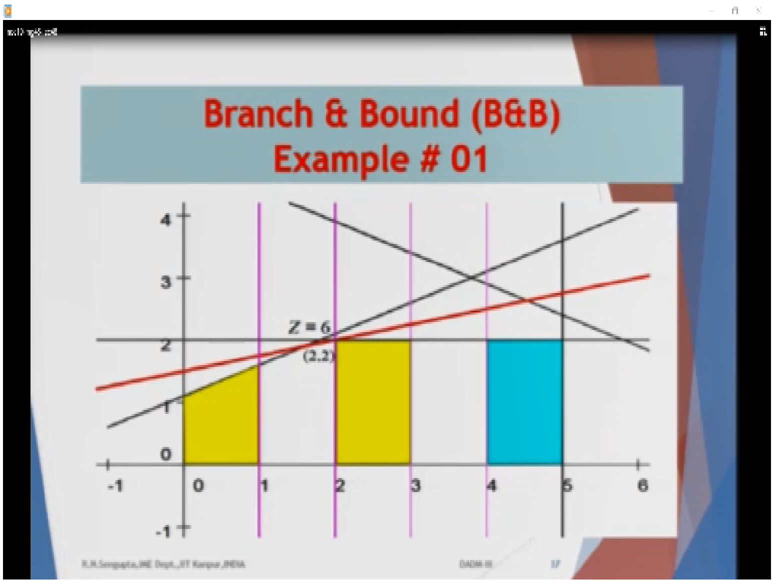

So, this is the value. Z is 6 which I just drew. 2 and 2, X1 is 2 and X2 is 2. Another value Z is 4, X1 is 4, X2 is 2 and the third contender in this area has to be solved which I just mentioned briefly, I will come to that. So, one is one contender, one is second contender. But the second contender has already beaten the first one.

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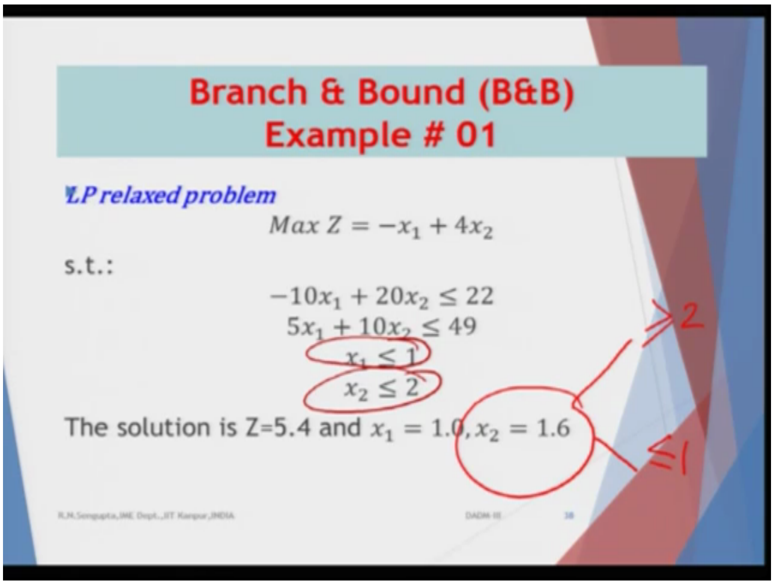

Now, we will basically have the third part. So, the actual problem formulation is like this, this integer one. Maximization problem remains the same, objective function, constraint remains the same minus 10X1 plus 20X2 is less than equal to 22and the second constraint is 5X1 plus 10X2 is less than equal to 49 and the integers are like this X1 value, we are taking the third area, is less than equal to 1 which is 0 and 1 for X1 and X2 was less than equal to 2 so obviously X2 values would be0, 1, 2.

When we relax it, the problem formulation remains the same. Maximization, constraints remain the same which is minus 10X1 plus 20X2 is less than equal to 22. The second constraint is 5X1 plus 10X2 is less than equal to 49 and here the relaxed values are written in linear programming. X1 less than equal to 1, all values 0 into 1 inclusive. X2 is less than equal to 2, all values inclusive. When you solve this problem, you get the answer X1 is 1, X2 is 1.6 and Z value comes out to 5.4.

Now look here, technically, we would proceed considering X1 is 1, X2 is 1.6, so X2 can values can either be less than 1 that is 0 and 1 and other values can be greater than equal to 2. Greater than equal to 2 is this only one value because X1 already by the previous constraint which I am just trying to highlight using a pointer is less than equal to 2. So, when this region is broken down, so it will be 2 regions greater than equal to 2, less than equal to 1. One would have basically 0 and 1. 2 values will have only 1 because it is less than equal to 2, greater than equal to 2 so obviously one value is there which is 2.

