Data Analysis and Decision Making-3
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Lecture 4

A welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of globe and as you know this is the DADM-3 which is Data Analysis and Decision Making-3 course under NPTEL MOOC series and this total course duration as you know is for 12 weeks which basically goes into 12 into 5 which is 60 lectures, each lecture being for half an hour.

And this course so as you know would basically have as I am just mentioned 12 weeks which means that after each week you will have an assignment and related to whatever is being covered and after the end of the session you will have a final examination based on whatever topics have been covered. And my good name is Raghu Nandan Sengupta from the IME department, IIT Kanpur.

Now if you remember optimization in a very simple sense like what we mean by the boundaries, what we mean by the concepts of the constraints. Now before I go into all the general details I would like to basically dwell on a topic which may not be it seems it or you may have a feel it is not directly related to optimization but it will have some bearing later on when we basically discuss the problems and there that has a historical background for that.

So whenever you are discussing the optimization, the point of concept or utility comes up time and again. So basically whenever you are trying to solve a problem from the economics point of view and you meant if you read do remember I mentioned the name of Koopmans, so it was basically trying to optimize some utility function and that came out to be the fact that it was the optimization of the expected value of the utility based on some constraints and if you want to (opt) also obviously in that case if you want to optimize the expected value of the utility we trying to basically maximize.

On the other hand, if you want to basically find out the minimization concept it would be related to the variants of the utility and the problem formulations even though they would be different type, will basically consider the concept of optimization very simply from the utility concept and I will go very simple there and then basically increase the level accordingly.

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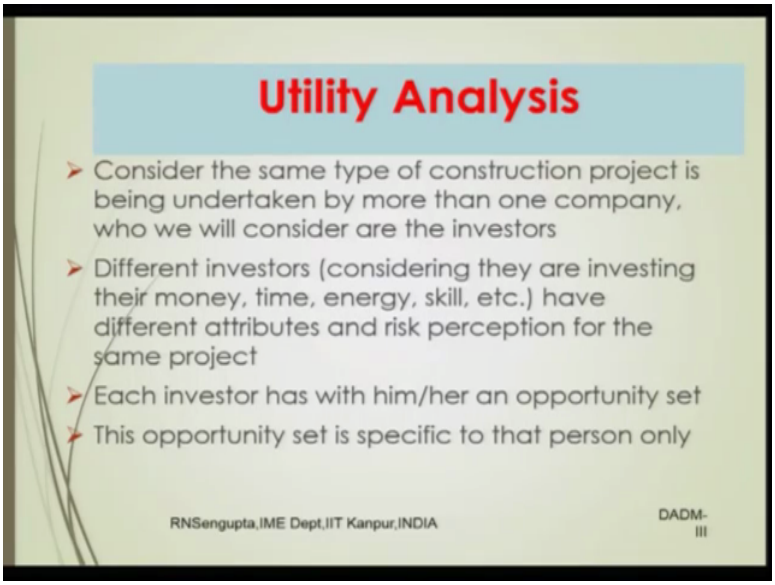

So consider the same type of construction in a project is going on and this being undertaken by more than one company, so who we will consider are some investors, they want to invest some money. So obviously money means you want to optimize your output, whatever the output is. It can be per unit of investment you want to increase it or on long term basis you want to increases what whatever these are and then dynamic concept of time would should also come but will not consider that accordingly.

So different investors considering they are investing their money, time and energy, skill etc have different attributes and risk perception for the same project. So the word risk I am using for the first time but you will later see that it will be true according to the concept that how the word risk could be utilized. So each investor has with him or her an opportunity set, so opportunity set would basically be collection of different sets or single sets only, single values only.

So that will depend on the type of problem whether it is a univariate case or the multivariate case and you will basically have the problems framed accordingly. This opportunity set is specific to that person only and based on that opportunity set the person would basically take a decision. So, if the opportunity sets are consider I am basically given by the word O, so O 1, O 2, O 3, O 4 till O n are the members of that set, so obviously that O 1 to O n are unique to that person who is going to take the decision.

Now you may ask that whether it is possible for the other person when he or she takes a decision to have a different opportunity set? I would say yes because what is feasible for me

may not be feasible for you or vice-versa but the universal set would besides that we have to pick up, we means the investor or the decision maker has to pick up some opportunity set from the universal set based on which it meets his or her criteria of the optimization based on the constraints.

