Data Analysis and Decision Making - 3
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Lecture 47

A very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe and once again welcome to all the participants, all the students in the DADM 3 course under the NPTEL MOOC series. DADM means Data Analysis and Decision Making 3. We have already (done) DADM 1 and DADM 2 where DADM 1 was mainly in statistics, DADM 2 was in other non-parametric methods and DADM 3 is more into optimization and operation research.

This course total duration contact hours is 30 hours which is spread over 12 weeks and considering each lecture in for half an hour. The total number of lecture is 60 and each week we have 5 lecture of half an hour each. After each week we have assignments.

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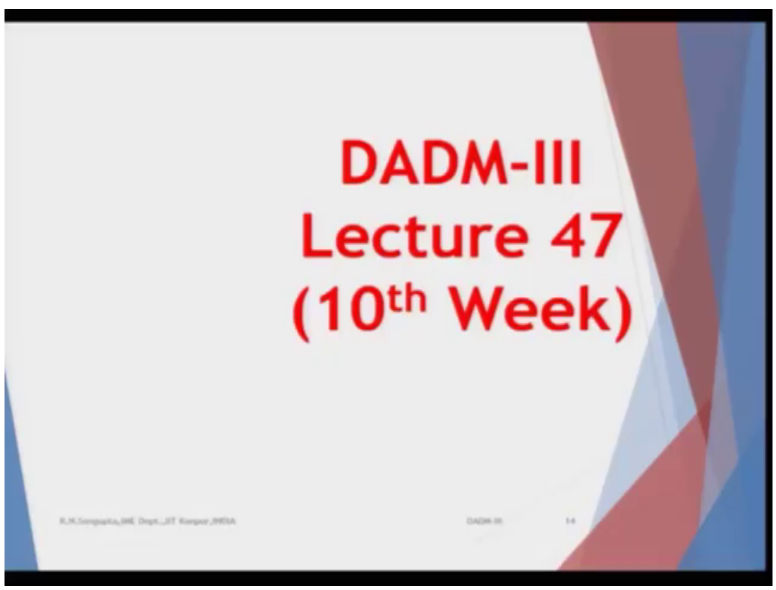

So we have already if you can see from the slide we are in the tenth week. So we means we have already completed 9 weeks and their assignments. And, we are in the 47^{th} lecture which is the second lecture in the 9^{th} week. And my good name is Raghu Nandan sengupta from IME department at IIT, Kanpur.

So if you remember we in the last two classes we are just given some results. And I have been mentioning quite a few times that I will go into the area of trying to understand the concept of branch and bound which can be utilized very nicely in different type of problem. So we will consider 2 type of problems. One is integer programing and one is binary variable values of the decision variables.

Binary means it can only to take two values like 0 and 1. So very good example would be I am giving one thing like you want to build up a factory and other option is you do not want to build up the factory. So, if you remember in that one of the problems where you had to basically build or manufacture the cars. There were in thousand in numbers and in case if it was less than thousand it was not allowed. And in the other problem we had considered that a binary variable x was converted, x was the decision variable which was converted into the binary variable in which was basically y.

So Y I is depending upon the number of y decision variables are I is equal to 1 to n, Y1 can be 01 y2 can be 01 and so and so forth, either 0 or 1. So this is the binary programing, so we will consider a problem solution for that also using the branch and bound. And I will draw the graphs how the branching are done and it will be given in totality with the branched has being highlighted as we proceed from step to step. And then hopefully in this week we will also cover some part of quadratic programing.

And in the, this is the 10^{th} week so in the 11^{th} and the 12^{th} week we will basically considering the concept of reliability optimization robust optimization some part of stochastic programming. So, these are the slides which you have already gone through but I will quickly go through them. This step, this algorithms branch and bound I have already been discussed by like by giving the numbering a step 1, step 2, step 3.

But, as still rehearse it in order to make it much more logical in the discursions which I will have with you considering that we would be going into depth into two problems. So branch and bound is the divide and conquered rule.

