Data Analysis and Decision Making 3

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Lecture 42

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you and as you know this is the DADM-3 which is Data Analysis and Decision making-3 course under NPTEL mock series which is 30 contact hours which when broken down into number of lectures is 60 because each lecture is for half an hour.

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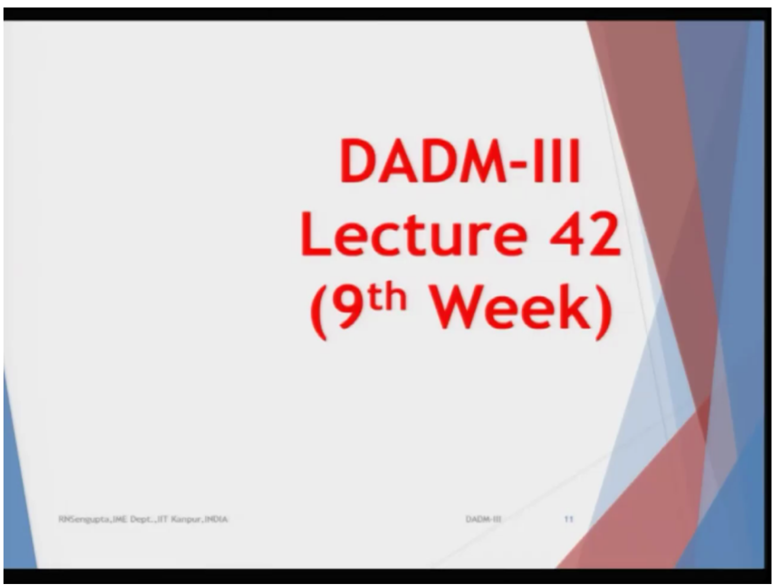

And we have if you can see the slide it is the 9th week going on, so you have already completed 8 weeks, each week with 5 lectures of half an hour each and you have already taken such 8 assignments after the ninth one you will have the ninth and so and so forth till the 12th one and after the end of this course you will have the final examination.

My good name Raghu Nandan Sengupta from the IME department at IIT Kanpur, so if remember we were discussing about general formulation of integer programming and I am again repeating that the concept of trying to formulate the problem in the right perspective with the decision variables, objective functions, constraint is the main idea in operation research but we are more discussing in this course about the methodology of solution. So I will try to basically highlight few of the points for modeling which will also give you some idea that how innovative the modeling can be.

In the last example which we did in the 41st lecture was basically trying to plan the location of the fire brigade considering there are 6 cities in a district in any part in India and there is some minimum time or maximum time which the fire brigade engines can take to reach from city 1 to city 2 or so and so forth and based on that we found out that where the fire engines would be build up.

At the end of the 41st lecture we started the discussion about Indian Auto company and is basically going to manufacture three different types of automobiles or cars and we did not finish it, so I will again restart the examination of that problem such that you will understand the nitty-gritties and the innovativeness of how the overall methodology of formulation is done, while the methodology of solution is already discussed which is basically the (())(02:35) and the branch inbound I will come to the branch inbound later.

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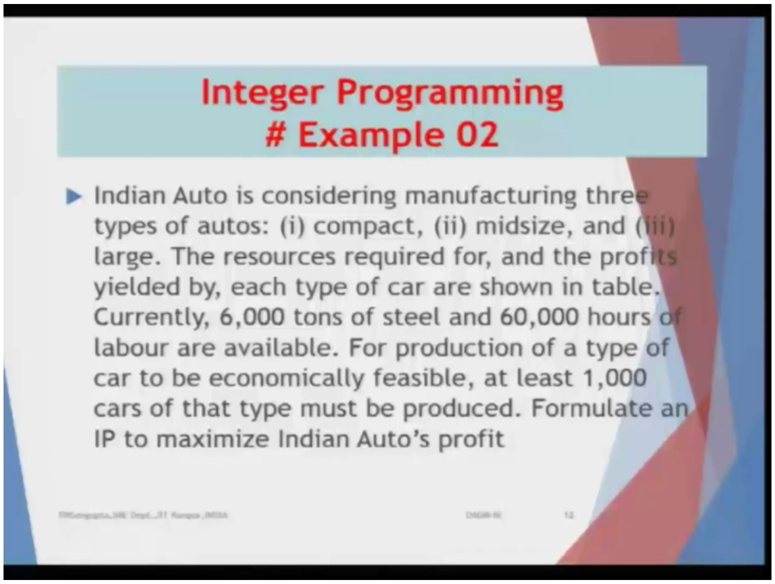

So the problem is again like this you have already seen it but will still repeat, so please bear with me, Indian auto is considering manufacturing three types of autos or cars which are the compact, the midsize and the large one depending on the sitting capacity, the mileage, so and so forth. The resources required for and the profits yielded by each car are shown in the table which we will follow.

Currently in the next slide, currently 9000 tons of steel and 60000 hours of labours are available, so that requirement of steel, requirement of labour can be per unit time, so that unit time can be either week or a month or a quarter whatever it is. For production of a type of car to be economically feasible, at least 1000 cars of that type must be produced, so if you are producing compact or midsize or large you have to produce 1000 or more in order to be profitable because for manufacturing each car there is a cost plus there is a selling price.

So over and above thousand, thousand and above would definitely see, make it into a reality that there is some profit. We will need to basically formulate an integer programming problem and in order to maximize Indian auto's profit, so here the overall idea that what should be the objective function has already been stated is basically maximization problem and we need to basically find the overall profit for the same.

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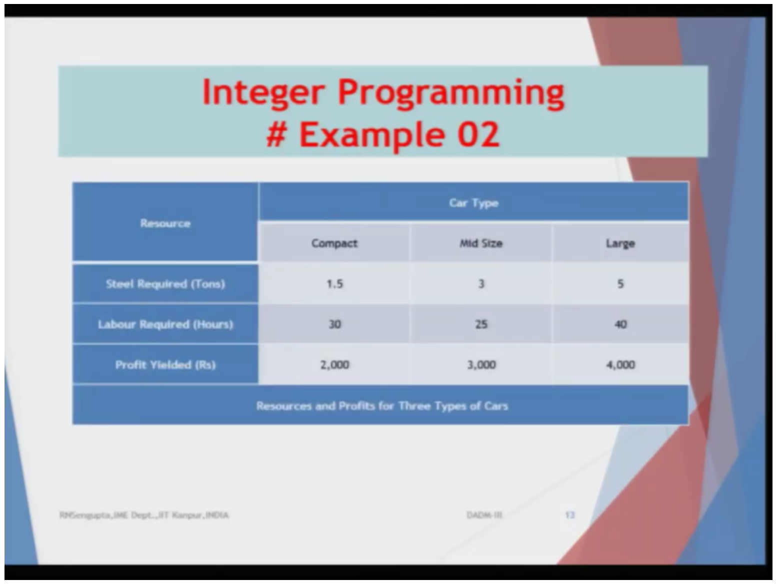

So as I just mentioned when we were reading the last slide the details about the compact, midsize and large the steel requirement, the labour requirement and the profit yielded in rupees would be basically discussed and here are the figures of the data. So in the first column you have the steel requirements in tons, labour requirement in hours and profit yielded in rupees and on the topmost row you have the car type which is compact, midsize and large.

