Data Analysis and Decision-Making-III
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Lecture 40

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe. And this is DADM III which is Data Analyses and Decision-Making III under the NPTEL MOOC series and as you know that this is the third part of the lecture series where we first had the DADM I which was statistics and all multi mediate statistical methods.

Then we had DADM II which was to do with different concepts related to the utilization of heuristic methods, metaheuristic techniques so and henceforth and then we have the DADM III which is more to do with optimization and optician research. So this total course is for 12 weeks which is 60 lectures which will convert into number of, contact hours is 30 because each lecture is for half an hour in each week we have 5 lectures each for half an hour after each week we have an assignment, so if you can see from the lecture we are in the eight-week.

So you have already taken 7 assignments after the end of this lecture 40th number you will basically take the 8th week assignments and in totality you have 12 weeks and after that end of the course will take the final examination. My good name is Rahul Nandan Sengupta from the IME Department and IIT Kanpur.

If you remember we are basically discussing the concept of integer programming problems and solutions and how the Gomory cuts would basically be brought into the picture. So just to recap you had one problem, very simple problem which was x1 minus x2 which has to be solved maximization and there were constraints of the less will then type both of them were.

And I drew the diagram in the last class which was in the 39th lecture to show that the integer point was inside the feasible region but about not optimum one or the corner points if you consider the linear programming problem. So first we solve the problem using the concept of linear programming and we saw x1 and x2 were the actual decision variables while exterior next 4 are the slack surplus as needed to be added depending on less than or greater than sign.

So in this case the problem formulation was that we added x3 and x4, x3 the first constraint, x4 for the second constraint and once you solve the problem you found out x1 value was nonzero but not an integer. x 2 value was 0 and obviously we can consider that as an integer. x3 and x4 which technically should be 0 in the best optimum solution we found x3 was nonzero and x4 was basically 0. So here was the solution.

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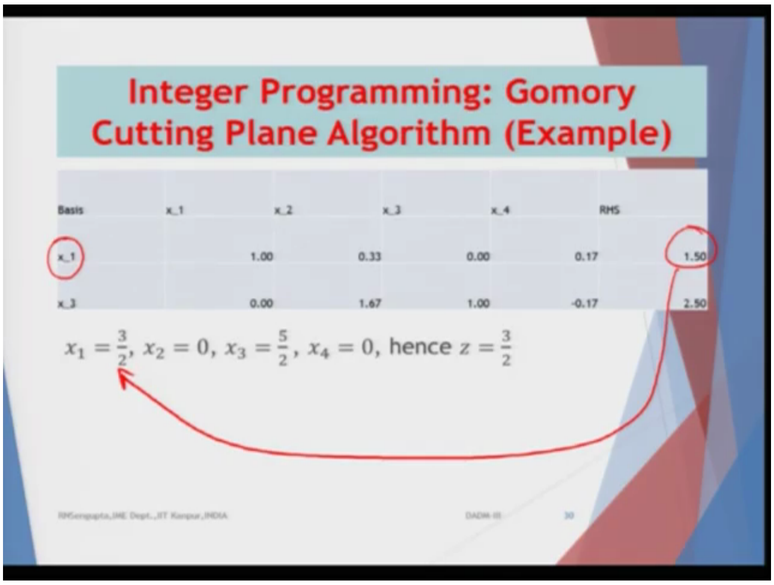

I should have also brought it as a part and parcel of the 40th lecture but still this was the last slide which we are discussing. x 1 3 by 2 actually should not be 3 by 2 some integer I will use. What we do not know we have to solve it. So obviously the integer values can be 2 or 1 or 0 or for whatever it is. 3, x 2 is still 0 if it is nonzero integer it will be interesting to see. x 3 and x 4 actually when you solve the problem should actually be 0.

But it may not be possible considering the constraints how they have been formulated and the Z value is 3 by 2. Now see here very interesting thing. If you consider the first constraint and actually which should not, x 1 value is basically 3 by 2 as we found out. So this was basically 3 by 2. So it should not be, so we will try to basically reformulate the constraint using the concentrate Gomory cuts in order to ensure that the concept of integer programming idea is being brought into the problem formulation.

But why not x 3? Because x 3 is technically is one of the slash surplus where the values how when you end the problem is not going to be basically, it’s basically the leftover part of the access amount you are going to utilize depending on greater than on less than sign and this is not our main concern for us, so you only concentrate on the first constraint.

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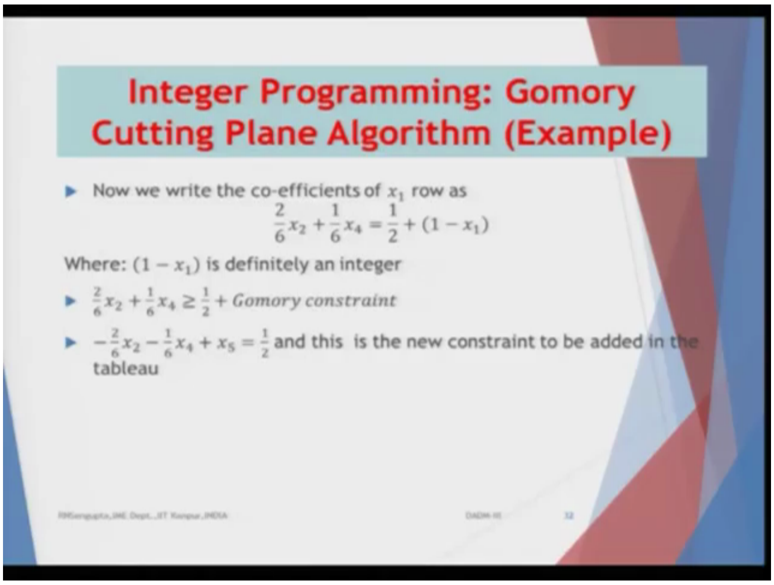