So the constraint may change, the opportunity set would obviously change because the objective function would be different from me which would be different from your case, your means the other person who is going to take the decision.

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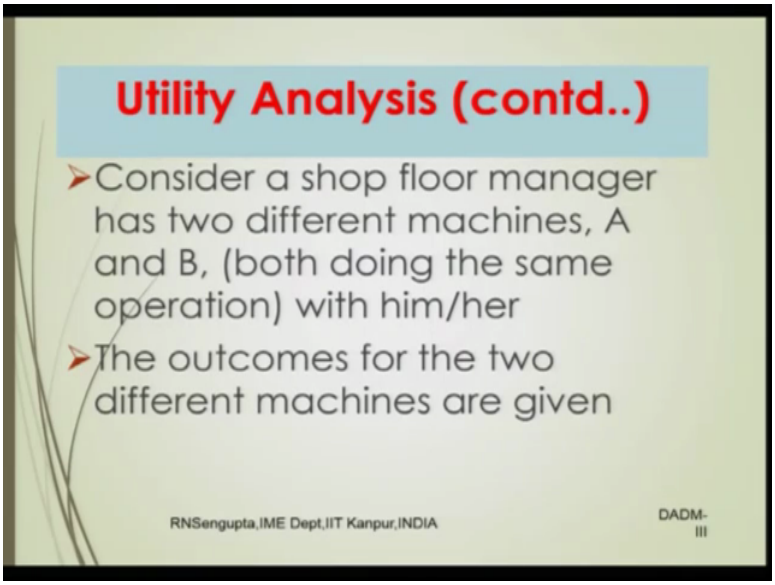

So consider shop floor manager has two different machines A and B, both doing the same type of operations, so drilling machine or a lathe machine or a turret or the planer, shaper, CNC machine, NC machine whatever it is they are doing or it can be trying to cut a gear, warm gear or a bevel gear whatever it is or it can be right trying to basically utilize this machine to produce paints or trying to basically produce different qualities of chairs and tables or different type of machines as it is.

The outcomes for these two different machines they are producing are given now, the output does not mean the numbers of output it basically means some value, what is that value I will going to come to that later because that value would have our bearing on your objective function. So, what the value I am trying to analyse would basically have an bearing on the objective function and this type of objective function may be specific to the person who is going to optimize or it may be very general in nature.

We will not try to go into the very specific examples, we will go into the general formulation of the or the objective function based on the fact and that we are trying to optimize some utility or some net worth. So I want to increase my profit which means I want to increase my net worth.

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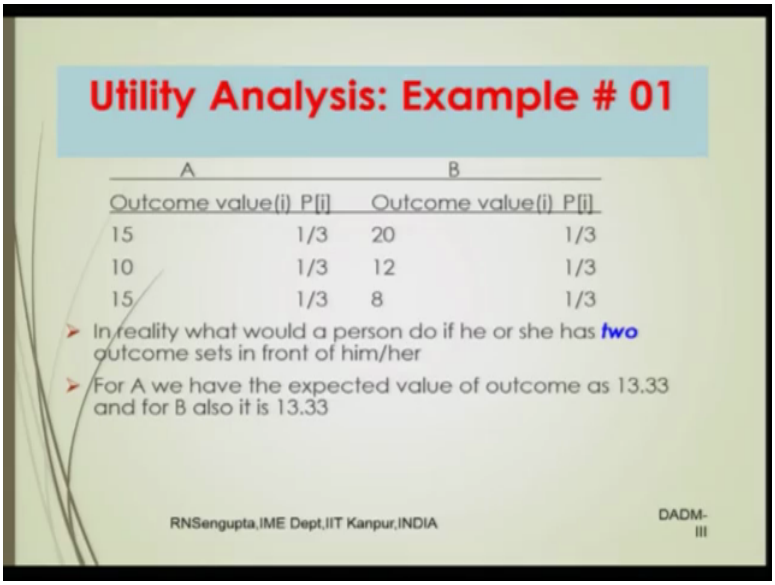

The outcomes like this for machine A, so outcome value I again I am saying the value does not mean a number, does not mean the rupees, does not mean dollars, yens whatever, it is just a value based on which we will try to optimize. So it will change depending on the situation I am going to come to that. So, the outcomes are 15 with a probability one third, 10 with a probability one third and 15 with the probability one third.