The corresponding values for steel requirement I will write read the values, it is 1.5 obviously the units are already given in the first column so I am not going to repeat it, it is 1.5, 3 and 5 for compact midsize and large. The corresponding values for this type of cars for the labour requirement is 30, 25, 40 and the profit which comes out that means selling price minus the cost price when all the cost component included which is subtracted from the revenues is basically for the each car for the variety of compact, midsize and large are respectively like these values 2000, 3000 and 4000.

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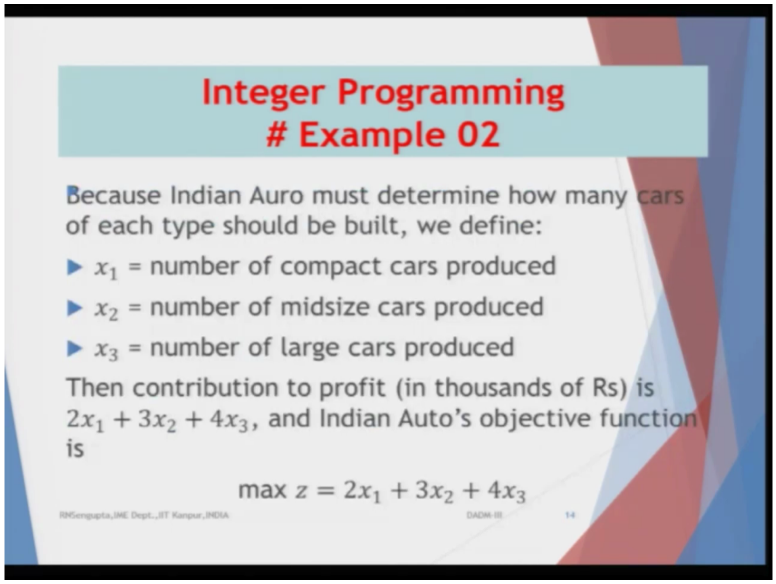

So because Indian auto must determine how many cars of each type should be built, so we define this numbers as follows. x suffix 1, so the 1, 2, 3 whatever I mentioned I have already told time and again they are basically the suffixes and depending on the problem formulation, so you have x1, x2, and x3 which denotes the number of compact cars being produced, number of midsize car being produced and number of large cars being produced.

So obviously, if the profits are 2000, 3000 and 4000 and your main aim considering the profit maximization is the main idea of this problem, the integer programming objective function would be 2x1 plus 3x2 plus 4x3 has to be maximized and we need to find out those values of x1, x2, x3 based on some constraints which will basically be restricted, considering the number of labour hours you have, considering the number of tonnage of steel which you can utilize and also remember that there is a minimum capacity build up which has to be done for each and every car of the three variety in order to basically bring profitability into the picture.

So hence we need to maximize 2x1 I am just repeating it 2x1 plus 3x2 plus 4x3, now what are the constraints? That will come in the next slide.

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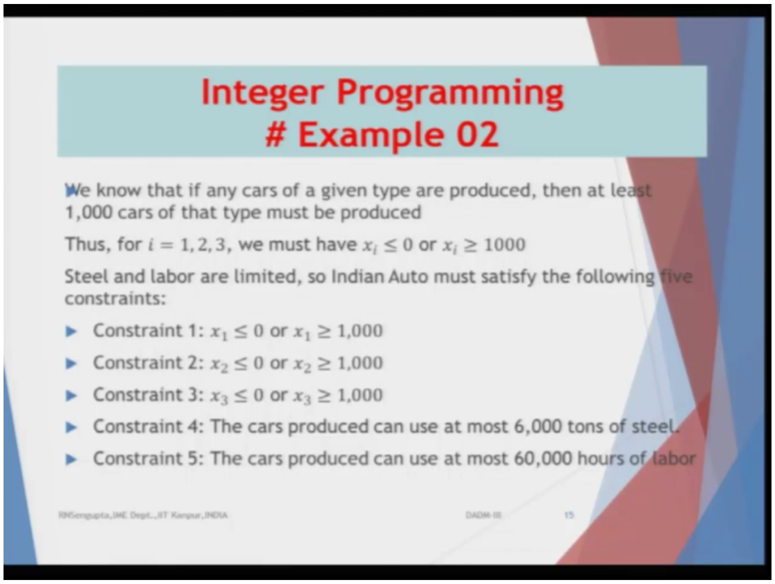

So we already know and we have read it very clearly that if any car of a given type is produced, then at least 1000 number of that car type should be produced in order to basically make a profit. So in that case we know that X1, X2, X3 are the decision variables corresponding to wrote that the type of cars which are being produced, so in that case X1 should be greater than thousand and in case if X1 is 0, so obviously that means it would not have any car produced.

So it can be 0 or some values over thousand like thousand, thousand and one, thousand and two so and so forth. So any number in between is not allowed because that would not bring profitability into the picture for the type of car which you are producing. So the constraints are X1, X2, X3 are less than equal to 0, so obviously they cannot be negative but there values would be 0 and other, in another sense the values of X1, X2, X3 are greater than equal to thousand. That is the first 3 constraints corresponding to the car 1, car 2, car 3 type.

The 4th and the 5th constraints are related to that the tonnage of steel being utilized and the number of labour hours being utilized, so constraint 4 would be the car produce produced can use at most 6000 tons of steel and the cars produced can use at most 60000 hours of labour. So this would also be brought into the picture as the constraints.

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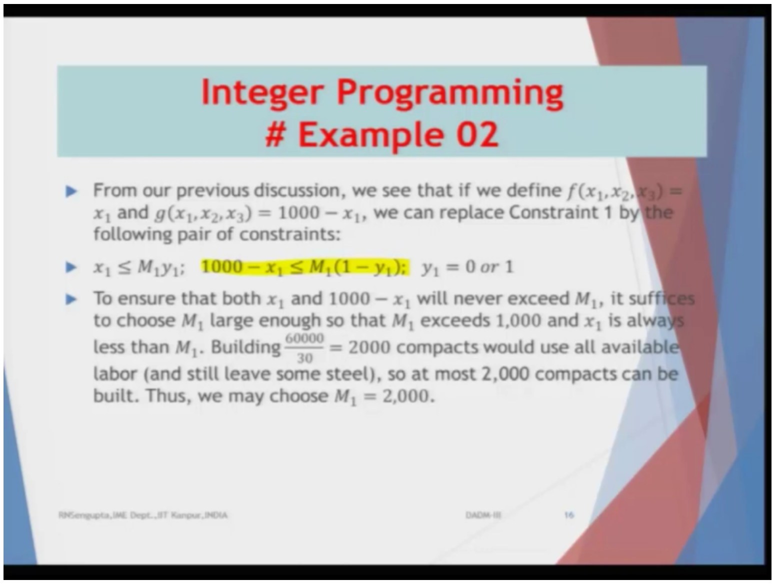

So let us define that will define the functions accordingly in order to make an innovative formulation. So what we will do is that we will replace the constraint 1 by the following pairs of constraints which is like this, we will define the a new set of decision variable Y1, Y2, Y3 and they would be done accordingly. So we will basically find out that X1 is less than equal to M1 into Y1, hence the first constraint would be done in such a way because what we are trying to do is that if you remember the X values were either 0 or they were thousand, thousand one, thousand two so and so forth.