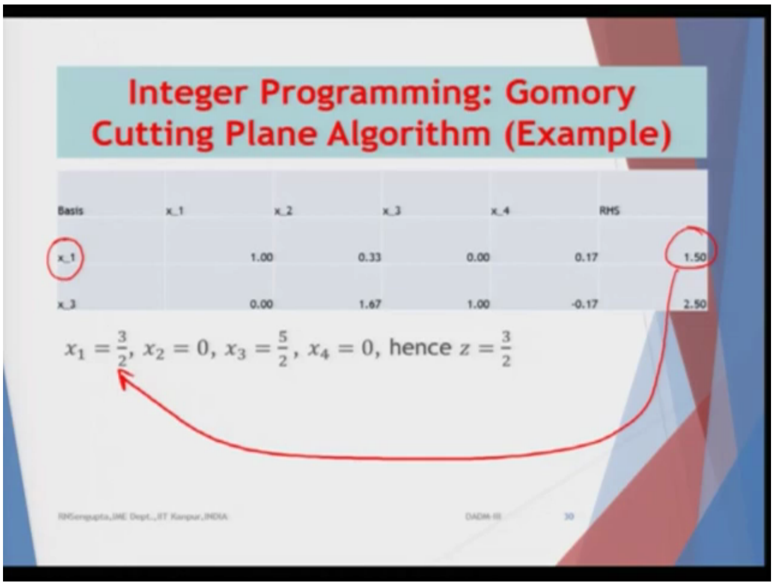

Now we write the first constraint and write this why 2 by 6? Because the problem formation is like this. When you solve the problem, so it's basically 2 by 6 for x2. And obviously another thing the x1 was on the left inside, I forgot to mention. So basically it was half x1, let us see. So it will be, once we bring the constraints and divide by any integer value, so which is basically x2 is one third, x 3 is not there and x 4 value is 0.17. So this is one third.

I am just multiplying by 2 and this value, x 4 value is 0.17 you can find later on it comes out to 1 by 7 this half value is on the right-hand side. So the half value, once we basically bring it on to the right-hand side it is half, so if you solve it and x1 is when you taken to the right-hand side that is x1 by 2.

Now if you solve this problem and if you look at this problem. So we write to the coefficient x1 in this way, if you check this problem the actual value here which is 1 minus x1 should definite to be an integer. So once you have this if it is integer plus some half value, so it is basically increasing by 0.5.

So obviously you will have that the problem formulation in order to solve x2 because x4 is not of prime importance for us. To solve x2 you will have 2 by 6 x2 plus 1 by 6 x4 would be greater than half plus the Gomory constraint, the constraint that we are going to put which is corresponding to the fact that x1 is integer. Now in order to solve this and obviously bring in the less than sign I multiplied by minus.

So it is minus 2 by 6 x2 minus 1 by 6 x4 is less than equal to half and the Gomory constraint whatever. Now it is less than equal to and x5 which is to be added as a new decision, decision variables means the concept of slack and surplus and it will come into the tableau. So once it comes into the tableau you will start a solution, the standard solution constraining the basics feasible solution, so now the basic feasible solution once we do is like this.

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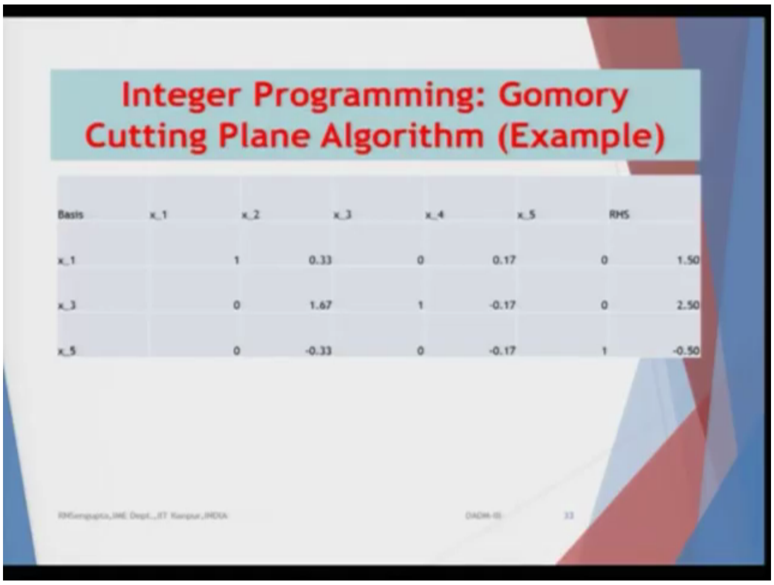

Once we stop, so once we stop you will basically x1 is still 3 by 2. x3 is 5 by 2 and x5 which has been added as the added constraint the gomory, the new constraint has been added or changed depending concept of the gomory cuts is basically minus 3 by 2. Now see here the corresponding value of Z which you have. So let me consider.

