Data Analysis and Decision Making 3
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology, Kanpur
Lecture 43

Welcome back, my dear friends a very good morning, good afternoon, good evening to all of you where are you in this part of the globe. And welcome to these DADM-3 which is Data Analysis and Decision Making-3 courses under NPTEL MOOC serious. And you know this course total duration is 30 hours, contact hours which is 60 lectures spread over 12 weeks. Each week we have 5 lectures after the 5 lectures we have an assignment and in totality you will have 12 assignments and as you can see from the slide we are in the ninth week that means you have already completed the eight assignments and you will go into the ninth assignment after the end of this week.

So, if you remember in the last example we discussed the production of the cars but very innovatively we converted and brought into the picture a set of 0 1 variables in order to basically divide the total range of the x1 , x2 and x3 which is the production number of cost to be produced for type 1 of variety in which compact similarly , for the type 2 and type 3 and they were basically a discrete region, discrete is not I am talking about that discrete numbers, discrete region in the sense 1000 and above for all the 3 cars and generally or the car they can be 0 or 1000 nothing in between based on when we solved.

We found out that the total profit was 3000 into 2000 depending on the number of car which is being manufactured for only one type but the moment we remove the restriction and the number of car has to be 1000 and above then we see the total scenario changes. So, obviously it will depend on how the problem formulation on the constraints are. Now we will basically as happened in the 41st lecture we discussed started discussing Indian Auto example but I switched to and basically discuss the whole problem 42nd lecture.

Similarly in the end 42nd I was trying to basically start off the discussion of new example again I will start 43rd lecture. So there would be few slide repetition but that will if I did not do that then that will break continuity that is why I am doing that. So, please bear with me.

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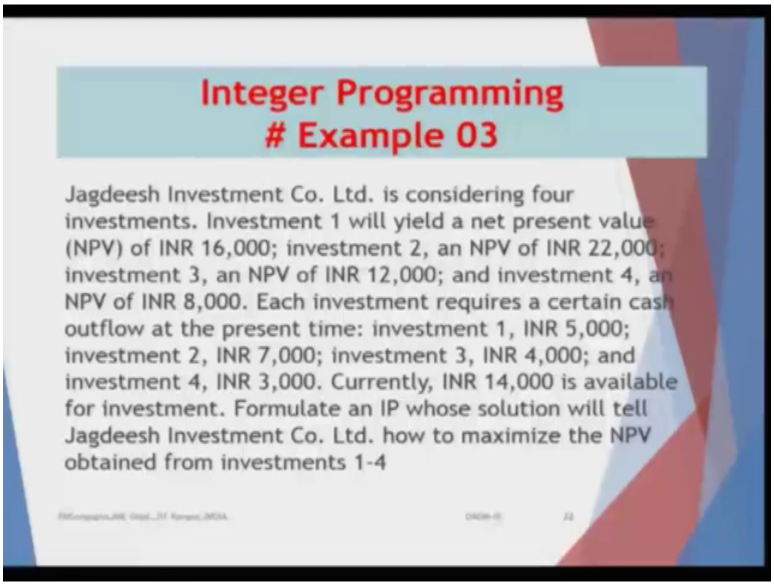

So, overall problem I am again repeating you have heard been 42nd lecture but so that is just repetition and it will make much more clear to you. Jagdeesh Investment Company is a company which is considering 4 investments the net present value from invest 1, I will not consider 1000 value. So it is basically I may or may not depending upon how the problem formulation are been stated by me. All right, discuss the problem. So net present value from investment 1 is 16000, investment 2 net present value is 22000, net present value means the value as of now today.

Similarly, for investment 3 it is12000, investment 4 is 8000. Each investment which is investment 1, 2, 3, 4 requires a certain cash outflows at the present time. So even basically, have to give some cash or cash would be going out of your pocket or out of the pocket Jagdeesh Investment Company limited. So, they are as follows over investment 1 is of now of it is 5000, investment 2 is 7000, investment 3 is 4000 and investment 4 is 3000.

Currently total amount of money which is available in your pocket or available in Jagdeesh Company's 14000 which means that 14000 has to be (proportioned) proportionally depending on what the investments are. So is like this, if you invest in investment 2, 7000 and then you invest in investment 4 which is 3000 and then you can invest in investment 3 which is 4000. So total 7 plus 3 plus 4 is 14000 so you would not have any amount of money left for investment 1. So there can be different combinations.

So let me continue reading it, currently in14000 is in Indian rupees is available for investment formulate and integer programming problem whose solution will tell Jagdeesh Investment

Company Limited how to maximize the net present obtained from the investment numbers 1, 2, 3, 4.