So there was a gap or a discreteness but what we will try do is that we will try to formulate the problem in a such a way that rather than having X we will basically bring Y, so Y would now be a problem formulation of the zero one type, so any production of zero or thousand and above would give the value of Y as say for example 1 and any other value which is not feasible for profitability to be true, in that Y can be 0.

So it can be other way round also that for (not) no profitability Y1 can be or Y's can be 1 and for profitability it can be 0 but we will basically formulate it as for profitability we have Y's as 1 and non-profitability we will have Y's as 0, so this how we formulate.

So if you remember we had thousand as the minimum number, so if I look the equation I will highlight it, if we look at this equation, so consider thousand minus X1 so any values below the value of thousand obviously would not be profitable, so in that case if Y is 1, so in that case M1 is 0, so you will basically M1 into 0 is greater than thousand minus X1 which would basically violate and not violate depending on the equation values of Y1 is equal to 1 and 1 we have.

So any value of Y is equal to 0 or 1 would basically be satisfied by this equation, because now we are changing the decision variables from X1 to Y1, X2 to Y2, X3 to Y3 like this thousand minus X1 is less than equal to M1 into one minus Y1, so if Y1 is 0 or Y1 is 1 you will basically have the values of X1 define accordingly.

Now let us read it further, to ensure that both X1 and thousand minus X1 will never exceed M1, M1 is basically a maximum value of which we are trying to put for the values of X1 or thousand minus X1. It is sufficient to choose M1 as large as possible such that M1 exceeds thousand and X1 is and they this values of X1 would always be less than M1, so if say for example we have the concept that we want to build, say for example 2000 compacts if you in case we want to build. So it will be basically would be available depending on the number of labour hours because the reason being.

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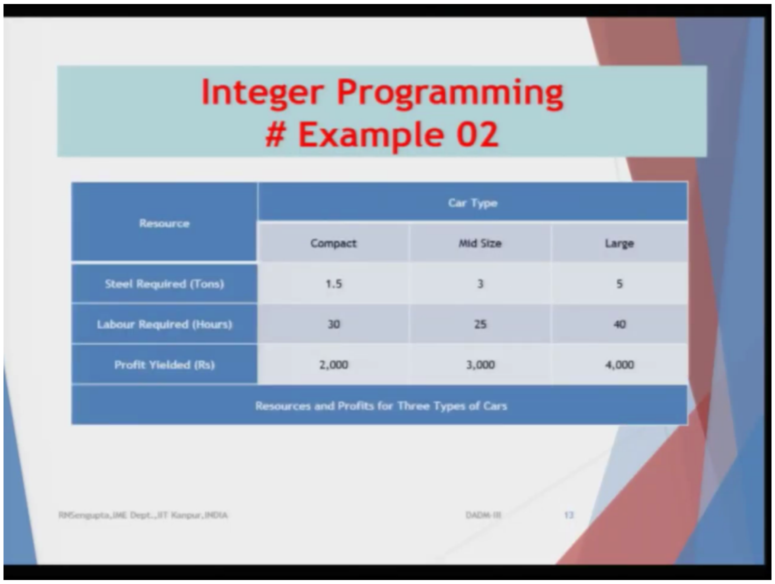

You have 6000 hour, 1000 to be produced, so you need labour hours, so if you basically multiply the labour hours for compact it will be 30 into number of cars which is produced is 1000, so the total number of labour hours utilize would be 30 into 1000 would be 30000.

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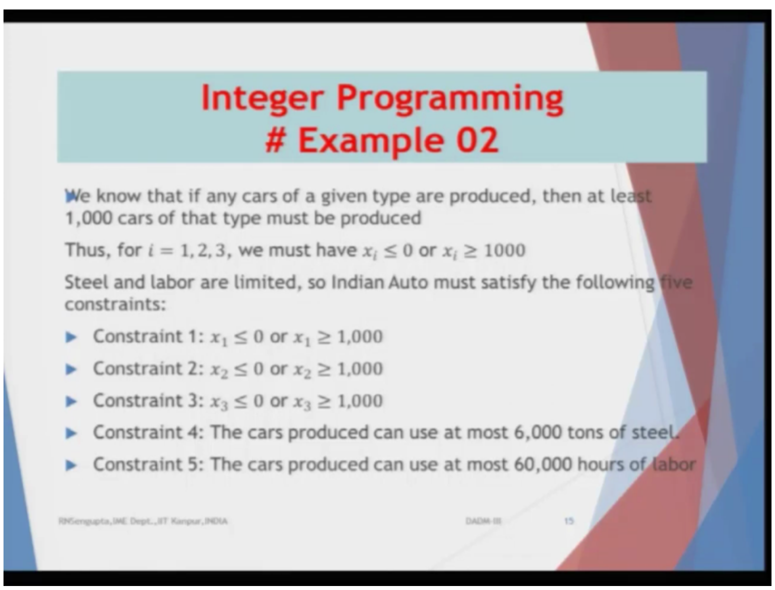

Now having said that you should pause, reason is we have already mentioned that the cars produce can use at most 60000 hours of labour, so depending on the combined one or the single one utilization labours hours it would never basically exceed 60000. Similarly would be the case of tonnage of steel which is being utilized.