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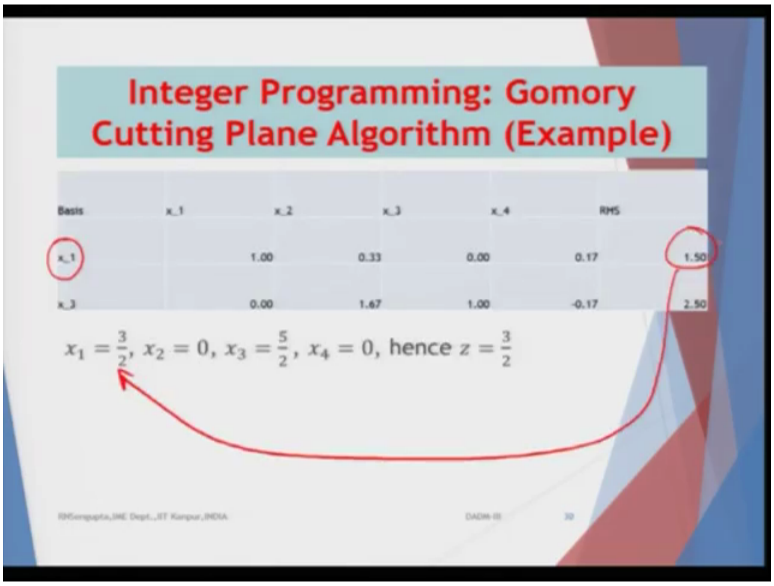

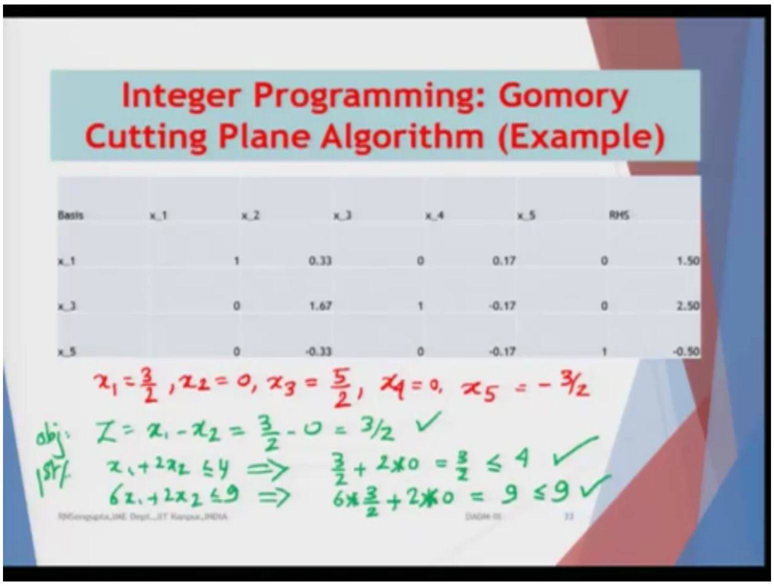

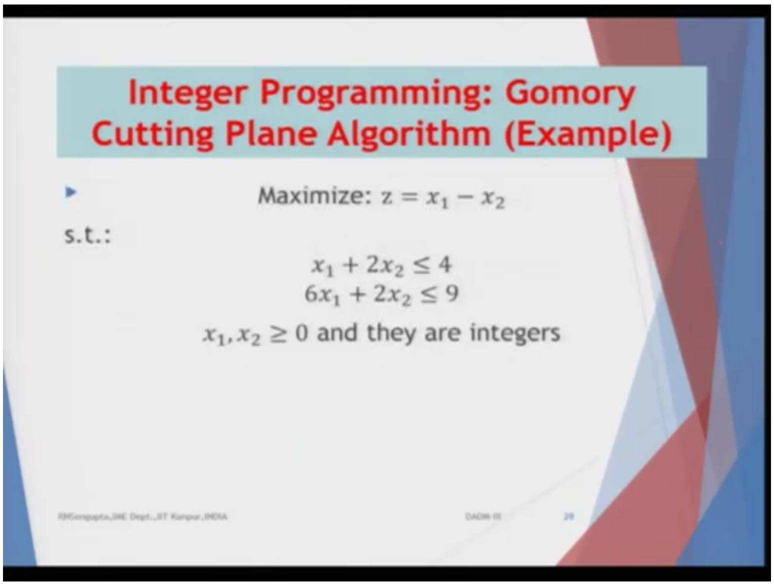

So it was 3 by 2, so it is x1 minus x2. So let us 6 x1 minus x2. So x1 is 3 by 2, x2 is 0, so these are the decision variables. I will use another colour. So Z is equal to X1 minus X2 is equal to 3 by 2 minus 0 is equal to 3 by 2, so this is the first part. So objective function was this. So the first constraint. So let us say the first constraint.

x1 plus 2 x2 is less than 4. x1 plus 2 x2 is less than equal to 4 times to 3 by 2 plus 2 into 0 is less than 4 which is right. so because x2 is 0 that second constraint is 6 x1 plus 2 x2 is less than 9, 6 x1, I should write it this, wait. I should write like this. this is a multiple relation signs. Now see the interesting part in the first constraint you had x3 as 5 by 2, let us put x3. x3 was there in the first constraint.