And for machine B which is the second scenario, the outcomes are given as respective as 20, 12 and 8 and the corresponding probabilities are also one third, one third, one third. So, these probabilities of being one third is just arbitrarily it can change also depending on the scenarios but we have taken only one third in both the cases in order to make our initial starting simple. Remember one thing obviously as it should be the probabilities should add up to 1 and it is right for both machine A and machine B which is case 1 and case 2.

Now, another question which will come up according to if you see the outcome for machine A, it would mean that for two different outputs of 15, why is it that I am basically trying to separate the probabilities one-third, one-third? The actual essence is that whatever the actual decision would be may be different but the outcomes are such that for both the decisions, the outcome total value of the outcomes are 15, 15 in this case.

So, that is for a particular person or for a particular decision maker. Now, if the same set of decision variables are taken by a second person, the outcomes need not be of equal values of 15, 15 they would be different, so this is to be remembered by each and every person when you are trying to solve the problem, do not be too much bothered about it this is just a general important fact which I am mentioning such then they can be analysed accordingly.

In reality, what would the person do if he or she has two outcomes sets in front of him or her? So these are given machine A machine B. For A we have the expected value of the outcome, so the expected value of the outcome as you can (fund a) find out would the multiplication of 15 into one third plus 10 into one third plus 15 into one third and you add them up is basically simple, multiplication of probability in the value.

So, then that expected value comes out to be about 13.33. Similarly for B also it comes out to be 13.33, so in case we are trying to compare both decision A and decision B which is machine A and machine B. So in that case the person who is taking the decision based on the outcome set which you have would be indifferent or cannot take the decisions accordingly.

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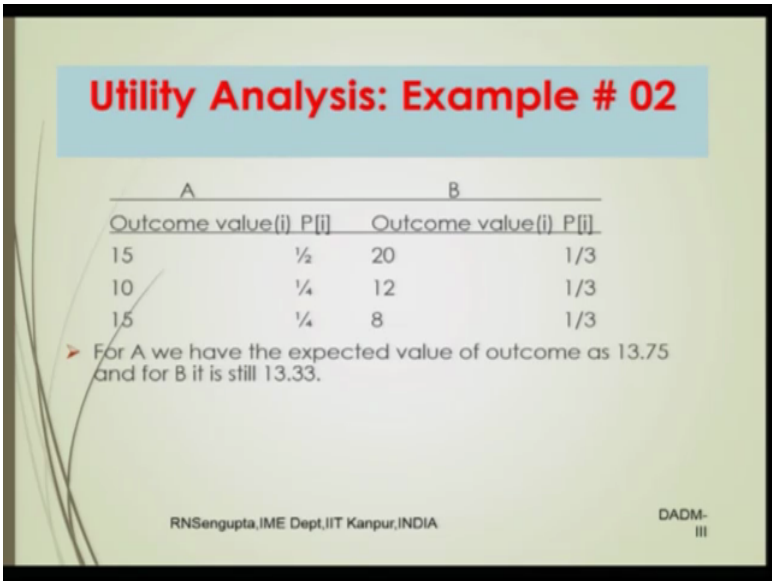

Now let us change the probabilities. So, the scenarios are almost the same again on the left most column for decision A you have the outcomes again I am mentioning they are not in values not the numbers again it is given as 15, 10, 15 and the corresponding probabilities are now changed to half, one fourth, one fourth which is 50 percent, 25 percent, 25 percent and addition adds up to 1.

For outcome B or machine B again the outcome values remain the same 20, 12, 8 and the probabilities are one third, one third, one third. So, if I find out the expected value for the scenario A which is machine A, the values are as it comes out to be 13.75 which is the (multiplication) addition of the values which is 15 into half plus 10 into one fourth plus 15 into one fourth, while for B it continues to be 13.33 as the last example.

So if now if you are comparing corresponding to the probability being changed for machine A or scenario A, will basically take a decision considering the expected value for A now is high.