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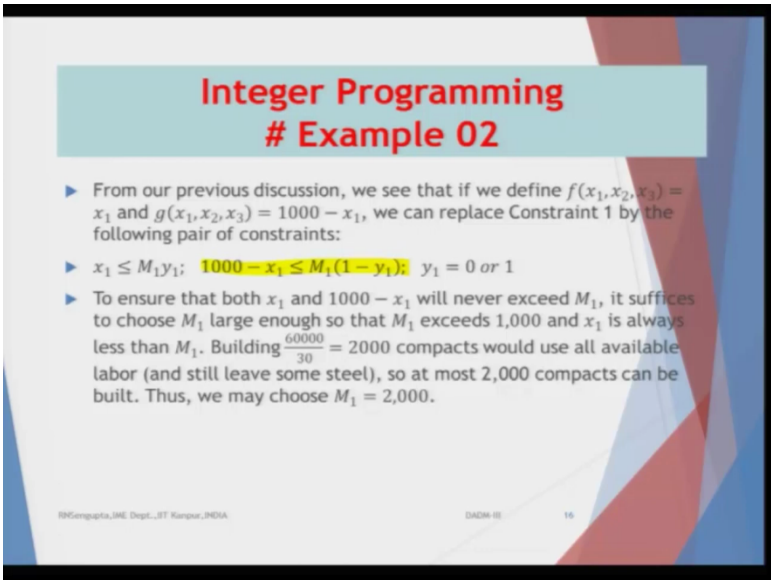

So building 60000 which is the total number of labour hours divided by 30 which is the numbers of hours required to produce that compact one, you will basically have 2000 compacts would use all the available labour, so it basically eats up, so in that case compact eats up all the labour hours number of hours, so the hence the number of labour hours available for the other two variety would be 0 or other two variety of the cars.

But having said that if you basically utilize the total number of labour hours and which means that you are trying to utilize the steel but still steel would be left because only you have eaten up the total number of labour hours not the total quantum of steel which is there, so this is what it states I will read it again building 60000 divided by 30 is equal to 2000 compacts would use all the available labour hours but it would still leave us with some steel, so at most 2000 compacts can be built not more than that.

Because it is violating one of those constraint, so you have to basically check that which of the other two constraints corresponding to the labour hours required or corresponding to the number of steel requirement utilize is being violated for the first time and obviously if one is violated you have to basically stop there.

So at most 2000 compacts can be built thus you choose M1 as 2000 which means that the values X1 cannot exceed 2000 it can be any value starting from 1000 to 2000, so in this you will try to basically find out the limits for X2 and X3 also.

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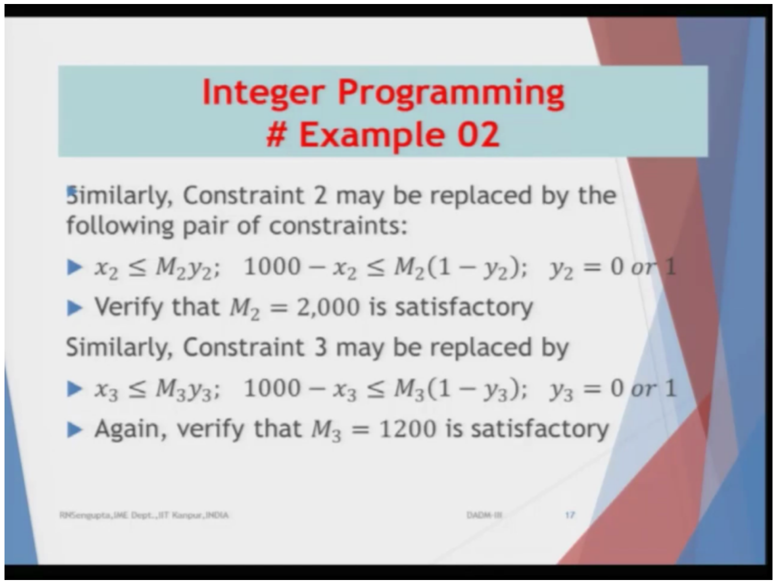

Similarly let us go to constraint 2, constraint 2 can now also be replaced by the same concept because if we remember we have said that we have to produce thousand number of cars in each of the segment in order to bring profitability. So the second constraint would be trying to consider that we are trying to bring Y2 into the as the decision variable in place of X2 such that Y2 is 0 and 1, so the actual equation becomes 1000 minus X2 is less than equal to M2 in the bracket 1 minus Y2, Y2 can be 0 and 1. So verifying that we can basically again verify that the total amount of M2 can be 2000.

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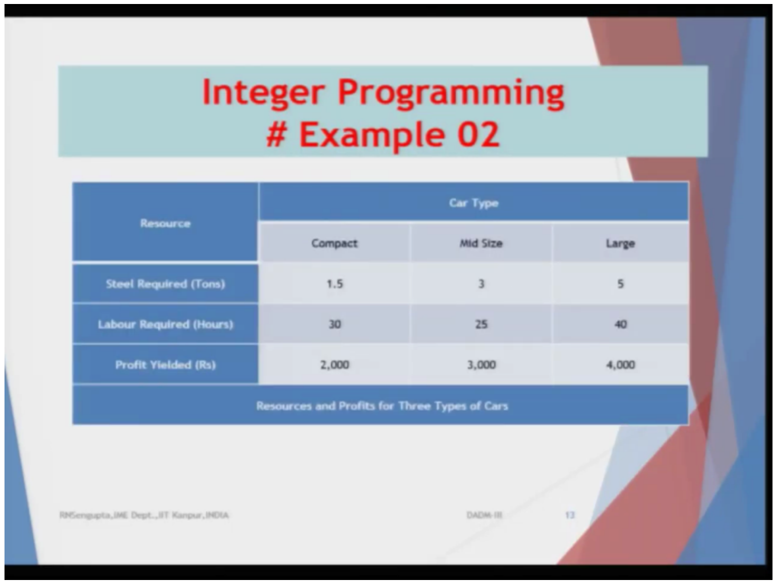

Resource	Car Type		
	Compact	Mid Size	Large
Steel Required (Tons)	1.5	3	5
Labour Required (Hours)	30	25	40
Profit Yielded (Rs)	2,000	3,000	4,000

So here the violation would be basically you have to check both the labour hours requirement and the steel hours requirement, steel required total values based on the that you will say that your requirement for the type, second type of car is basically again 2000.

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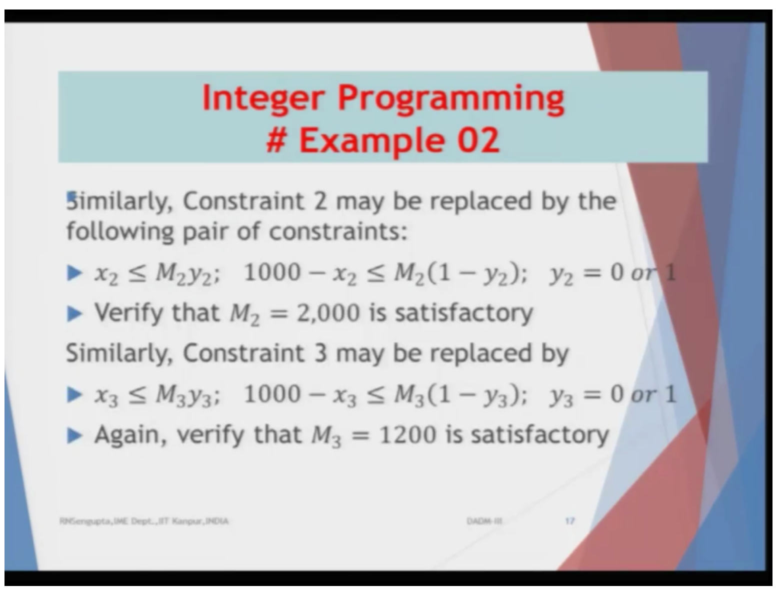

Now let us go to the third constraint, third constraint is corresponding to the third variety of cars, so that it became again 1000 have to minimum produce 1000 minus X3 is less than equal to M3 in the bracket 1 minus Y3 where Y3 can be 0, 1 variable. The moment you basically consider the constraint one at a time which is violated the first based on that you find out that M3 that is maximum number of cars which can be produced in the highest category, in the third category would be 1200.