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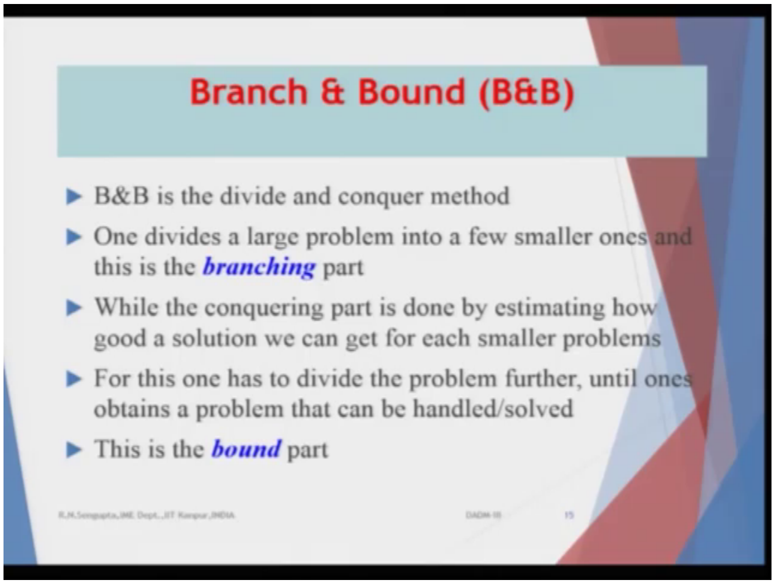

So you divide in which way you want to go there would be only two paths and one of them would be feasible you will basically go further down considering the results. Results means not the optimum results, the decision variables are not non-binary or non-integers you will go further down try to basically break it down into two arms. And if they are infeasible or if their values are not increasing, increasing or decreasing depending on the type of problem you are trying to handle.

These are the so called dead ends or not fathomable you will stop there. And you will find out that as you differentiate the results for the integer variables and the decision variables which is z will come out automatically. So in the branches I will write down which root you are going to take, what are the decision variable values and what are the z value. We will proceed accordingly. So one divides a large problem into few smaller once step by step is goes like a hierarchy.

And if you remember in DADM 2 we had done AHP Analytical Hierarchy Process where you divided the primary decision into nodes and then proceeded step by step but only change here leave apart the ranking system and all those things, only change here would be that the branches in AHP analytical hierarchy process could be more than 2 or consider that you have more than two number of decisions or criteria to consider but here we will only consider 2.

So, let me again read it, one divides the last problem into few smaller once and this is the branching part. While the conquering part is done by estimating how good a solution we can get for each smaller problems and then eliminating and considering the once which are feasible.

So we will eliminate which are infeasible consider only those who are feasible and obviously we will try to reach the optimal solutions. Now remember one thing the word optimal solutions would not be with respect to the linear programing solutions. Because if you remember I have been say mentioning a time and again that you relax the problem. Which will do it in each and every step solve it like a linear programing and then proceeds.

So we are not going to compare the results with respect to the linear programing we will only consider the results with respect to the integer programing, binary programing results which you get at different steps. Because linear programing would basically have continuous decision variables where if they are continuous they need not be integers. So, that would not suffice the actual problem formulation based on which we are proceeding.

So as we conquer conquering is done by estimating how good a solution we can get from each small problems. For this is what we have to do is that we divide the problem further on into layers and layers. Until one obtains a problem that can be handle and solved and we can compare the results accordingly. So this is the bound, bound means to what stage you will go whether is infeasible, feasible, unbounded whatever it is we have to stop at some stage depending on the stopping criteria.