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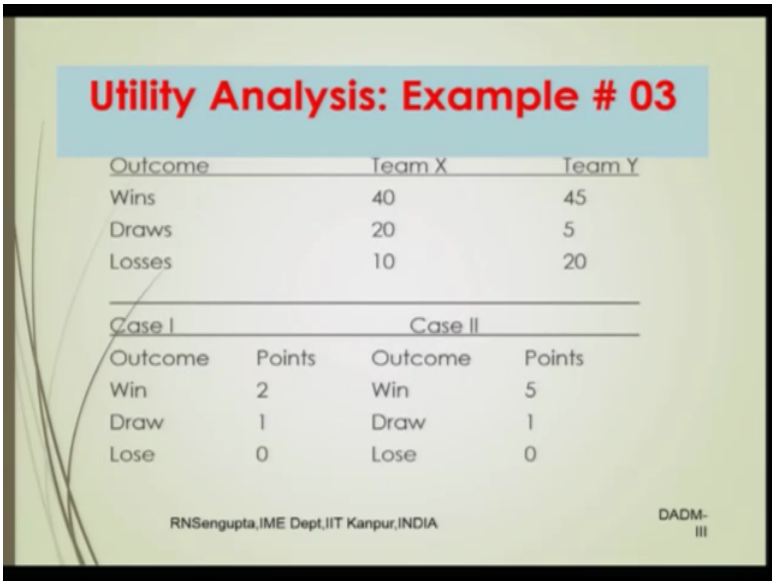

Consider an example number 3. Now you have 2 teams playing I am just arbitrary picking up2 teams, so they are playing a tournament along with other teams also. So, in that totality, in the same tournament the wins by team X have been 40 numbers they have been they have drew matches 20 numbers then they have lost 10 in numbers. The corresponding values for team Y are 45 for wins, 5 for draws and 20 for losses.

Now consider when you start the tournament it was decided that for outcomes which there is the win, you will get 2 points, for there is a draw you will get 1 point and for a loss you will get 0. So everything was going fine but suddenly the due to some reason consider we are not going to the details due to some reason the organizers says no, we are wrong to they have changed the marking system which means now according to the second case of marking the outcomes are again win, draw, loss is fine but the points accrued to the win, loss and draw are now 5, 1, 0.

So that means if you win you get a huge amount of points while in case 1 the amount of points was 2.

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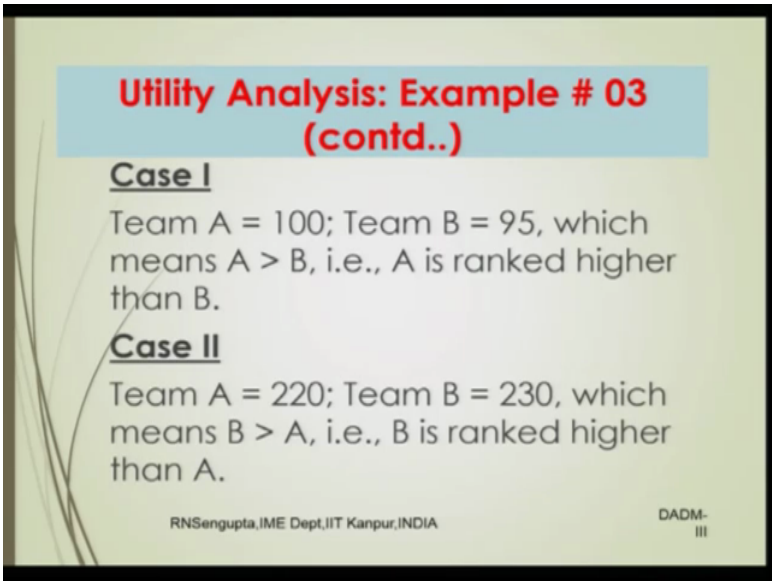

So if we consider the ranking system based on case 1, team A would have 100 points, team B would have 95, so if I am trying to compare the expected value of the points based on the outcome so B would have A would basically be better off so A would be on the top position. Now consider that means in the ranking system, now consider team A and B based on this the case 2, so in that case team A would have 220 points and team B would have 230 points, which would mean that when I am ranking the teams based on (scena) case 2 points it will we found out that B is ranked higher than A. So obviously the whole ranking system would change. So, you should be careful based on which your decision you want to take.