So 2000 for type 1 car, 2000 for type 2 car and 1200 for type 3 car depending on the violation of which of the constraints is important, so if you remember you have to check the violations as you have done it for the first case.

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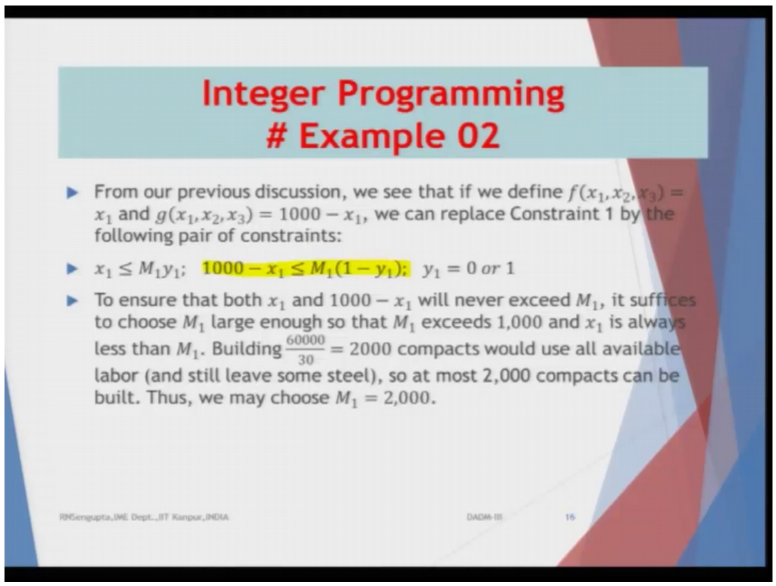

So checking 2000 compact cars will still leave you some steel

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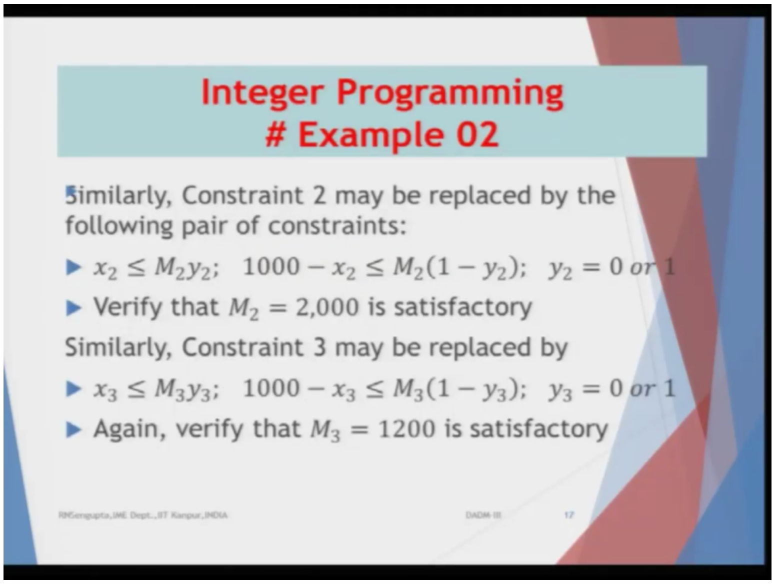

Then you will check again number of labour hours and the steel requirement which is violated first based on that you will say that yes M2 can be 2000, then coming to constraint 3 you will again check labour hour violation, violation means that constraint is not being made and the now amount steel requirement based on that which is violated or that constraint is not made based on that we find out that maximum value of the type 3 car is 1200.

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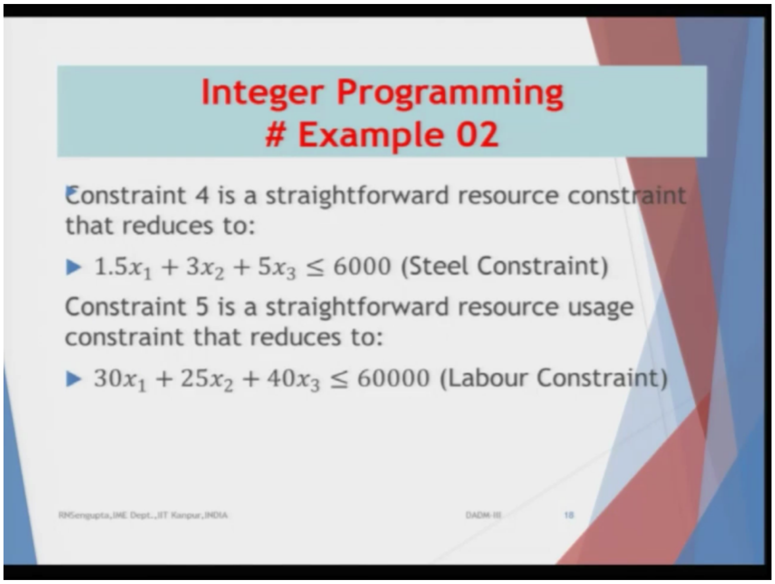

Constraint 4 which was constraint 4 and constraint 5 which was related to the number of labour hours and the number of quantum of steel which was being utilized, so constraint 4 is a straight forward resource constraint because if you remember, now where does this values 1.5, 3 and 5 come? So let me go back to the table.