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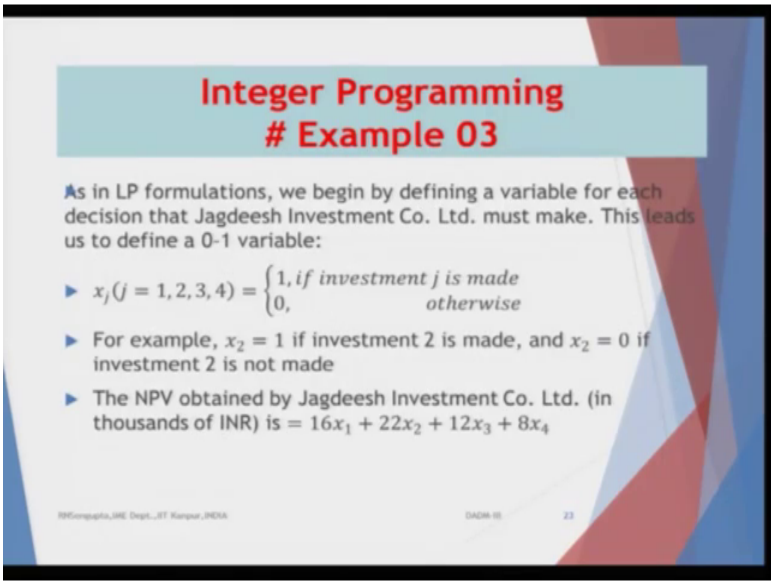

So, as in linear programming formulations we began by defining a variable for each of the decisions. So, what are the variables? It is x1, x2, x3 and x4. So, what are x1, x2, x3, x4? Technically they should have been consider as y1, y2, y3, y4 because we consider y as 01, 01 variables in the general formulation of the problem but we will consider x1, x2, x3, x4. As the decision that you want to invest in investment 1, x2 corresponding to the second investment, x3 corresponding to third investment, x4 corresponding to the fourth investment.

So this leads us to define a 01 variable where x1 is 1, if investment is 1 is made as I have said. Similarly x2 is 1 when investment 2 is made, similarly, x3 is 1 if investment 3 is made and x4 is 1 if investment 4 is made. So for example, I will repeat it x1, x2 is 1 in investment 2 is made x2 is 0 if investment 2 is not made.

So if I consider the total net present value coming out from investment 1, 2, 3, 4 which where respective value of 16000, 22000, 12000 and 8000 the total net present value obtain by Jagdeesh Company investment company would be 16 x1. So, x1 can be 0 on 1 if x1 is 1. So, obviously net present value is there. If x1 is 0, so there is no net present value from investment 1 plus 22 x2 plus (13) 12 x3 plus 8 x4.

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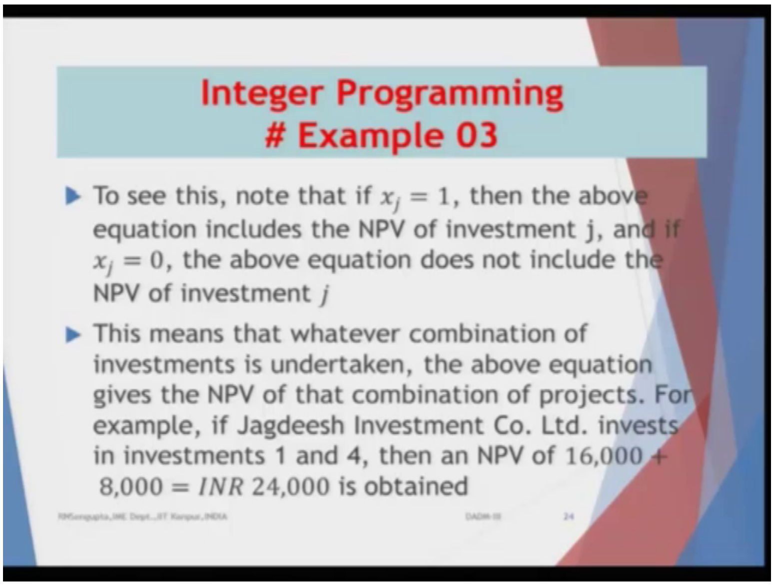

Now to see so if xj is 1 that can be 1, 2, 3, 4 anything then the above equation includes the net present value for the jth one. If Xj is 0 the above equation would definitely not (considering) consider the investment from the 1st, 2nd or 3rd or 4th as j is for the investment. So this means that whatever combination or investment is undertaking the above equation gives the net present value of that combination of the project or the combination of the projects.

For example, if Jagdheesh Investment and Company invest in investment 1 and 2 the net present value will be coming because x1 is 1, 1 and 4 sorry, x1 is 1 and x4 is 1 while x2 and x3 are 0. So in that case it will be 16000 plus 8000 so the total value is 24000 will be obtained.