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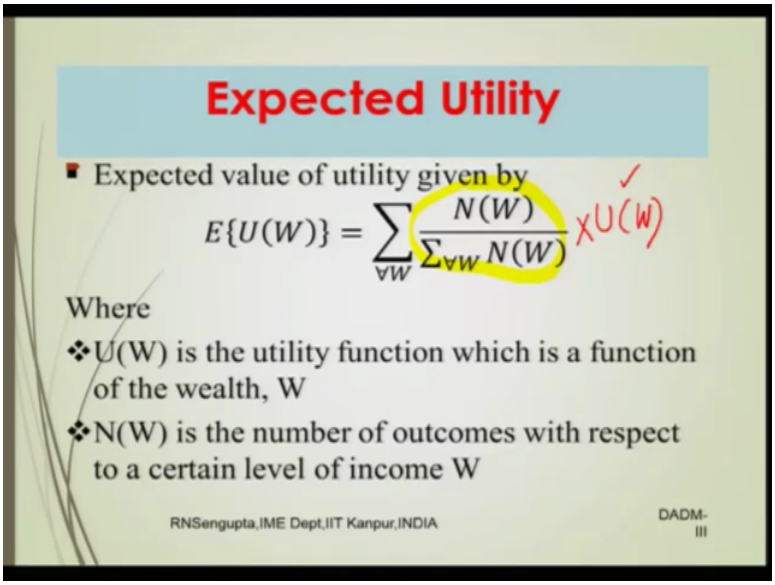

Now, whenever you are taking a decision I have been making mentioning very briefly the concept of utility function. So how do you find out the utility function? So utility function is found out by multiplying the two values, so you will basically find let me write it down I think there should be a U W here also, so this would be I just change it, wait let me change it will be easier for it, I will just mark it accordingly, let us continue.

So this expected value would be the multiplication of the utility which was that values you remember 15, 10, 15, 20, 12, 8 so those are the values so that is U W and if we (new) remember the probability, probabilities were one third, one third, one third in the second case it was one fourth, half, one fourth, one fourth, so those probabilities are going to come from here, let me mark it. Do you see it is a ratio of the number of outcome which is supporting it divided by the total number outcome which is actually plausible.

So here U W is the utility function which is a function of the wealth W and N W is the number of outcomes with respect to a certain level of income W which is there, so income I am considering that there is some input and you are getting some output based on that the input value which you are giving.

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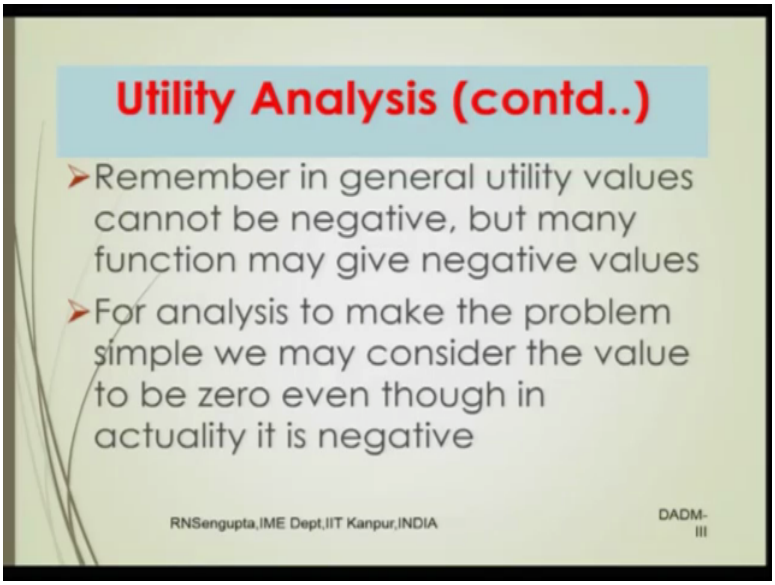

So remember in general utility functions cannot be negative but many functions may give negative values of the utility function so you have to be careful about them when you utilize that. For analysis to make the problem simple we may consider the values to be 0, that means if they are negative we will consider the values to be 0 even though in actually the real value which comes out to be negative.

So, negative obviously would be ranked lower than the value 0 but we will consider negative values of 0 in order to make our calculations very simple.