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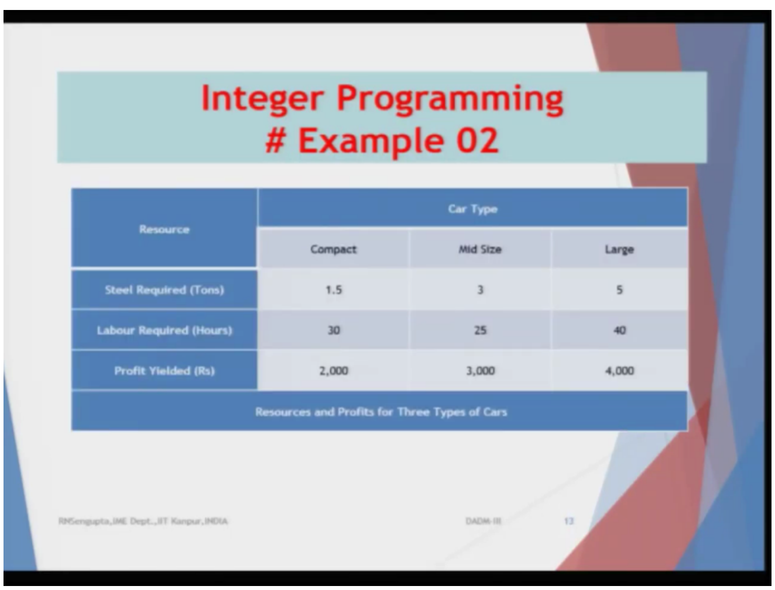

Resource	Car Type		
	Compact	Mid Size	Large
Steel Required (Tons)	1.5	3	5
Labour Required (Hours)	30	25	40
Profit Yielded (Rs)	2,000	3,000	4,000
Resources and Profits for Three Types of Cars			

So the table says that compact car, one car being produced will require 1.5 tons, midsize will require 3 tons and large will require 5 tons, so if you are producing X1, X2, X3 the requirement would be Y 1.5 into X1 plus 3 into X2 plus 5 into X3 and what is the total amount of tonnage of steel which is there? That is 60000, so we will put the constraints accordingly and this is how the constraint look like.

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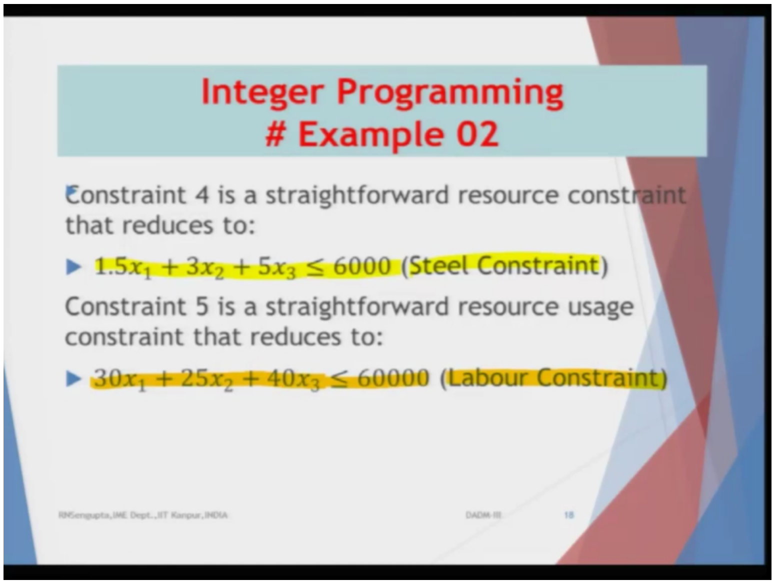

1.5 X1 plus 3 X2 plus 5 X3 is less than or equal to 6000 steel constraints which is there. Now let us come to the last constraint, what is that last constraint? That is related to labour hour requirement, we have already said that the total number of labour hour requirement is 6000 number hour, so let us check that what is the number of hours required for producing each and every variety of the car? So that is 30 number of hours for the first time, I am not going back to the table which you have already seen it number of times, so it is 30 numbers of hours for the type 1, 25 number of hours for the type 2 car and 40 number of hours for the type 3.

So obviously, the total number of labour requirement would be 30 X1 plus 25 X2 plus 40 X3 and that should be less than equal to 60000 number of labour hours which is the maximum which is available for you. So I will just I should basically utilize a different color, so this is labour constraint and this is steel constraint, so you are basically converted the labour constraint and steel constraint corresponding to the decision variables.

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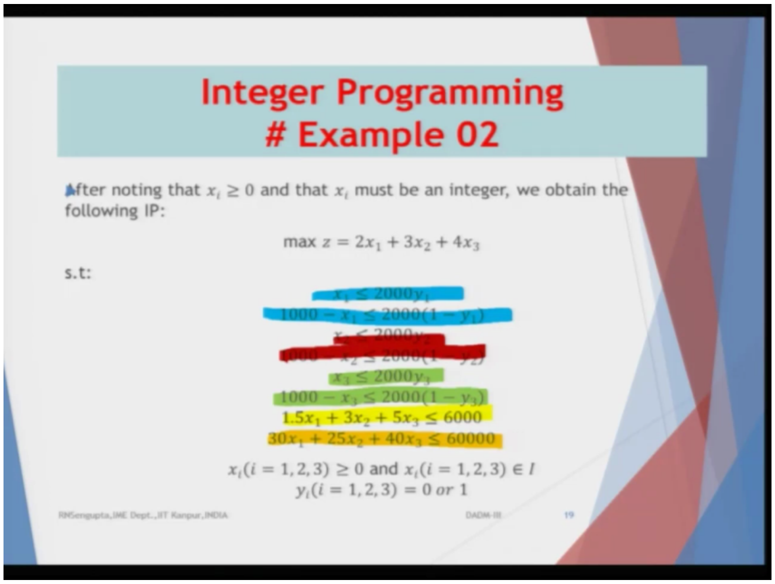

Now after noting that the problem X1 is greater than 0 and X1 must be integer we obtain the following inter programming which was maximizing 2 X1 plus 3 X2 plus 4 X3 is what we want maximize because you remember 2000 was the profit for selling one car of type 1 compact one.

Similarly 3000 in Indian rupees and 4000 in Indian rupees for the third variety and the corresponding values when you find out when you put the constraints, the constraints are like this you have Y1, Y2, Y3 as 0 and 1 depending on whether you exceed or not exceed and the constraint which have been included, I will start from the last part, so this one is the constraint due to labour hours so it cannot exceed 60000.

This is number of hours which is required for the number of tonnage of the steel utilized. So the green marked one light green one is basically corresponding the X3 quantum, so if you look at this values, so on the left hand side that is 1000 is the maximum minimum of requirement of cars we have to produce, so it would be corresponding to X3 or X2 or X1 and here are the data. This for the second one, let me check this are very dark so I will try to utilize this, so this for the first one based on that we solve the problem.

So this is the integer linear programming, integer programming and also zero on programming you solve it using any other methods and you get the answer. So here remember X1, X2, X3 are greater than 0 and similarly you have brought the decision variable Y's which is 0 and 1 depending on whether you proceed, not proceed.