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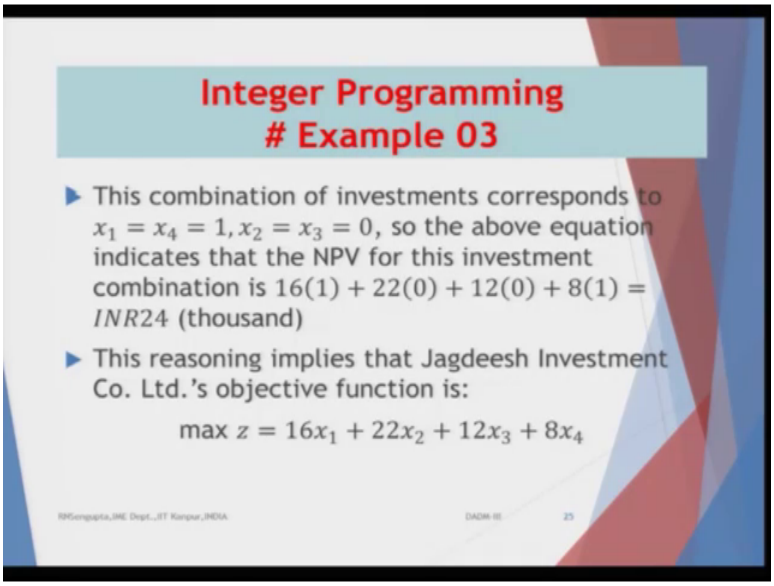

This combination of investment which we just discussed would mean that x1 is equal to x4 is equal 1 and while the corresponding values of x2 and x3 are 0. So the above equation would yield the combination which I just repeated 16 into 1 because x1 is 1 plus 22 into 0 because x2 is 0 plus 12 into 0 because x3 is 0 plus 8 into 1 because x4 is 1. So the total value is 24000. This reasoning employees that Jagdheesh Investment objective function is already as I have stated 16 x1 plus 22 x2 plus 12 x3 plus 8 x4.

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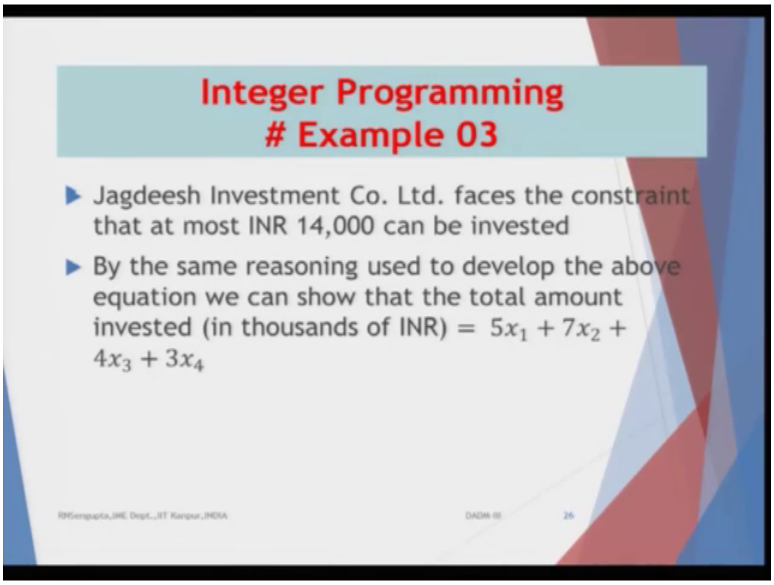

Now the constraints, so which are very simple constraints. So Jagdheesh Investment Company faces the constraints that at most 14000 can be invested. So at any point of time it cannot take out 14000 from it more than that from its pocket. By the same reasoning we use to develop the above equation as said we can show the total amount invested in thousand hours in rupees would be this equation would be true. So go back to the initial problem and stat that why this is equation is true.

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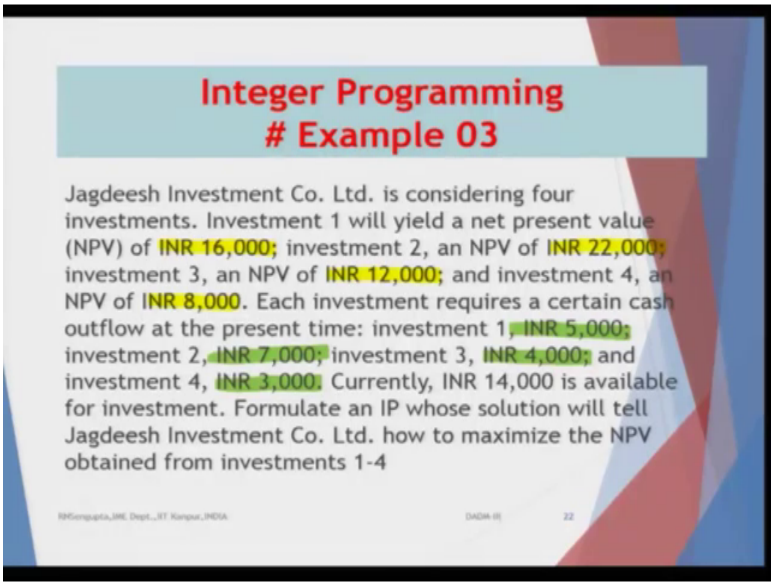