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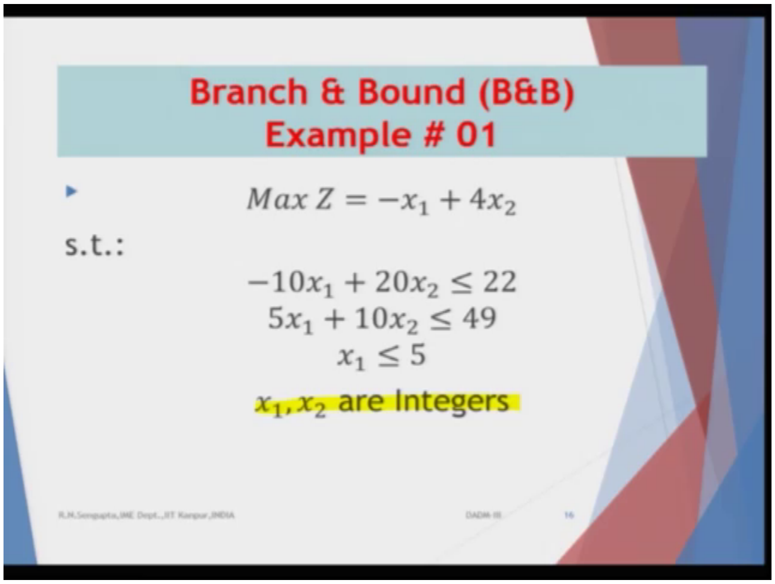

So this is the first problem branch and bound and here we will consider there are 2 variables x1 and x2. Why I am taking two variables is because I will be able to draw and try to basically give a picture of stage by stage like solve the problem and all show the feasible region and what is the optimum points. So one side by side if they proceed so that means one after that I have proceed you will get a better picture. But if go to the higher dimension trying to solve them using 3-dimensional figures would be difficult in the next problem which is example number 2.

We will have such a problem whether the number of decision variables are more than 2. So we have a maximum problem of minus x1 plus 4 x2. So, obviously in this case as it is mentioned in the last point which I will keep highlighting many of the points accordingly so it is mention that x1 and x2 are integers. So the what are the constraints? Constraints are minus 10 x1 plus 20 x2 less than equal to 22 less than type and plus 5 x1 plus 10 x2 is less than equal to 49 and another one is that x1 is than equal to 5.

So x1 has values if we solve the problem using the integer programing would be 0, 1, 2, 3, 4, 5 and for x2 it can 0, 1, 2, 3, 4, 5 and continue till infinity and but they are all integers point 1. Point number 2, we are not taking values less than 0 not in the negative values because as for the assumptions x values have to be basically positive because that is very logical, because if you are trying to formulate a problem where the number of trucks are important.

Where x1 is the number of trucks you are going to utilized to ship products from Bombay to different cities in east India. The number of trucks cannot be non-integers they have to be integers. So the problem is there are 2 constraints, two variables maximization problem. Now, before I proceed one few things should be mention very categorically. When we convert the problem into linear programing relax it.

We will be always utilizing the same concept which you have studied in the first 5 or 6 weeks of the lecture which was to do with the concept of how you solve the North West corner method, the Vogel's approximation method considering that you have the network problems. How you solve the linear programing primal problems, dual problems, simplex methods all this concept will be utilized time and again.

So this is I would not repeat that I will only highlight that where the integer programing and the branch and bound would be utilized in between whatever you do would all be the simple concept or simplex algorithm.

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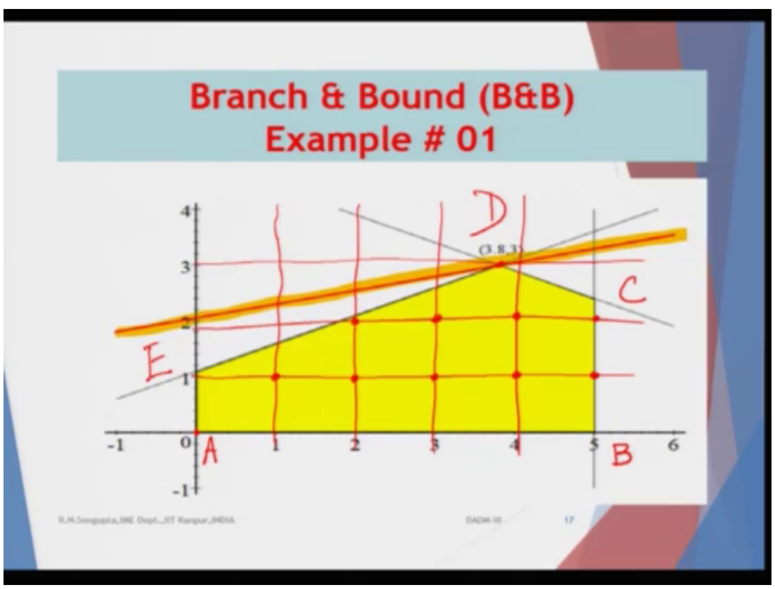

So, this is the feasible region, the yellow one is the feasible region and you have x1 and x2 drawn accordingly in the Cartesian coordinate and this orange one I would not highlight it okay I will high light using the orange color if it is yes, yes so this is the objective function. And if you remember for the maximization problem you try to basically move outwards and the interior points. Where you will try to solve would be point A which is the origin, point B, point C, point D. point E.