So 3 by 2, so 4 minus 3 by 2 will be what? 8 by 2 minus 3 by 2 is 5 by 2, so 5 by 2 is x by 3 which is matching. So left hand side and right-hand side matching. Now in the second constraint you had x3 and x4, let us take. So the left hand side is 9, right-hand side is also 9, so technically x4 should be 0 and this x5 minus 3 by 2 is the added constraint value of the slack slash surplus which we are bringing into the new constraint as required. So to function is 3 by 2 and we solve the problem.

Now again when we do the iteration, so initially we had x1, x3, X5 as the point when we have stopped, not the optimum point the eternity point where you have stopped and objective function was 3 by 2 and X1, X2, X3, X4, X5 were given by this, the mark in the red. So once we solve and we stop the problem using the simplest method whatever concept you have done.

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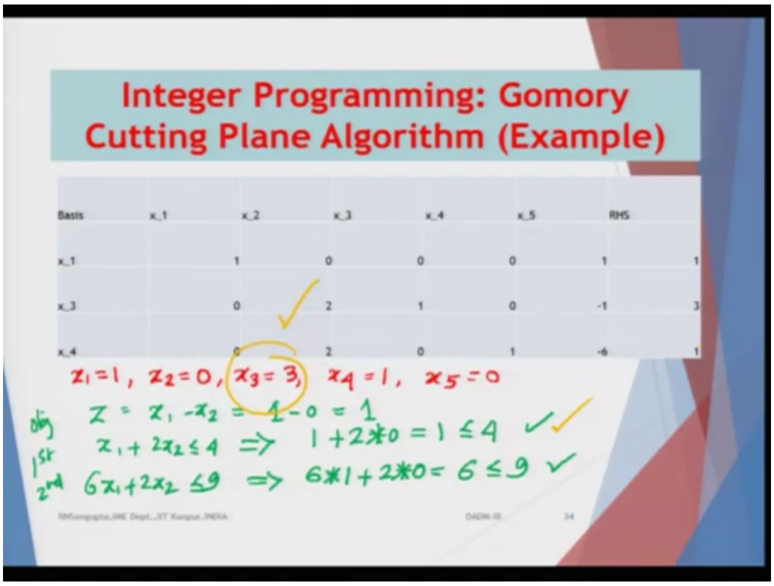

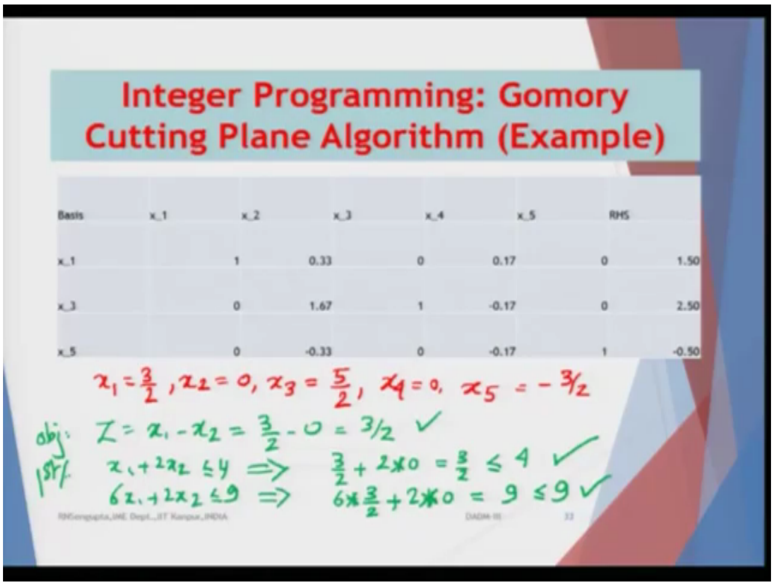

So the values were x1 is equal to 1, x2 is equal to 0, so see here very interesting fact. The objective function which we have solved using the linear programming problem concerning there were no integer constraints or integer concept brought into the decision variables was 3 by 2, now it is 1. Obviously it should be technically less because if you remember the diagram which you have drawn the feasible region, the points which were inside the feasible region considering that those are integer points was less than 3 by 2 actually as it should be for Z, so it's 1 here.

So x1 plus 2 x2, it was 6 x1, so I should have written it second 6 x1 plus 2 x2, now let us check. In the first constraint, x1 is 1, x2 is 0, so if you put everything in the first constraint I have one, so what should x3 be? Because on the right-hand side I have 4, on the left inside I have 1, so x3 the slack slash surplus is 3 as it should be. It matches to this.