So here I will use two different colours, for the first one is net present value. So, I will use the yellow colour. Now is the total amount of investments, I will use the (green) light green colour, investment 1 5000, investment 2 7000, investment 3 4000, investment 4 3000. So it will be if you invest, so 5000 from 1, 7000 from 2, 4000 from 3, 3000 from 4. You have to take out from your pocket corresponding to the fact that the total amount cannot exceed 14000. So let us see, the values are I am omitting the thousand it is 5, 7, 4, 3.

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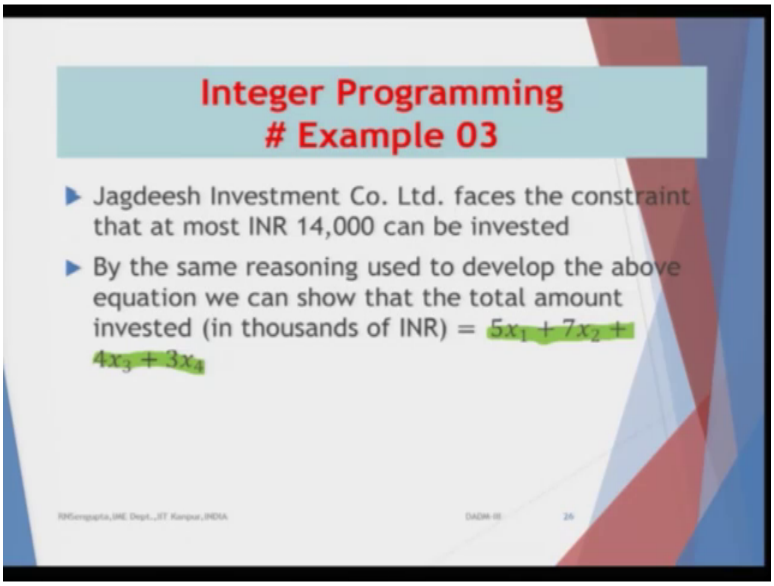

So here it is, 5 into x1 if it is investing in first, 7 into x2 if it is investment in second, 4 into x3 if it is investment in third and 3 into x4 if it is (investment) invested in forth. So, x1, x2, x3, x4 can be 0 or 1 depending on the problem ,not on the problem like the decision which is been taken.

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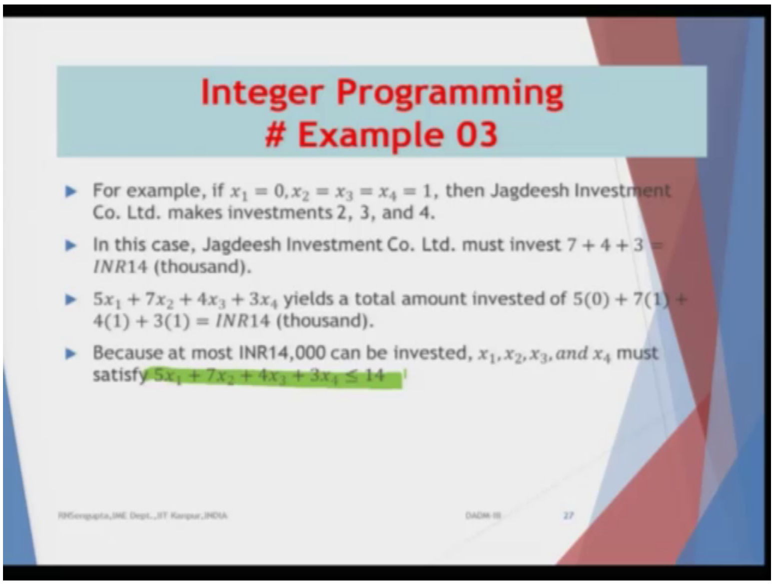

Again, x1 is 0 and if x2, x3, x4 is 1 then means the Jagdheesh Investment Company has made investments in second, third, fourth decisions and not in the first. So in that case Jagdheesh total investment would mean that should invest 7000 in the second, 4000 in the third, 3000 and in the forth. So the total amount of investment would be because it is not investing in the first one.