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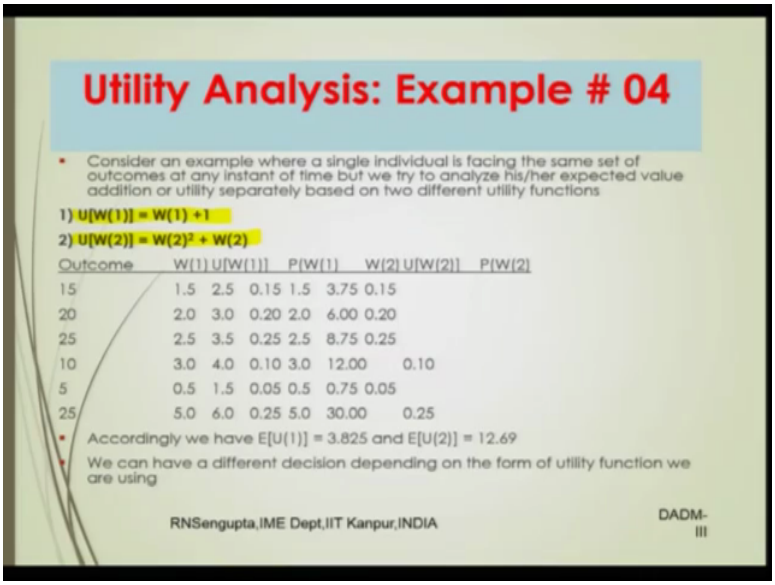

So consider an example where a single individually facing the set of outcomes at an at any instant of time but we try to analyse his or her expected value of addition or utility (special) especially or separately based on two different utility functions. So, what we are trying to do is that, will take one human being and he will (trace) basically try to analyse the problem based on two utility functions and we will try to find out that as the utility function changes the concept of decision based on which the that person will take a decision either for scenario 1 or scenario 2 would change.

So we will go step by step as per the problem. So, the utility function 1 is basically a linear one while utility function 2 is a quadratic one. That means it is just like I will come to the quadratic utility function later on just wait. So, now we will consider the outcome, so the outcomes are 15, 20, 25, 10 and 5 and 25 and the values of the wealth for scenario 1 is which I will consider is (swift).

So, scenario 1 means I am transiting the utility function 1, so this is 1.5 that means for a value of 1.5 what that value by (or) I will consider it to be either Rupees, Yens whatever it is and based on that we want to find out the utility. So it is 1.5, so 1.5 would give you a utility of what? So (one) if I consider the utility function 1 it will be W 1 plus 1 which is basically 2.5. So if you see the second values of the columns I am not marking it I am just hovering my this pen so the values are starting from 2.5 to last value 6, why 6? Because 5 plus 1 will be 6 depending on the utility function which is the first one which you are using.

Now when I go to the case of the probabilities, so if you see the outcomes here I am now changing the notion, so now I will consider the outcomes to be the actual numbers which have occurred. So, if you consider the outcome, the total of the values is 100 so the probability would be corresponding to the utility which (you are) that would be the probability for the utility. So, in case if it is basically 2.5 which is the utility it would mean that that utility function has technically occurred 15 number of times out of a 100 number of sequence or the experiment whatever you do to. So, the values are given, values are 15 I will not mark it 0.5, 0.2, 0.25, 0.1, 0.05 and 0.25 so this is the overall probability for the outcomes.

Similarly, when I go to W 2, W 2 is the utility function 2 (and) so which is a quadratic one, so again I take the values accordingly. So the values taken are 1.0 for W the values are 1.5, 2, 2.5, 3, 3. then it comes to be the values of so let me consider 1 minute 1.5, 2, 2.5, 3, then you will basically have 0.5 and 5 so these are technically the values which am taking. So, based on that we will basically proceed what with, so I am basically taking these values.

Now consider that the corresponding utility would be found out by the utility function 2. So, utility function 2 is W 2 suffix 2 square plus W 2 which is a quadric one and if you have to basically find out the utility function, the values would be coming out as the value 1.5 whole square which is 2.25 plus 1.25 would come out to be 3.75, so this 3.75 is basically the value of that utility function considering the quadratic one. Similarly, if I go step by step I can find out the quadratic utility function.