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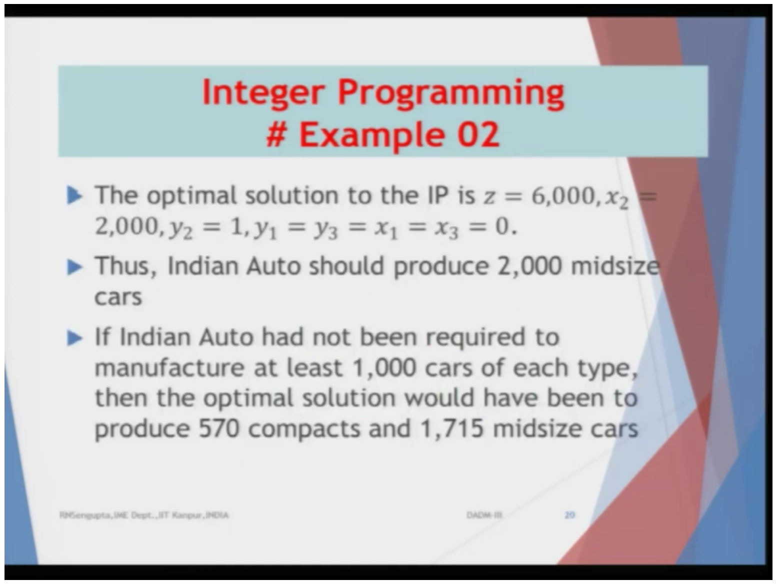

When once you solve the problem the optimal solution to the IP is you get a profit of 6000 and the corresponding values of X1, X2, X3 are like this, X1 and X3 are 0 and X2 is 1000, so which will mean that in that case Y2 which is corresponding to the 0, 1 variable for the second decision variable which is X2 is 1 because you are producing cars there while Y1, Y3 would be 0 because corresponding values of X1 and X3 are 0 that means no cars are being produced other type 1 and type 3.

So let us double check if it X2 is 2000, so what is profit which we had from each and every car which is being for type 2 quality?

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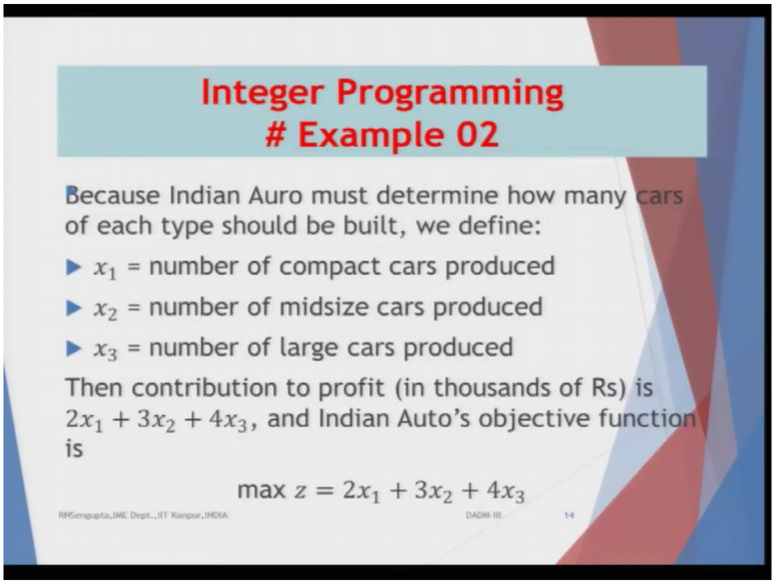

We have seen it was 3000 here in the Indian rupees I am not going to highlight it I am hovering my pen here. So 3000 into the number of cars which are being produced will give you the profit which is 2000 number of cars.

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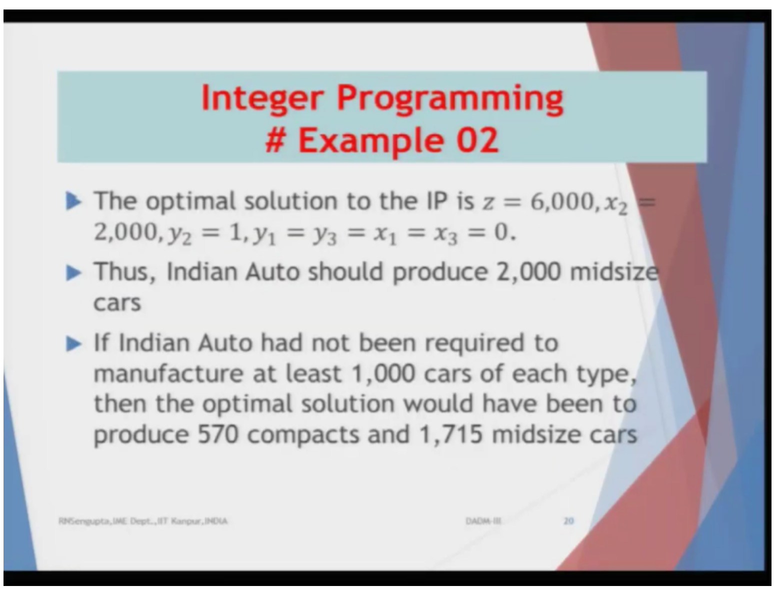

So you will basically have profit accordingly, so that would be 2000 into 3000 and you will basically give 6000, thousand that means six million or sixty lakhs. So thus Indian auto should produce 2000 midsize cars, if Indian auto not been required to manufacture at least 1000 cars of each type, oh remember, so here the constraint was that each car has to be of thousand variety or more.

So anything less than thousand also not allowed, so in that case if it was not there that restriction was not there then the optimal solution would have been that means any number can be produced for the cars would have been (5) 74 compacts and 1715 for the midsize cars, so this is totally a different scenario and output which you will get depend on the constraints which are changing.

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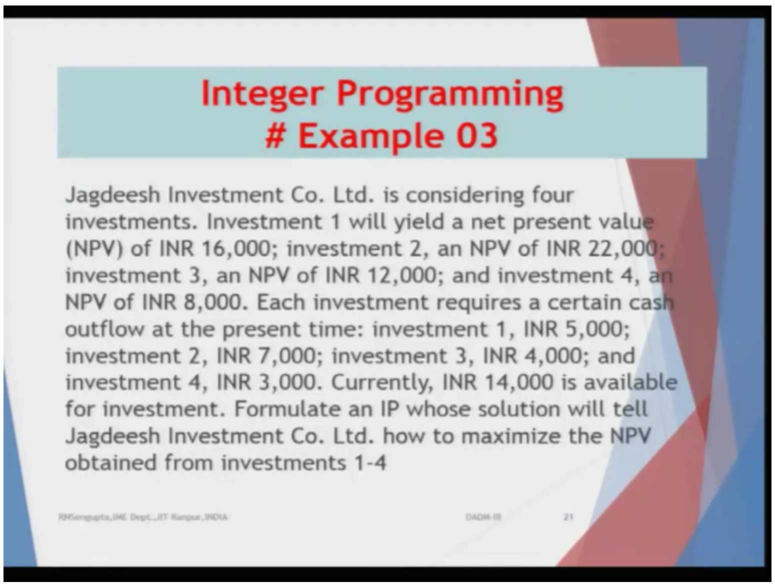

Next we will consider the investment problem and here is the problem which is being formulated, Jagdeesh investment company limited is considering 4 investments, so they want to invest in different investments. Investment number 1 will yield a net present value or NPV of rupees 16000, investment 2 will give a net present value of 22000, investment 3 a net present value of 12000 and investment 4 a net present value of 8000, so each investment requires a certain cash outflow at the present time, so you want to basically utilize that money, so investment number 1, number 2, number 3 and number 4.