In the next constraint I have which was 6 on the left hand side, right-hand side was 9, so if I basically find out the values accordingly, so if these are changed variable, so obviously you will basically have 1 for x4 and x5 is 0 because that shifts have been there corresponding the changed Gomory cut planes have been added accordingly. So once you solve the problem your values are 1 3 1 for x1, x2 and x4 while x2 and x5 are 0 and objective function basically should be one and you solve the problems accordingly.

So you are basically by shifting the corresponding main constraints which was corresponding to the first constraint in the tableau you change the constraints and basically you should solve the problem using the linear programming to obtain the optimum solution in the integer programming case. I will come to one problem solution for the bunch and bound method also but which is the next method and I will discuss about that in details first.

Now integer programming problem when you are solving the concept of Gomory cuts basically you are taking cuts are shifting the overall boundary. Basically like you are slicing the cake and slowly doing away with some of the actual feasible region which was applicable for the linear programming part and then making the overall space much more restricted in the sense that the corner points would be corresponding to the integer solutions only for the decision variables.

Now in the branch and bound method the concept remains as the word says, you basically branch out at each and every steps. So as you branch out you will basically try to ask the question that the branching which you are doing our whether they are permissible, if they are not you stop and you put some bounds depending on which direction you will go.

So if it's the maximization problem we will go in the direction that the objective function will keep increasing depending on the integer values which are under decision variables which are integers and in the case if the branching is not possible and you have reached a dead end. Dead end considering the concept of degeneration points are there, unmounted concept or whatever are there, we will stop there.

Andy will branch out in that direction for the minimization problem in exactly the same way that they will basically try to reach the minimum point. So this is the generation, basically a start from the initial solution from the linear programming and then branch out depending on the decision variables which are integers and basically go into the direction where it gives you the best optimum solution for movement.

So that means it is basically a tree. Primary than secondary and so on and so forth. So we remember we discuss the AHP problem analytic hierarchy process problem in DADM II, so they were hierarchy, so you will basically consider that sort of hierarchy with a branching goes and gets more enhanced and more detailed as you go down the tree.

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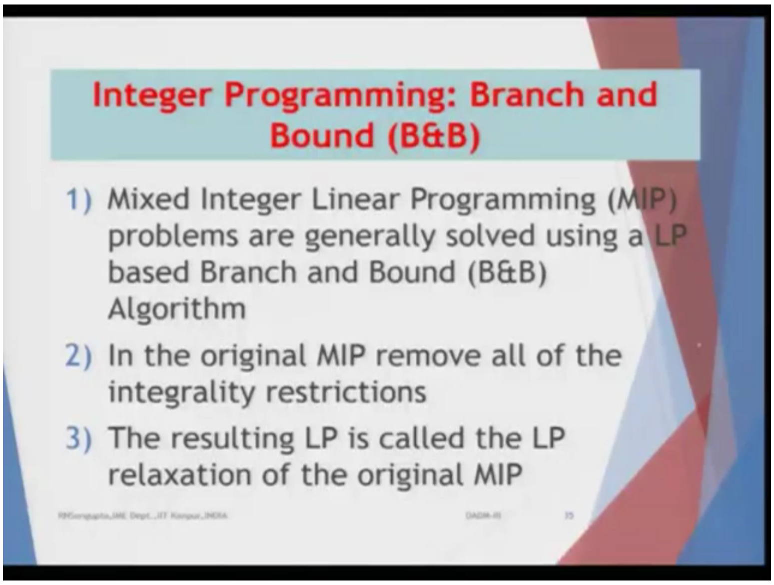

So the Mix Integer Linear Programming or MIP problems are generally solved using the concept of linear programming based on the branch and bound technique. So branch and technique which I will basically discuss accordingly. So in the original MIP mixed integer linear programming you remove all the integratility restrictions. Integratiliy constraints are basically corresponding to the fact that x1, x2, x3, x4 are integers you will basically remove them and you will solve them.

So that means you are doing the same method, same concept as you have done for the Gomory cut method. So that resulting linear program which you will get it would be basic of the LP relaxation of the original mixed integer programming problem. So the objective function remains same, the constraints and all these things remain same, you solve it using the simple linear programming and get an answer.

So this problem solution which you get is for the LP relaxed problem for the original MIP which I have solved. So basically from there you will start. You will basically solve as I just said it's all the linear programming that is the LP relaxation on the original MIP.

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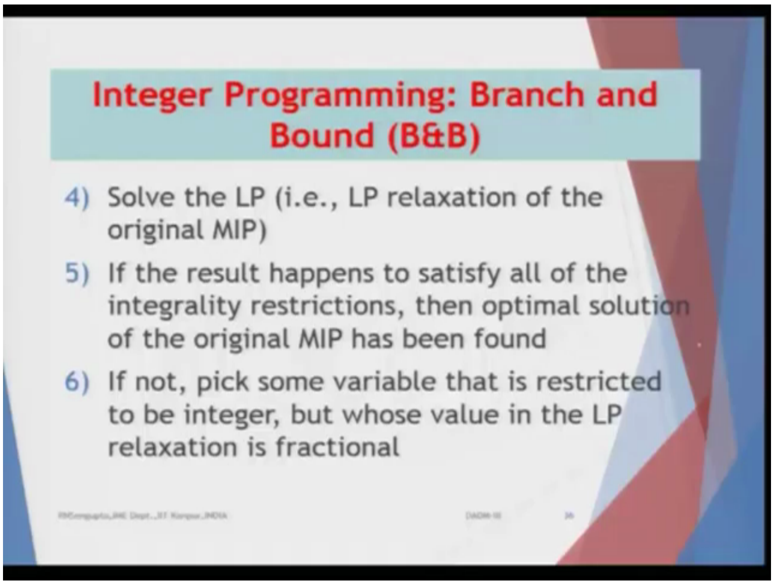