So it will be 5 into 0 plus 7 into 1. So this 0 and 1 are the corresponding values of the 0 and integer values you are getting, 7 into 1 plus 4 into 1 which is for the third one and the last one is 3 into 1 which is for the fourth one. So the total value of investment which is coming out of your pocket is 14000 and what is the total value available? It is 14000 in this example also. So if I formulate the constraints became 5 x1 plus 7 x2 plus 4 x3 plus 3 x4 is less than equal to 14 you cannot exceed but you can basically spend less.

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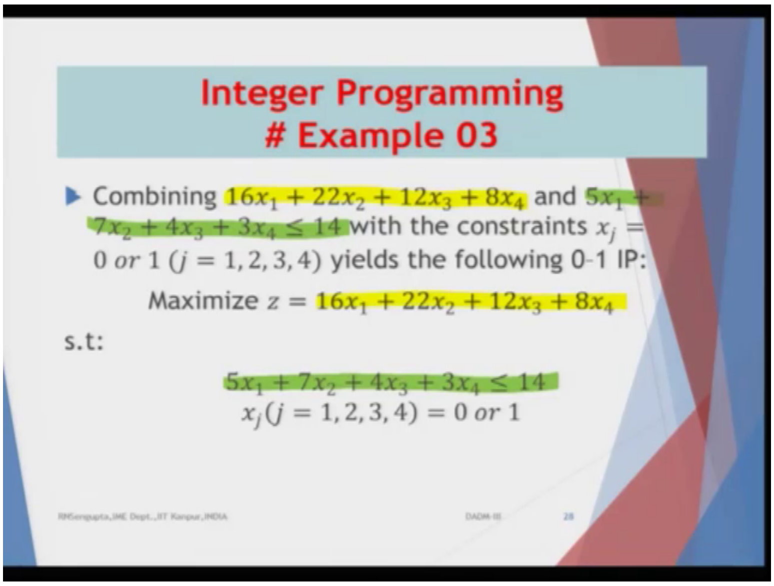

Combining these equations, what are these equations? One was corresponding to the net present value for each and every investment which was there, which was corresponding to 16 x1 plus 22 x2 plus 12 x3 plus 8 x4 plus 5 x1. For 8 x4 for basically the objective function. So I will use the colour yellow as I have been using, so this also comes here and the constraints are corresponding to the green colour which will be less than equal to 14, is this true. So this yields this 01 programming and you have four decision variable 0 and 1 based on which you can solve.

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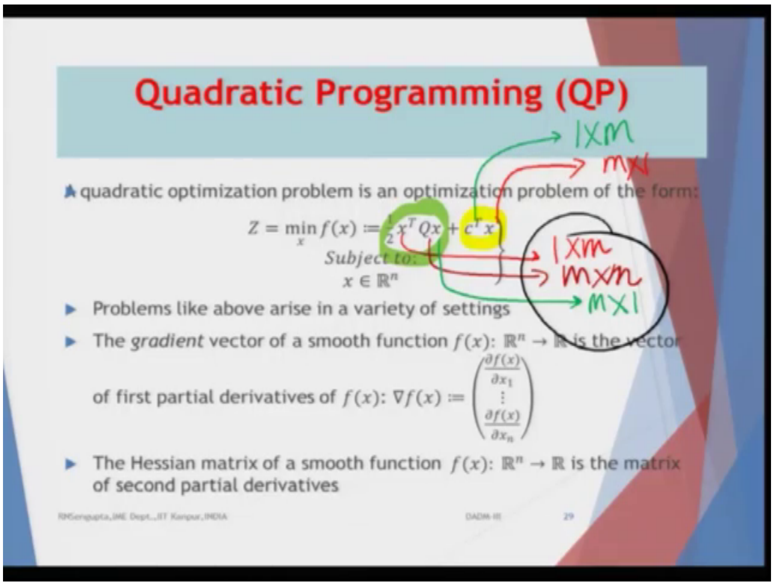

Now I will come to the concept of quadratic programming and based on the general formulation some ideas I will just build up the ideas and then go into the concept building of that. Now where quadratic programming would be important? And there is a background for that. So consider that you are trying to solve a problem. So whenever you solve a problem you have to either maximize or minimize. When you are trying to maximize or minimize forget about optimization. Generally, what we do?

We basically try to use the concept of dy dx or differentiation try to find out that the value is 0 and then at the second derivative try to find out where there is positive and negative based on that we can pass on the judgements that weather is maximization or minimization. Now if you also remember that if the second derivative is also 0 it is basically the point of inflection where the rate of change of that function is changing from positive to negative and negative to positive. So if you remember seeing the normal distribution curve.