Now remember this vertical line BC and goes up basically the fact where x1 is less than equal to 5, so the actual values of x1 are 0. So one value would be I will use the red line, so this is 1, this is 2, 3, 4, 5 and the corresponding values of x2 are, so this 3.8. So obliviously to evaluate it should basically (match) so somewhere going like this. So the actual points of solution in the feasible region if it is an continuous variable they are I am repeating A, B, C, D, E but if you want to find and all the inside region.

This are the A, B, C, D, E are the points where you do the search and the actual feasible region are infinite in number considering that is the (your) yellow region inside. But if we solve the integer programing, integer programming points are this can be a solution can be, this can be a solution provided is inside, this is can be a solution, this is not I am not going to highlight, this is not because obviously it is not in the yellow region also and also it is outside.

But also remember these points are applicable for the linear programing continuous variable case not for the integers. This is not, this is yes sorry, this is yes, this is okay, this line should have been 3.8 this is technically a point which seems very interesting from the fact that x2 is an integer but x1 which is 3.8 is not. This can be a contender, can be a contender, can be a contender, can be a contender.

So you have to find out what is the point based on which we can solve the problem. So now we will proceed, so you have seen the problem we have seen the overall diagram of the feasible region both from the perspective or linear programing as well as the highlighted points are from the concept of integer programing. Which are the feasible points where you will do the search now we will start all the procedure on the branch and bound.

So first what we will do is that if you remember as per the algorithm which we have stated, step 1, step 2, step 3 so and so forth. We will first find out the LP relaxed problem solution. So what is the concept of linear programing relax problem solution would be very simple. The problem solution remains the same, you first relax the decision variables make them continuous and solve them. And the solution techniques are all what we have learnt in linear programing simplex methods so and so forth.

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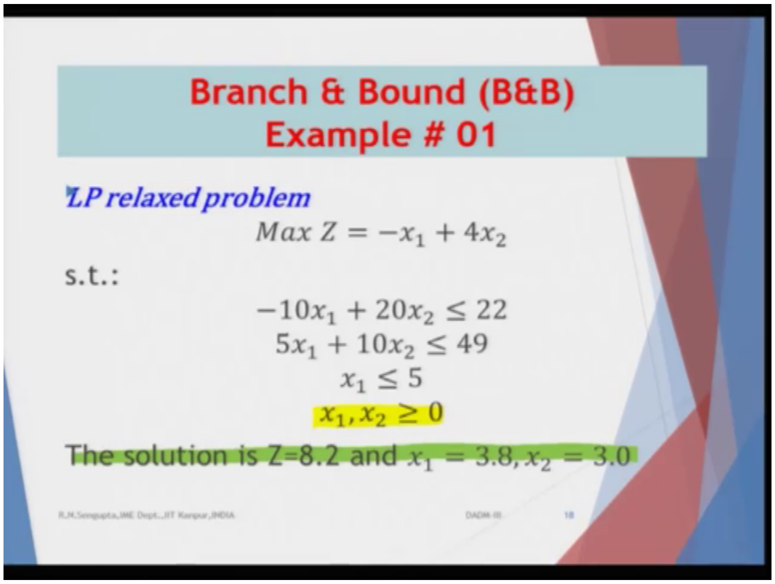

So, the problem formulation is like this maximization of the same function which is minus x1 plus 4 x2 such that constraints are also the same, let me go one by one very slowly. Minus 10 x1 plus 20 x2 is less than equal to 22 which is same, 5 x1 plus 10 x2 less than equal to 49 which is same, x1 less than 5 which is same. The main part which has changed considering this LP relax problem that is why I have highlighted in blue bold color, is this one.