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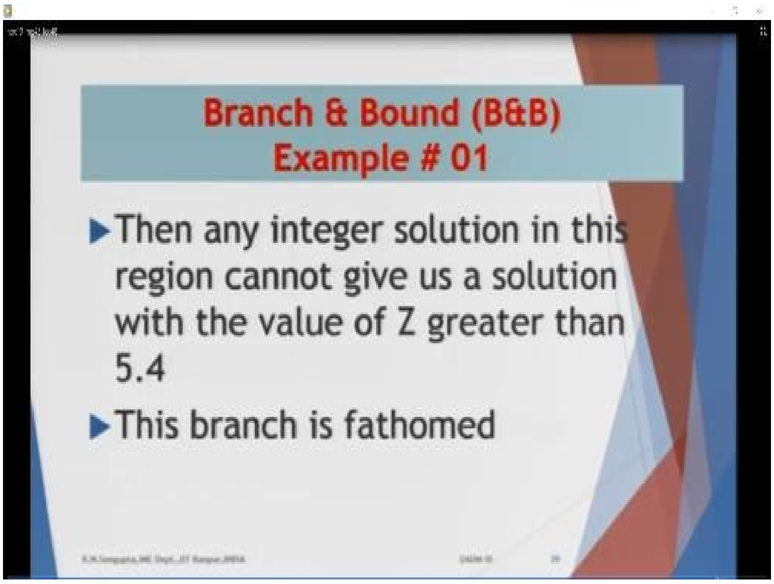

Then any integer solution in this region cannot give us a solution with the values of X greater than 5.4, so the values of the branch is fathomed to we have to basically stop

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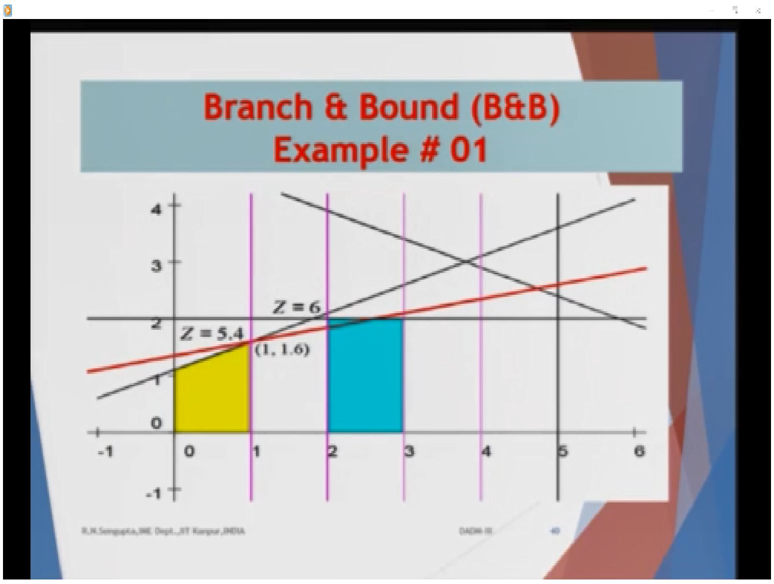

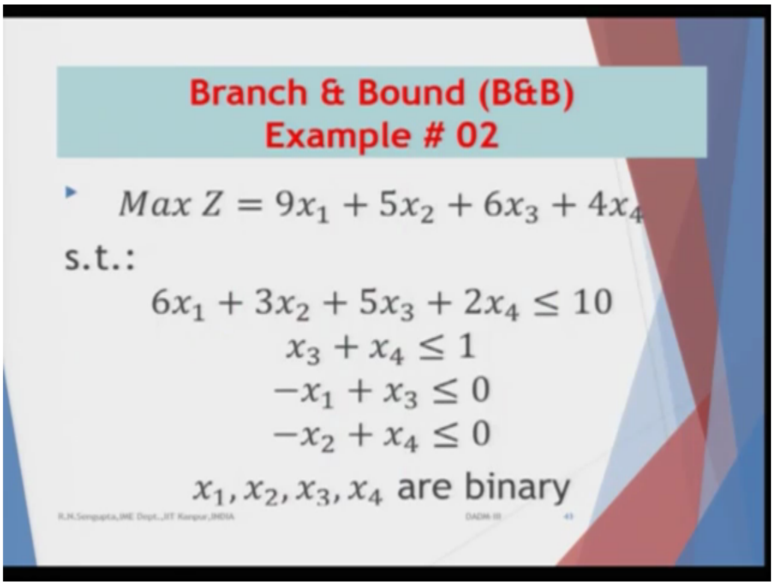

So, here it is. This blue region has been eliminated because the Z value was coming out to be 4. So here X is 6, Z is 6, X1 and X2 is 2. This yellow is still fighting and trying to get into the picture whether it can be able to give us a better result considering the integers. So, Z is 5.4, 1 and 1.6. So, if I consider, the contenders are not plausible, they are not fathomed so you will stop there and basically report this result.

I just showed the slide which was for the second example of the Branch and Bound for the Binary Variable which I will consider in the next lecture.

So, now we report the values of Z, so in the branches, we have only proceeded, in such case, we branch out and find out where they are infeasible, fathomed or not fathomed and take the bounds accordingly and stop and report the answers accordingly. Thank you very much and have a nice day.