So, the normal distribution curved and I am talking about the standard normal distribution curve the mean value is 0 and the standard division is 1. So the point where on to the right of the value of 0 if you are going to the right hand side or you are going to the left hand side. One standard division movement from the mean value or the median or the mode also because for the normal distribution mean, median, mode are the same values.

So, one standard division moving to the right and one standard division movement to the left would basically bring those points in the curve where the rate of change of the function changes from positive to negative and negative to positive depending upon which side of the equation you are looking that means of which side of the curve you are looking on the left hand side or the right hand side.

Now, in quadratic programming we will try to utilize I am not talking about the point of inflection which you will be utilizing in quadratic programming but we will try to utilize the concept that the rate of change of the function whether if it is dy dx is positive or negative and if it is 0 then in the second case we will try to utilize the second derivative. So but, the issue is that in many of the cases we have basically the multivariate cases.

Multivariate cases means there are more than or multivariate decisions variables that means there more than one number of decisions variables it can be any number that means x1, x2, x3 till xn depending how the problem formulation has been done.

Now, a quadratic optimization problem is a optimization problem where there would be two parts. One would be the quadratic part which is this and one is the linear part which is this one. So you have the quadratic part has the half. So, this t x to the part xt is the transpose remember that. So that would be there be a vector column vector or the row vector depending on that how the formulation has been done but remember one thing if that if x q and x transpose are all of them are like these.

So if x is basically a vector column or row that is immaterial now. If an x transpose obviously as x is a row or a column vector and q is a matrix. So in that case the multiplication which you are going to do should basically have the similarity or the compatibility or the concept or rows and columns would be similar. Which means if x is (the value) and obviously the z value which you are going to get at the end would be one value. So you want to basically maximize one scalar quantity. So in the case if you want to formulate.

So x transpose would be say for example of size (m cross m) 1 cross m then the size of q would be m cross n but here even though I am writing n it not need has to be and it has to be the of the same size because or else the matrix multiplication would not be compatible. So let me write in m and the third case would basically be the value of x. So if x transpose is m cross 1 so it will be 1 cross m it will be m cross 1. So when I multiply the total values it will be 1 cross m into m cross m is 1 cross m and when I am multiplying 1 cross m into m cross 1 it will be 1 cross 1 which is a value.

Similarly, when I go to c transpose into x, I already know the value of x value means the size of x as m cross 1. So the value of c technically would be 1 cross m is a transpose. So 1 cross m into m cross 1 would be another 1 value. So you have basically a quadratic part because this is a square and 1 is the linear part. So what can be, I will give you an example. I will come to the general solution mythology later on. Consider you have, I had discussed a very small problem in DADM 2 which I will go into details in DADM 3.

So consider you have one portfolio and the portfolio consists of n number of stocks in very simple sense and it is given that the prize or the returns of each and every stock is given on an average and I want to find out the decisions variables like this I have 100 rupees or I have say for example whatever amount it is there. I want to invest in this stocks. Consider I am formulating the problem as the proportion of investment. So the proportions of investments are W1 to Wn and this 1 to n are all are suffixes which I have already mentioned when I was starting the last lecture.

So all are suffixes, so what I want to find out is that the average return of the combination of the investment which I am going to make in the proportions of W1 to Wn in the stocks 1 to n where the returns are given by r1 to rn that should be maximized. So that would be corresponding to the yellow part which is there, the highlighted one which is the linear part. Now what is by the way do not concentrate on the minimization, maximization I am only talking about the problem formulation. What would be the quadratic part?

Because we know that whenever we want to basically do such combinations in portfolio management. We need to basically minimize the risk. So what is risk? Technically risk is considered as the variance. So if I have basically n numbers of stocks the total variants would basically be considered taken into consideration the variances of each and every stock plus the co-variances which exists between the ith and j stock. When I am talking about the co-variances. Generally I would have the co-variances has a n cross m matrix because n is the number of stocks.

So where the principle diagonal is the variances of the first stock. So the 1 cross 1 element is the variance of the first stock, 2 cross 2, 2 cross 2 means the second element 2, 2 is the variance of the second element. So and so forth till the n, n which is the variance of the nth element and the off the diagonal element are basically the co-variance of the first ith to the jth and the mirror image of that would be the co-variance of the jth to the ith. So if I consider this then it is technically is a quadratic function because generally, the variance concept is what?

It is basically the expected value of x minus mew whole square. So that means I am trying find out the expected value of the differences of the random variables from its mean value. Now the cons, the idea which I have told to not to basically consider is in depth and ignore is because the minimization concept or the idea which is written. So generally, whenever you are investing in stocks your main idea would be to maximize the return which is the yellow part highlighted.