So this has been converted from a integer to a simple linear programing considering x1 and x2 can be any values continuous. So x1 now can take any number between 0 and 5 including non-integers and x2 can take any values greater than equal to 0. Now you solve it, you check the problem solution, the solution is already shown in the graph just we finished checking the graph. So you will just take one of the corner points basic feasible solution take the concept of the tableau.

Consider what is, which is the exiting variables, which is the entering variables continue solving it step by step. And it is easier for us to understand this problem because it is visible to us in 2D solve the problem. And the point where you will get the optimum solution is where x1 is 3.8 and x2 is 3. The actual value of z is 8.2 I will basically highlight it. The solution is this z is 8.2, x1 is 3.8, x2 is 3. Now you will ask the question is it a solution integer programing? Answer is no.

Because x1 is 3.8, x2 is 3 which is fine and z is 8.2. Now, what we will do? Now here where you will take a decision, so if you remember in one of the very simple brute phase a logical method or I can use the what brute force in the sense I enumerated all the values and highlighted the points that technically I would basically start with x1 can be either 4 or x1 can be either 3 and then solve the problem that means I have started the branching.

So this is from I will start the actual branch and bound. Now, we considered two cases because x1 is 3.8 so x1 could have been 4 or could have been 3.

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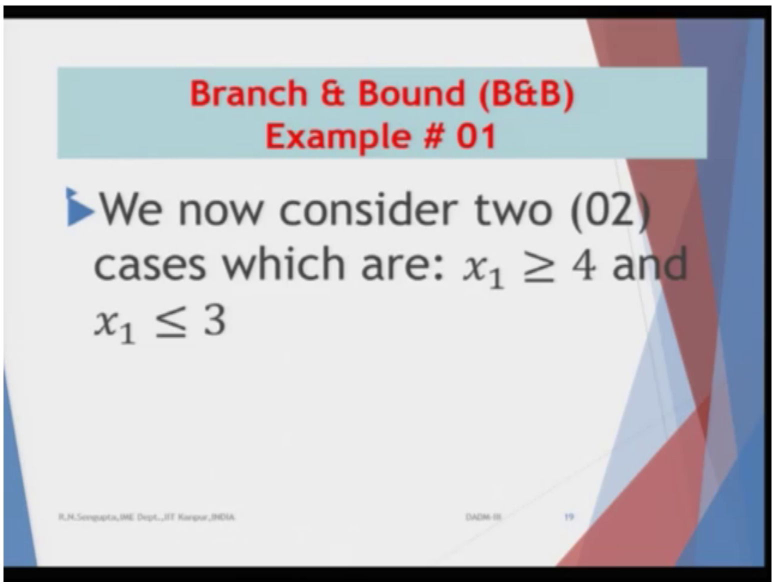

And we will basically branch out from there in one aim branch will take x1 as 4, (x2) x1 as other x1 is 3. And the value of x2 which is already 3 would be considered as it is and we will proceed.

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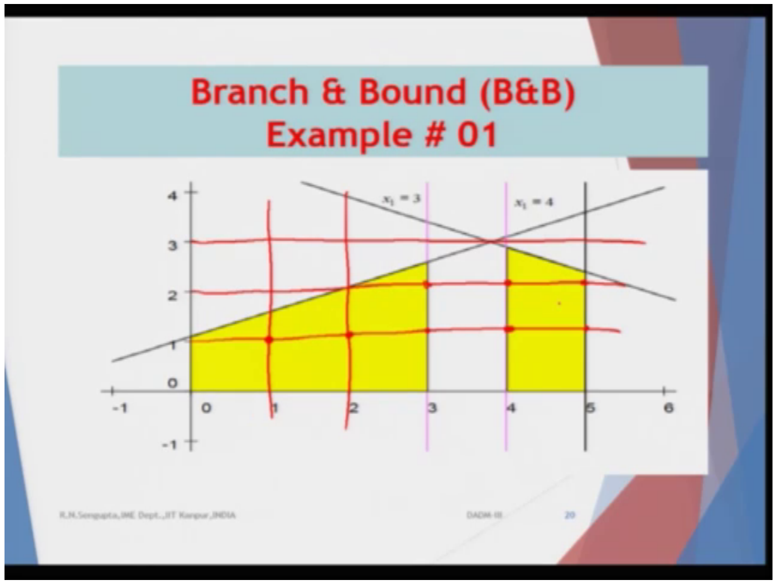