If the results happens to satisfy all the integrality constraints restrictions which were there. So what were the restrictions, if you remember? So the restrictions were that x1 is integer or x2 is integer it may be possible that x3 x4 are not integers they are continuous, so we basically check everything and if it meets all the criterias corresponding to that and you have basically reached the optimum solution, so that is a solution based on which we will report answer.

They would be integers, non-integers or continuous and the optimum value for the objective function depending on maximization or minimization. So the way you have solve it is exactly the same and the linear programming concept using the tableau method, the simplex method.

So if the results happens to satisfy all the integrality restrictions than the optimum solution of the original MIP has been found and you stop.

If not pick some variables that is restricted, restricted in the sense they are integers but whose values in the LP relaxation are fractional. So the fractional value say for example 3 by 2, so you will basically consider the solutions at points of 3 by 2 the 1 integer value less and 1 integer you will have more, it will be 1 and 2 annual solve them at this points. Check for which case the objective function is increasing, decreasing depending on the problem formation is the maximization and minimization.

You find out and then direction was going to increase or decrease depending on the problem of formation you will basically follow that and you will basically take up one at a time those decision variables which are 2 integers.

Say for example initially you have considered x1, next step say for example you have to consider x2, so in this way you will branch out and basically obtain all the integer values which basically when formula, so you are trying to basically take all the combinations and stop in the direction where there is no increase in the problem formulation, increase or decrease and go in that direction where the optimum answer would be reached as required. So if not become some variables that is restricted to be integers but whose value in the LP relaxation is fractional.

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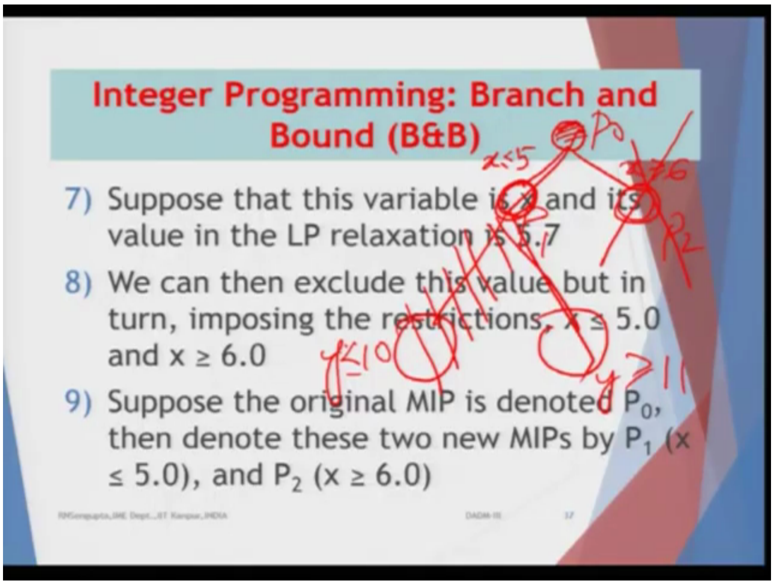

Now suppose the value which I was saying, suppose the variable is x. So there are many can be x, y or x1, x2 or x3 or whatever. Consider the variable is x and its value in the LP relaxation is 5.7 you have obtained 5.7 we are not considering the other decision variables which are there they can be non-integer also but we are not bothered about that. We can then exclude this value but in that place we will basically once you are at 5.7 we exclude that value then you basically branch out.

One of the branch would be for the values of less than 5 or equal to 5, so it can be 5, 4, 3, 2, 1, 0 any value is possible and for values of greater than 5.7 integers would be 6, 7, 8, 9. So you are basically partitioning the set into 2 distinct regions and do your research accordingly in this regions such that you know that the answers which are going to get would basically be integers.

Say for example once you solve then you find out, say for example the value of the next variable, so it was x, so the next variable y was coming out to be 4.3. So now you will basically take the values of 4, 3, 2, 1 and 0 for Y and the other set, other brands would basically be 5, 6, 7, 8, 9.

So obviously it would mean that in which direction you have gone for x, x is no fixed at value of say for example 5 and then Y would basically be. Be it optimized accordingly by taking the integer values. We can then exclude this value but in turn imposing the restriction that X is less than 5 and 1 of the value x is greater than 5. So suppose the original MIP is denoted.

So original MIP which you have solved using the LP relaxation is P not, the value P not then we will denote the MIPs for the branches would be obviously they would remember they would be 2 branches only not more than 2. Each and every step which you do, so the branches would be denoted by P1 corresponding to the fact that you are taking x values is less than 5 and P2 is for the cases for zx value greater than 6.