I will write it in this lecture as I proceed I will create a new blank PPT and write it. And the second point is basically the green part which is highlighted which is basically the quadratic part which you have. So I come to the other three parts later on but let me first write down the problem in its practicality such that you can understand. So I will just discard it (so it will be removed)

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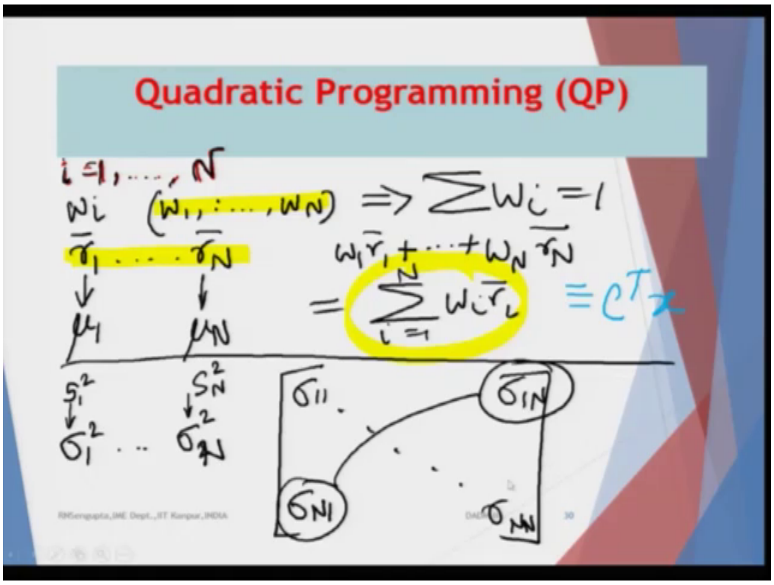

It is like this. So you have i is equal to 1 capital N or small n whatever it is. So I will use the black colour but I will to write then I will highlight it in order to bring concept of the yellow colour and green colour which have been use (let me use the). So these are i's investments and wi. So which means here w1 which will also imply summation of W i this are the proportions remember is 1. Because w1 till wn should be 1.

Returns of each stocks are given by r1 bar is basically the sample average you are aware of what sample is but I will use the bar or without the bar depending on that. So this is the best estimated mew1 which is the return from the population return from the first one or the first stock. So this is rn bar, this base estimate is Mu n. So when I formulate that c transpose x. So, x are basically the w1 to wn. So will highlight using the yellow colour. So these are C's and I need to find out I no I re-interchange it because if I consider x as the decision variables. So, I will consider w1 to wn as the decisions variables. So, these are the x's and if I consider the c which are the return I will consider this are the returns. So if I combine, it will be basically w1, r1 bar plus.

So I am basically writing all of. So this (will be) so this is the ctx, ct means, c transposes x. While if I come to so this is a kin to ctx (somewhere the colours are not very) so this is ctx which I have utilized. Now when I come to the quadratic part, so now I have the variances I am only talking about the variances not the co-variances. So, there best estimate would be s1 squares till sn squares so this is n. So this is and the co-variances variance matrix is also there so it will be like this. So I am writing in the population values.

So I can replace them with the estimates from the sample but in order to make life simple for us let us write so it is sigma 1,1 which is 1, 1 element, sigma n, n which is n, n element and off the diagonal element is given, So, this is basically n1 this is 1n. So, they are symmetric. So this is basically the variance, co-variance matrix which you have. So, what you want to do you want to basically minimize this. So, what the formulation is? I will create another one (keep)

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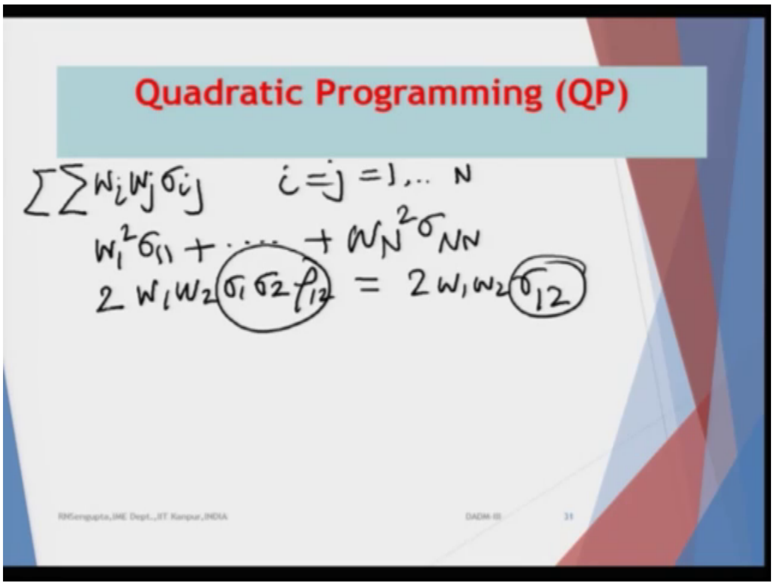

So your formulation is because weights will be coming. So, as it is a variance it will be squared. So what I actually would have for the variance for the whole portfolio would be Wi Wj sigma Ij double summation for i is equal to, j is equal to 1 till n. If i is equal to 1 and j is equal to 1, i is equal to 2, j is equal to 2 you will basically have the whole principle diagonal. So, those values I am going to write for our understanding W1 square because they are squares.