So, now here is the graph, we have tried to differentiate though overall area so now my solution technically would have been like this. So, this first part yellow region is now a particular problem with an objective function with some constraints. And this yellow region the first part would be the overall feasible region if I solve using the linear programing part. And if I get solutions technically where x1 and x2 are integers will take that and keep this beside and we would not proceed in solving that right part of the problem.

I am just giving you the scenarios and the left again you have a feasible region considering for the linear programming solve it. If you get the decision variables as integers then also you stop compare this two results and basically you give the optimal amount which is maximum one. Which is the this scenario which I thought which thought out experience sort of thing thought out process which I said is basically the best solution which I would have. Because we do not have to go any more branching.

But considering the right hand side and the left hand side the decision variables once you solve the linear programming are non-integers. Again we will branch them one at a time and then proceed according. So this are the second steps, third steps as you go as you branch out. So as just mentioned in the last slide x1 values would be also considered from 0 to 3, so that means actually they would be 0, 1, 2, 3 and the values of x1 for the other problem for the other branch would be 4 and 5.

And x2 remains as the values of 3 but interestingly this value of 3 which was there is now bound into the feasible region, infeasible region. So technically, so initially this point was there which is still true, this point was there which is still true, this point was outside I have not drawn it properly so it was not there, so this point true, true, true, true for the two sets so we will proceed now again.

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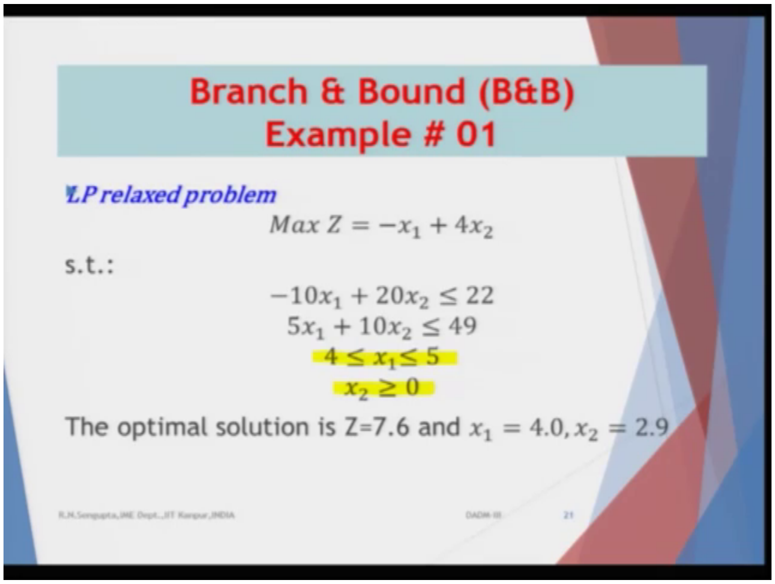

Now we start the problem for the case for the first right hand side where x1 is 4 or 5, why 4 and 5 why I am saying that? Check this, I am going to high light later on but check this or of you highlight it I can erase it later, this one. So actually that problem which you need to solve in the right side of region is still an integer programming with a following objective function of the constraint let me read it and then it will become clear.

So objective function is max of minus x1 plus 4 x2 which is fine such that constraints is minus 10 x1 plus 20 x2 is less than equal to 22 which is fine. Because these are the constraints which you want to take, this one. X2 is greater than 0 that means it will be 0, 1, 2, 3, 4 till infinity all integers, x1 would be only 4 and 5. Now having said that this is an integer programming we will relax it and that relaxed LP is written here in front of you which is maximization problem remains the same.

Objective function does not change, constraint 1 does not change, constraint 2 does not change. What changes are the following, this is I said I will remove, I will highlight this is the continuous variables for x1 between (0) 4 and 5 any values inclusive this is the values of x2 starting from 0 to infinity.