The values are 5000, 7000, 4000 and 3000. Currently 14000 this are all, all the values I am talking about are all in Indian rupees, so 14000 available for investment that company. Formulate an integer programming whose solution will tell Jagdeesh investment and company how to maximize the net presentation obtained from the investment what Jagdeesh company is making in the investment 1 to 4.

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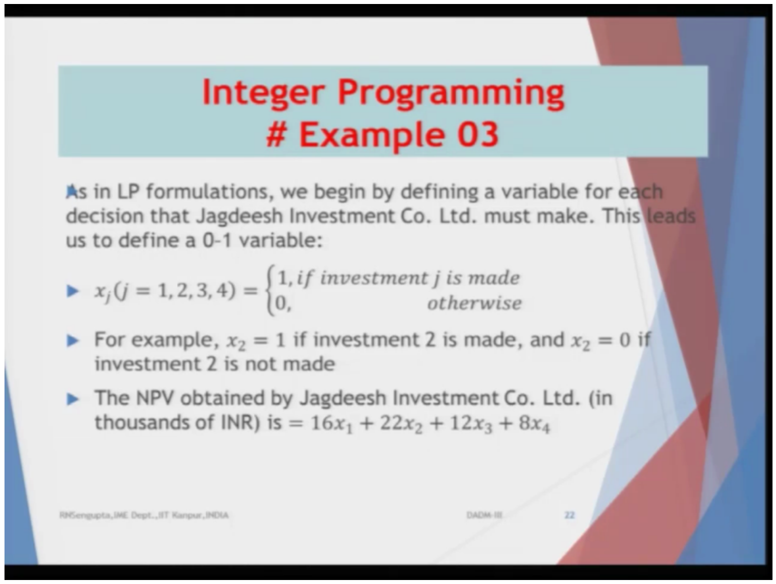

So as in linear programming formulation we need to define, so define either invest not invest the (())(25:21) quantum of investment which is required so and so forth. So as in linear programming formulation we begin by defining a variable for each decision that Jagdeesh investment company must make so this leads us to define a 0, 1 variable, decision variable. So which is what? X1 you will invest in investment 1, X1 if it is 1, X1 it is 0 that means you would not invest in decision 1.

Similarly X2 1 0, X3 1 0, X4 1 0 would mean that you are investing in second option, a second investment you are investing in third investment, you are investing in fourth investment. So all of them are one, actually it would mean that you are investing in all of the 4, if all of them are 0 it would mean that you are not investing in any of the investments for Jagdeesh company.

So for example it states that X2 is 1 and if investment 2 is made and if X2 is 0 if investment 2 is not made. Now comes the net present value, so net present value obtained by Jagdeesh Investment Company is like follows.

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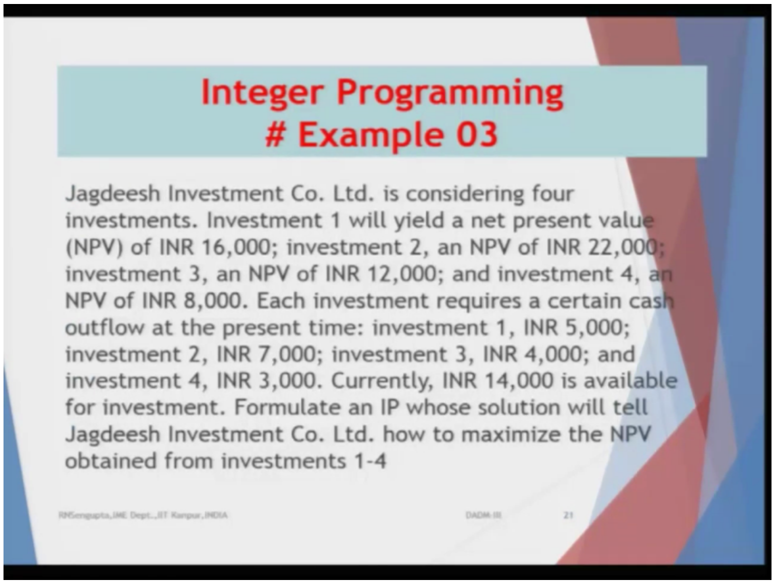

So if you have read the problem, so it gives the net present value for investment one is 16000, two is 22000, three is12000 and the forth one is 8000, so the values would be corresponding 16 into X1 (so let me) so it will be 16 into X1 depending on if it is 1 you will basically get 16000 if it is no investment you will get 0 because 16000 into X1 when X1 is 0 will yield a value of 0.

Similarly the net present value for the second one will be 22 into X2, similarly for the third one is 12 into X3 and the last one is 8 into X4, so all this thousand values are omitting from mentioning.

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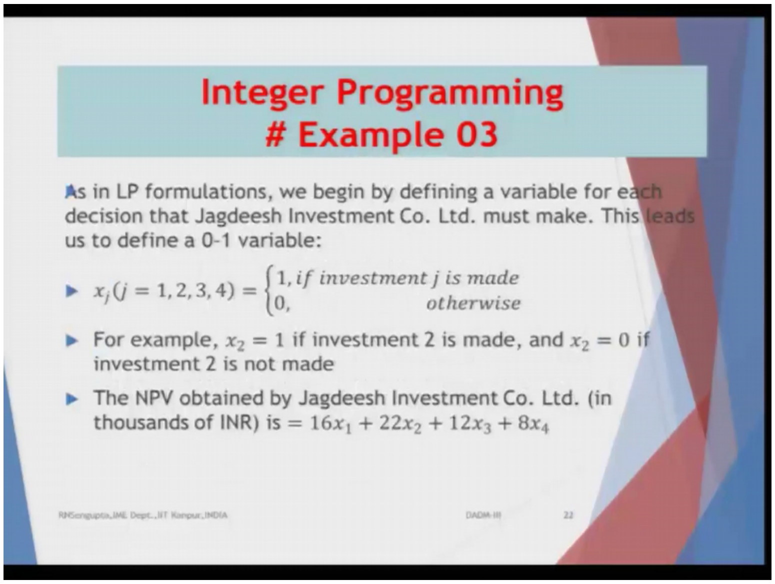

So hence the investment is given now you have to check the constraints also, so I will come to the constraint later on, so considering the time for this forty second lecture is already over I will stop here and discuss the problem in more details in the class which is the forty third class. Have a nice day and thank you very much.