For the quadratic utility function based on the values which are which I am considering, values means? The W values. Now in this scenario obviously the outcomes are to be found out we consider a similar scenario where the outcomes are given as it is and based on that we find out the ratio of n W by the summation of n W which is the probability, so those values I can come out to be the same. As 0.15, 0.2, 0.25, 0.1, 0.05 and 0.25 so that should basically give you the idea that how the probabilities are calculate.

Now accordingly when I want to find out the utility, utility which is basically for the utility expected value of the utility for scenario 1, scenario 1 is the utility function one which is the linear one the value comes out to be 3.825. If I find out the utility for the two, it comes out to be 12.69. So we can have a different decision depending on the form the utility, so if we change it the utility we can get different scenario. So, what is important to understand is that as the utility function changes your decisions based on the expected value also change.

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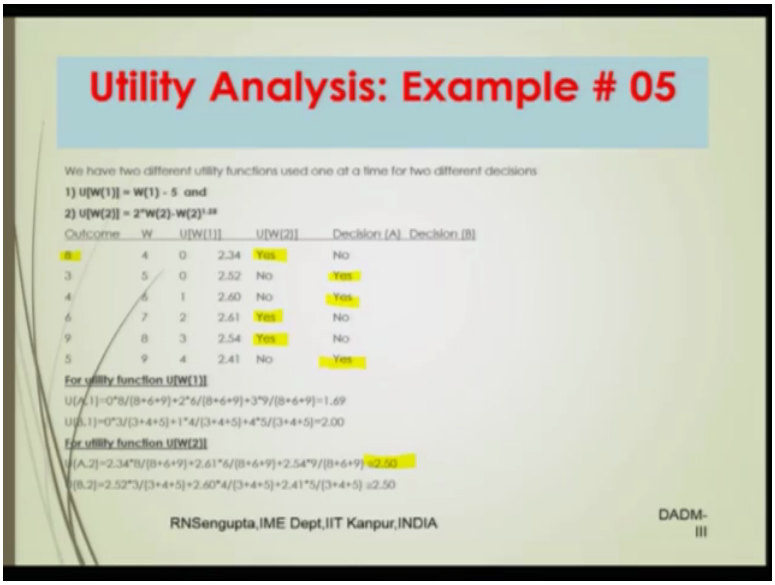

Now consider we had two different utility functions used, one at a time for two different decisions now. So now the utility functions1 is basically linear which is W suffix 1 minus 5 another is quadratic, so this is 2 into W suffix 2 minus W suffix 2 to the power 1.5 it is not quadratic it is basically polynomial. Now, we have the scenarios the outcomes are given, so outcomes at 8, 3, 4, 6, 9, 5 from the leftmost column and the values are alongside a given once you have W you can find out you can find out.

So you plug W once seen utility function 1, once in utility function 2 and get the output which are corresponding to the values as given. So, 4 minus 5 is minus 1, so we will consider as 0 because I had mentioned that negative values would not be considered. The next value would be 5 minus 5 which is 0, next is 6 minus 5 which is 1, 7 minus 5 is 2 so on and so forth. So, these values which I find out here are the utilities based on utility function 1.

Similarly when I go to the polynomial 1, the utility function are these. Now what is important to note is that we are considering decision A and decision B where yes or no would basically give whether it is possible or not possible. Now in this case if I consider the value yes here, for decision A and the outcome is 8 which means that outcome corresponding the probability will found out for the yes case. Similarly, yes here and yes here, so that mean 6 and 9 would be coming.

Now when I go to the decision B, the yeses are just one here, the yeses are for 3, for 4, for 5. So when I want to find out the utility it will be based on the fact that how many support this total utilities, as how many support means? How many are yeses are there? So for the first

case when is utility one, so the corresponding probability would be number of outcomes 8, 8 divided by the total number of outcomes which is favourable to the yes decision, so which is 8 divided by 8 plus 6 plus 9 and the corresponding utility is 0 so we will get that value.

So if we keep doing that for all the yeses for decision A and decision B under utility function 1 that means considering the utility function for both decision A and decision B are same, so we find out for A it is decision A it is 1.69, for B decision B it is comes out to 2. So obviously under the circumstances when I am using utility function 1, I will check and utilize the utility which is basically the for decision B.