Now, again you solve it using the linear programming concept once you solve it your optimal solution is like this you get x1 as 4 so your optimal solution is x1 as 4. So, here somewhere here so x1 is 4 and x2 is 2.9 but obliviously x1 is now integer x2 is not integer so you have to solve it and basically solve it means go for the next step, so this more branching is required. Whether we want to proceed in that branch area not would basically we will be considering in totality as we consider all the branches and the bounds.

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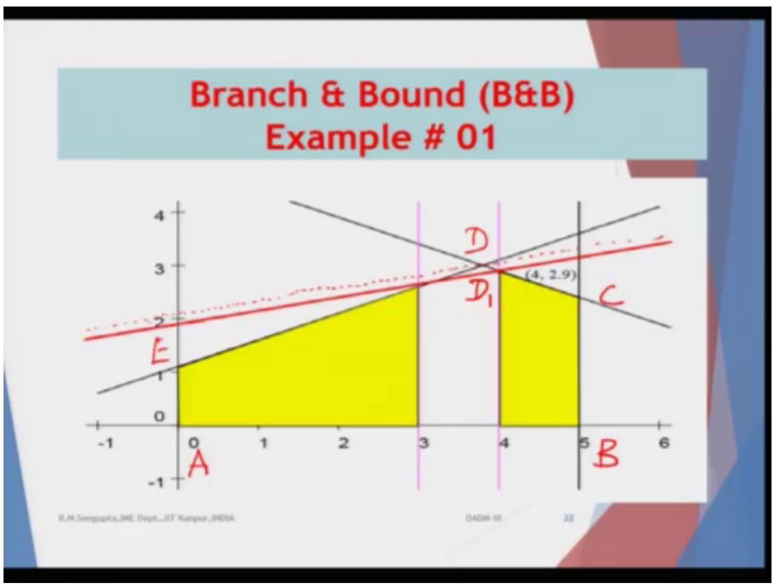

Now this is the solution, now very interestingly this red line which you see (sorry-sorry) I should not be utilizing this one, this red line is which you see is the same one which was earlier for the objective function, same objective function when the overall feasible region was the total yellow region. So it has just I have just brought it down in order to make you understand that when it crosses this overall area or the boundary area which is yellow in the right hand side.

And those points are actually this points you remember initially what is A, this was B, this was C, this was D, this was E let me check whether I have done it right. A, B, C, D, E (yes) but now D actually is not true this is a new point which is the D1, where we are trying to find out the optimum solution. Which you already stated so optimal solution is now z is 7.6, x1 is 4, x2 is 2.9 so this is the overall area.

Now we take I will tell you, so we have taken values of x is greater than 0 here. But if you consider here this values are only applicable for x is x2, is greater so x2 being let me check x2 being greater than 0 but if you consider x2 values can be 0, 1, 2, 3, 4 that will go on. So I need to basically change the feasible the search space of x2 considering that x2 has now have 2.9. Which means x2 can either be in the second branch it can be 3 or 2.

So technically x1 was basically 4 and now x2 was a non-integer now again the branching would be done. So first we will take the value of 3, so here you see x2 is 2.9 so hence it will be considered in the one of the branches.

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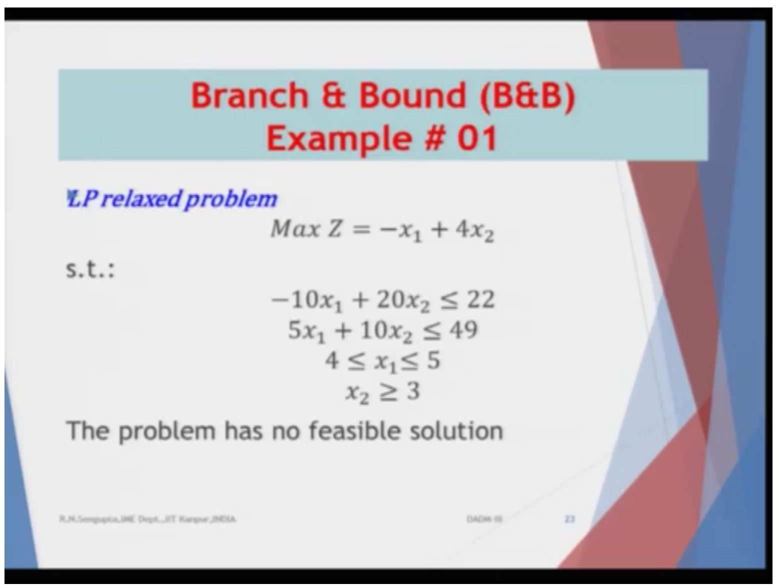