Ij are equal is sigma 1, 1 till the last in that principle diagonal would be Wn squares sigma n, n and of that diagonal elements are mirror images they are same. So obviously our a sign 2 will come and the values if I consider the first row W1, W2, sigma 1, sigma 2 into row 1, 2 or else is equal to W1, W2 row 1, 2. So, basically this is this.

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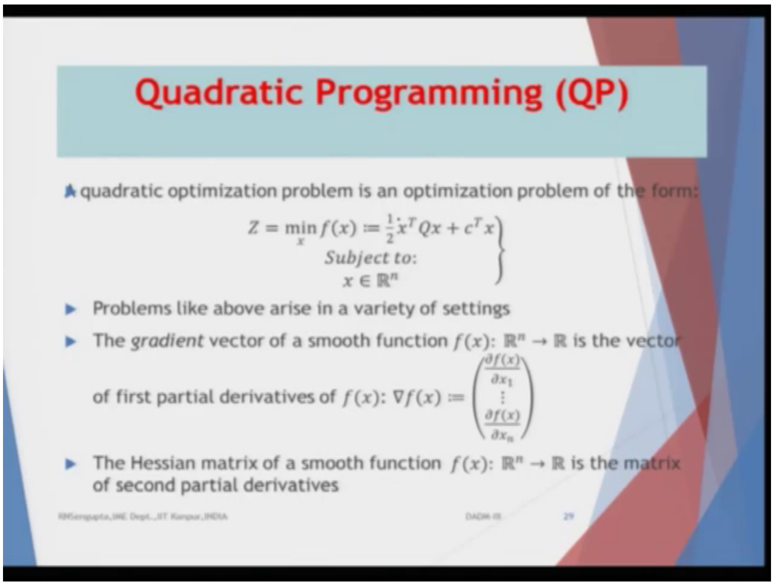

So, if you remember this value of two you have seen was here. So, it is divided by half in order to basically eliminate that.

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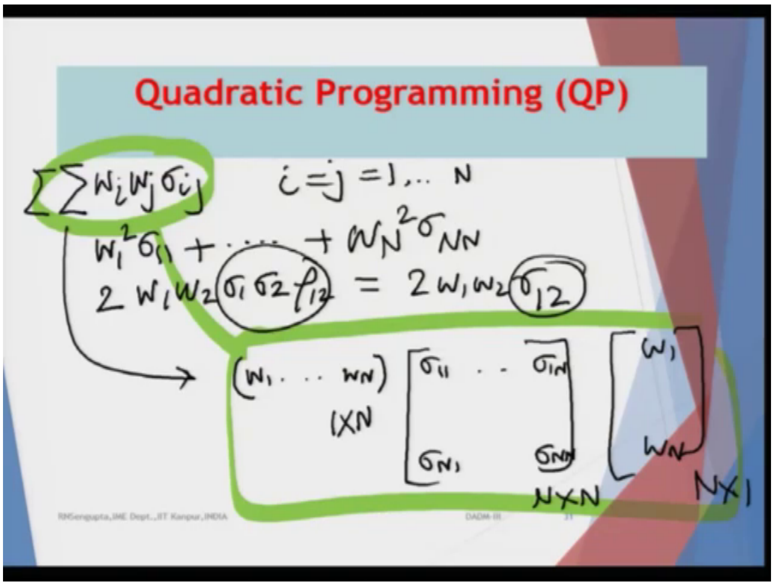

So you will basically utilize this in a sense that you will minimize this whole thing so you will minimize this. So half W1, W2 in sigma 1 Ij. Now, you are thinking how does it make sense with x transpose into q into x? It is very simple, this is what if I basically think as a term. So I would have basically I am just drawing the vectors. So this is 1 cross n, this is n cross n, this n cross 1 what are these? W1 till Wn sigma 1, sigma 1 and sigma n1 sigma n, n W1 Wn.

So if you see the whole value here this whole thing is exactly this or equal to that half sign is coming half into x transpose q into x. I will continue discussing such problems later on but I thought I will give you in practical example and come into the general properties based on which we will proceed. Have a nice day and thank you very much.