So what you are doing, so this was P naught, you have basically broken down into 2 branches, I will come to the diagram later on, I am just (())(24:09). So here if the values of x was less than equal to 5 and here the value was basically equal to 6, 7, 8, 9. So this was P not, this was basically P1 and this is P2. So out of them one is basically excluded. Say for example this is excluded and because of value which you are going to get for the maximization problem would be higher say for example for values of x less than 5.

And then you will basically branch out here depending on the y values and y values say for example one is coming up less than equal to 10 and y values coming out to less than equal to 11 and consider that this value is ruled out depending on the objective function is maximization you will basically take this road and then go accordingly lower down primary, secondary, tertiary and then list the solution. And each and every step you are calculating the objective function. Suppose the original MIP is denoted by P not then denote these 2 new MIPs by the values of P1 which is x is less than 5 and P2 where x is greater than 6.

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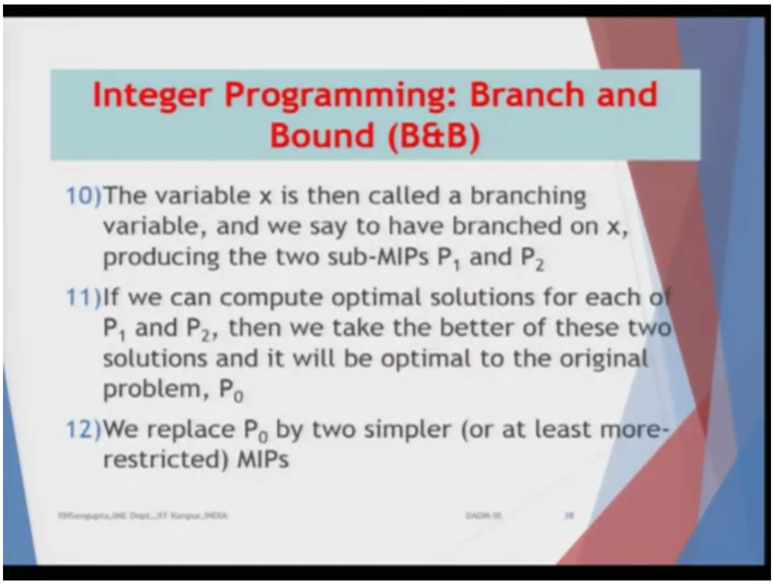

The variable x will be called the branching variable. So each and every step there would be one branching variable, so it would not be 2. We will basically consider one at a time. And we say to have basically branched on x producing 2 sub MIP problem solutions which is given by the values of P1 and P2.

Then say for example you go for y, it will be say for example P3, P4 and then go accordingly. So P1 may take the route, P1 is excluded, say for example, so P2 will take the route again P3 and P4. Say for example P4 is excluded, so it will be P1, P2 and P3 then P3 again goes into P5, P6. 5 is excluded, so you will basically go from P2, P3 and P6 and basically going that direction and each and every step.

If we can complete the optimal solution for each of this P1 and P2 then we will take the better of these 2 solutions. Better is the word which we are using depending on whether it is maximization problem or the minimization problem then we take the better of these 2 solutions and it will be optimum to the original problem solution which was P not. So we are basically trying to find out the best solution nearer to P not depending on the decision variables if they are integers.

So P not was basically the solution for the linear programming problem which is the best solution if we could basically have all the x as non-integers. So in that case either P1 and P2 would be closer to P not and then we will go down in that direction as I say P3, P6 and so on and so forth. We replace P naught by 2 simpler or at least more restricted MIPs. So you are restricting MIPs in the sense that you are slowly restricting the values of x to be integers only rebuild you will basically go around the direction and go from the top down method.

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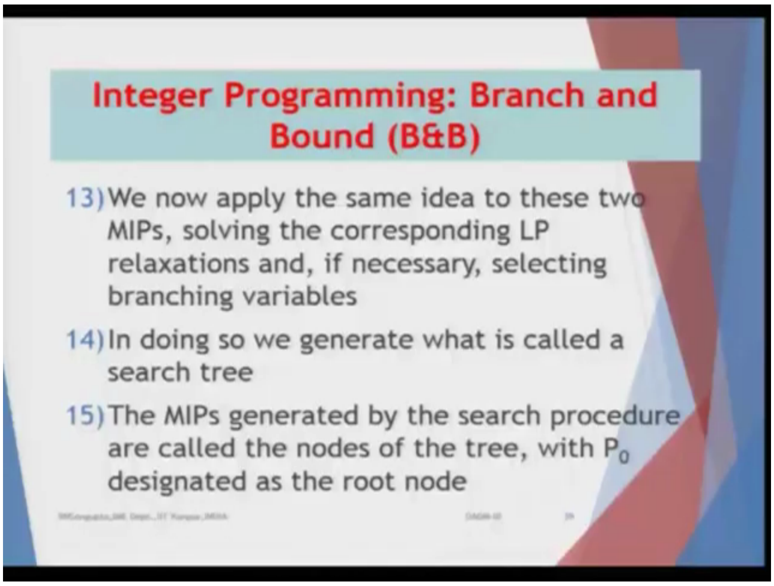

We now apply the same idea to these 2 MIPs which you have got, so one of them is giving you a better answer. So with respect to the others but it's not degenerated or not suboptimal. Suboptimal means it is not infeasible. So we will basically consider them and then branch out but in the process when you are doing the branching you will find out that some of them would be not feasible or they would degenerate or they are infeasible points.