Let us put it, we are actual problem when in the relaxed or the LP problem formulation is like this. I will come back to the actual integer problem formulation once I repeat the relax formulation. So relax LP formulation is max z minus z x1 plus 4 x2 is remains the same minus 10 x1 plus 20 x2 less than equal to 22 remains the same, same means as with respect to the earlier constrain. 5 x1 plus 10 x2 is less than equal to 49 remains the same, x1 is continuous remains the same in the linear programming relax form.

What is interesting to note down is that we have taken x2 divided into 2 regions, 2 less 3 more here we are taking x2 is greater than 3 and if you solve it there is no feasible region, feasible solution. Which is very interesting to note down form the diagrammatic point of view also, go here the point of 3 when x2 is 3 is not inside the feasible region so diagrammatically also you see it is not possible it is outside the feasible region and when you solve it also would not get any feasible region.

Feasible region concept we have already done linear programming and simplex method so you will get it verified by solving the problem. Here I just wanted to highlight the diagrammatic perspective also. That is why I mentioned when you started the problem will take a 2 dimensional problem. So for us to understand it clearly. So the problem would have no feasible solution so obliviously we will try to not obliviously not proceed.

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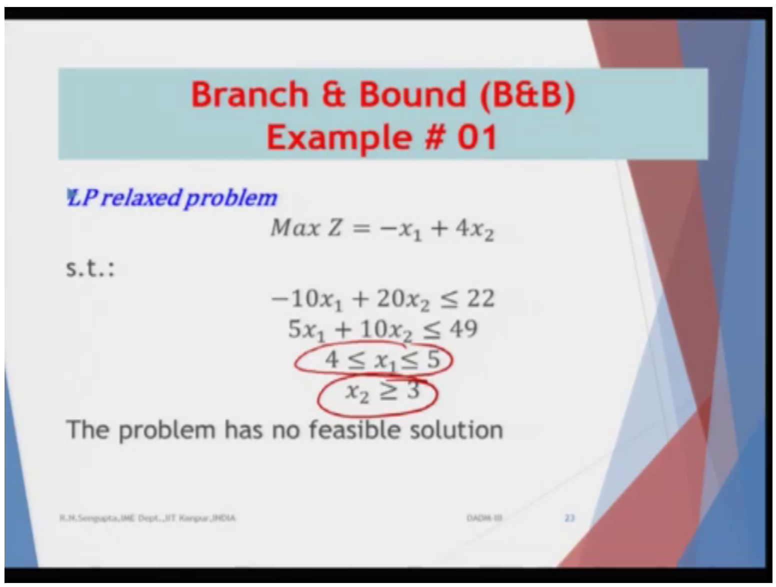

Now by the way what was the I would come to the other solution in the subsequent talk which is continuing, the non-relax I would not using the word non-relax but the integer programming actually is maximum programming is the same, constraints 2 are the same. While x1 has only values 4 and 5 and x2 has values of 3, 4, 5, 6 until infinity. So, that has been relaxed and we have obtained this and this for the linear programming.

So now is what is left as we considered 3 will considered also x2 in the integer programming values of less than 2 less than equal to 2 So, that means it will consider the values of x2 as 0, 1, 2, x1 will remain as 4 and 5 we will relax it continue by solving the linear programming and give the solution according. So, we will proceed in the branch and then take a decision whether there is bound we stop it and proceed in the other direction. Thank you very much and we will continue discussing this problem and the other branch and bound problem in the subsequent 3 lectures definitely in the 10^{th} week, have a nice day and thank you very much.