Now when my go to utility function 2, so now the scenario also changes but what are the information? Now you have to only check the yeses so if it is yeses so for that decision B so for the utility function A the decisions are given, so if I find out the decisions for A, so A would be yes for the case when it is outcome is 8, yes for the case when outcome is 6, yes for the case when outcome is 9.

So I find out the corresponding probability and multiplied it by the corresponding utility which I am going to get for the scenarios. So it is 2.38 multiplied by 8 divided by 8 plus 6 plus 9 and so when I suppose, when I add up the values it comes out with 2.5 and when I go to decision B, so now the yeses would basically go to other outcomes, so when I find out the expected value it comes out to 2.5. So in this scenario when you are taking the quadrant the polynomial 1 will find out that the decisions can be indifferent, that means based on the process how you are basically analysed it.

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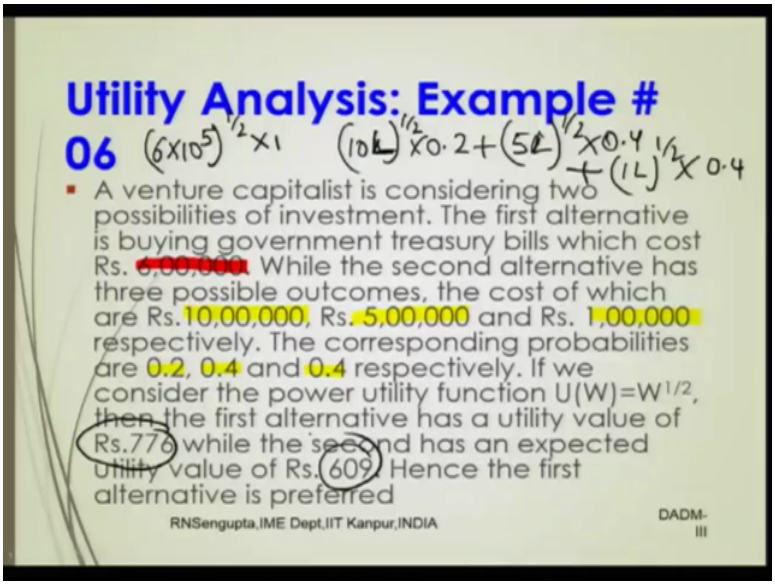

So, let us consider an example, a venture capitalist is considering two possibilities of investment. The first alternative is buying new government treasury bills which costs 6 lakhs so I am giving the values. While the second alternative has three possible outcomes the cost of which is are 10 lakhs, 5 lakhs and 1 lakh. The corresponding probabilities are given as 20 percent, 40 percent and 40 percent. The utility now based on the fact that what is the outcome in value terms, the utility is basically W to the bar half. So, if it is (10) 100 then the utility function be square root of 100 which is 10.

So if I use this utility function for the first alternative which is basically the government bond so treasury the basis of the governments with the cost was 6 lakhs, so I will find out the square root of 6 lakhs will give me the utility and multiplied by the probability whatever it is. So obviously in this case probability will be 1 because this is certain values where because there is only one decision. Now when I go to the second case, so it will be, so there are three scenarios square root of (1 lakh) 10 lakhs multiplied by probability is 0.2 plus square root of 5 lakhs multiplied by the probability of 0.4 and square root of 1 lakh multiplied by the property of 0.4.

So once you find it down the value of the utility expected utility comes out with 609 while for the initial case it would be basically as I am just repeating it, it would be square root of 6 lakhs into 1, so 1 is basically the probability because there is only one outcome. So hence the first alternative would be preferred because the value comes out to be 776 and the value which is here comes across corresponding to the gamble or lottery is basically 609.

So I will just write it down, so I should basically use the different colours wait, so this would be going out the probability 1 and this is yellow, so this will, so technically the first one would be I should use the different colour wait, so this is 6 into 10 to the power 5 square root into 1 that is the probability, that value comes out to 776.

Other case is 10 K to the power half into 0.2 plus 5 K not K ,sorry lakhs I am sorry next to the power half multiplied by 0.4 plus 1 lakh to the power half multiplied by 0.4, so if you basically calculate it comes out to 609. So with this I will end this fourth lecture and continue more discussions about the utility functions as required, thank you very much and have a nice day.