So those points at which you stop infeasible degenerate you stop there, you do not basically go down further the tree. But you will only restrict your searches to the ones which are feasible points. We now apply the same idea to these 2 MIPs solving the corresponding linear programming relaxation and if necessary selecting the branching variable accordingly in the next up.

In doing so we generate what is called a search tree. So first up, second step, third step and as I mentioned like the AHP diagram. The MIPs generated by the search procedures are called the notes of the tree. So the tree which is basically starting P naught, so they are the notes, with P naught designated as the root note based on which you want to find out the best solution which is nearer to, actually as close as possible to P naught.

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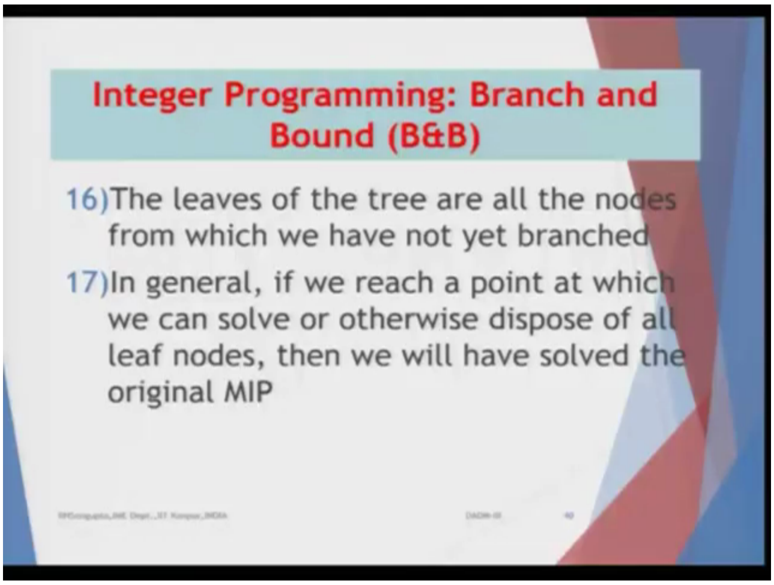

The leaves of the tree are all the nodes from we have not yet, the leaves of the tree are all the notes from where we have not branched out and they will basically branch at accordingly will stop or terminate at some points where there is no more feasible solution. In general, if we reach a point at which we can solve for otherwise dispose of all the leaf nodes then we have basically been able to solve the MIP problem.

So considering we are going down and down and we will stop at one point, let us say for example the rth iteration process and considering there are a number of nodes. Consider only one node is feasible and some optimum solution you have reached other nines are not applicable because if they are in feasible degenerate or so and henceforth, so we will consider that in order to be the best optimum solution with respect to the P not solution from where we have started.

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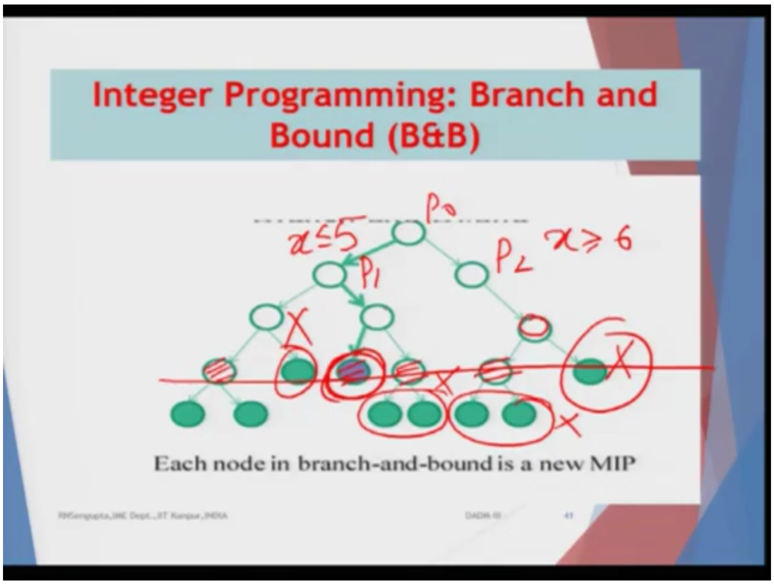

So let's consider this. This is a diagrammatic representation, so for example we have P naught, you have gone to P1, P2 depending on the values of x is less than 5, x is greater than 6 then say for example when I go from P2, I go into this second node. So obviously the branches are there and one of them are degenerate stop. Degenerate stop. Degenerate stop. Degenerate stop that means it is not possible to go further now then obviously when I stop at this position I will take the one which is the best solution out of these.

And consider this as the best solution. So and this will be ported and find out with respect P naught. So with this I will end this 8th week lecture and considering this branch and bound method I will start the solution with the branch and bound and also try to compare some of the methodologies other than branch and bound on the Gomory factor. Have a nice day and thank you very much.