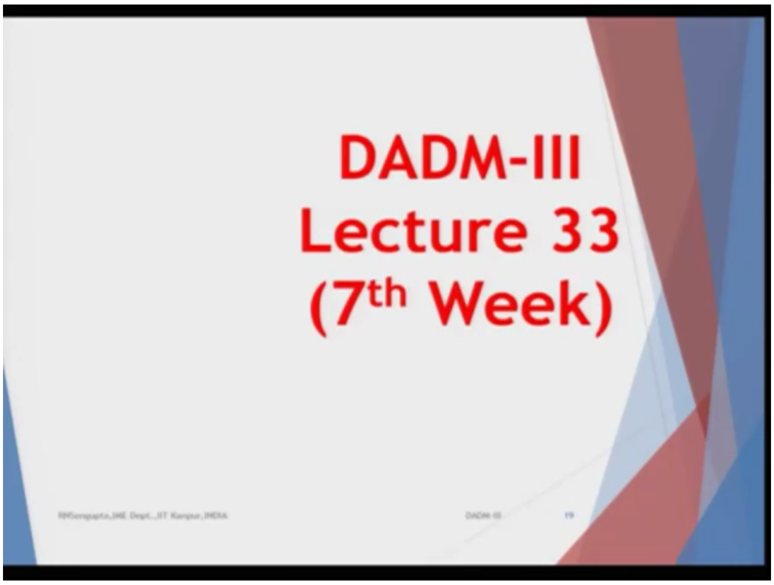
Data Analysis and Decision Making- III
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology Kanpur
Lecture-33

A welcome back my dear good friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe.

(Refer Slide Time: 0:33)



And as you know this is the DADM 3 which is data analyses and decision-making-III course under the NPTL MOOC series and as you can see from the slide this is 33rd lecture which is the third lecture in the seventh week and the total duration on the course is contact hours is 30 hours which when broken down in the number of lectures is 16 number and each lecture is for half an hour.

And each week we have five lectures, each being for half an hour and after each week you have one assignment. So as you can understand we had already taken 6 assignments you will totally take 12 assignments in totality. And then we will basically appear for the final examination. Another point which I wanted to mention I should have mentioned when I was starting the thirty first lecture which was just the first class in the second half of the session so and so for called. They would be a live session and the date and all these things would be announced by the NPTL course coordinators where I will come and try to basically answer questions depending on whatever the portion which I have covered.

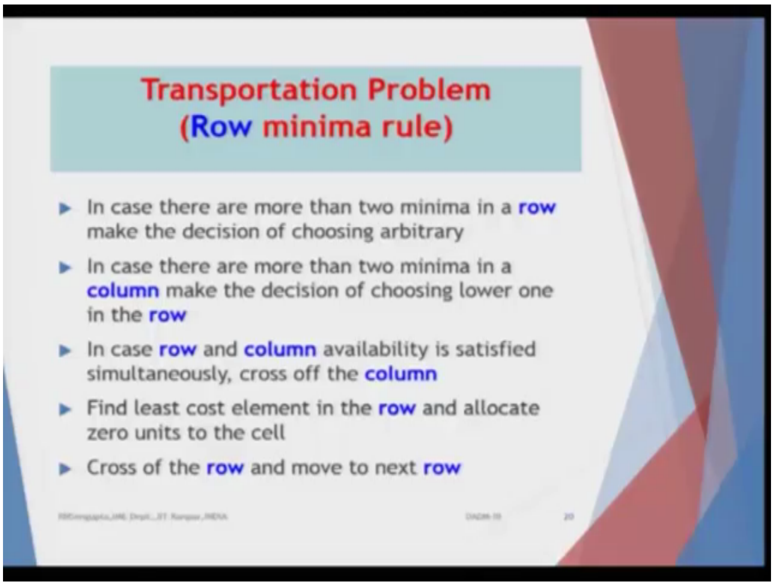
And obviously they would be many questions which would be directly answered by me or if any reading has to be done, I will definitely encourage you to read that. And also basically for the problem solving part we will try to basically if there are queries that if we need to solve more problems in a form of order for you to understand. I will definitely tackle all these things to the best possible manners as that all of you basically get the benefit of the interaction which we will have. And and we will fix the dates as that it is the best possible manner where all of you can take part in this interactive session, online session.

Now coming back to the course and as you see this is the thirty third lecture which I mentioned and my name is Raghu Nandan Sengupta from IME Department of the IIT Kanpur. Now in the mythologies which I am discussed we would have be discussed initially Vogel's approximation method then we are gone in the north-west corner method then we have basically go to the row minima concept. Then we will come to the call minima concept.

In the row minimum concept, I will just be paid which we just was the last slide well before we closed the thirty second lecture. It was basically a concentration were all on the rows and in this case our rows were basically the destination that we are only concentrating trying to basically fulfill the demand, requirements there. If the requirements are not met will basically go to the next column in the same row obviously and then try to basically allocate the numbers such that the first main concentration is to meet the requirements of the rows.

If the rows are met then we go to the columns and if the columns are also met then we go to the next row and basically do the allocation accordingly. Once the rows and columns are met we basically remove them such that no other allocations can be done. And as we have seen that in the Vogel's approximation method and the north-west corner method. So continue your discussion in the row minima allocation concept. Now further on in case, if there are more than two minima in a row, it is possible but as you saw the cos can be 60, 60,60 or it can be 40, 40 depending on the co structure on the practical situation.

(Refer Slide Time: 3:47)

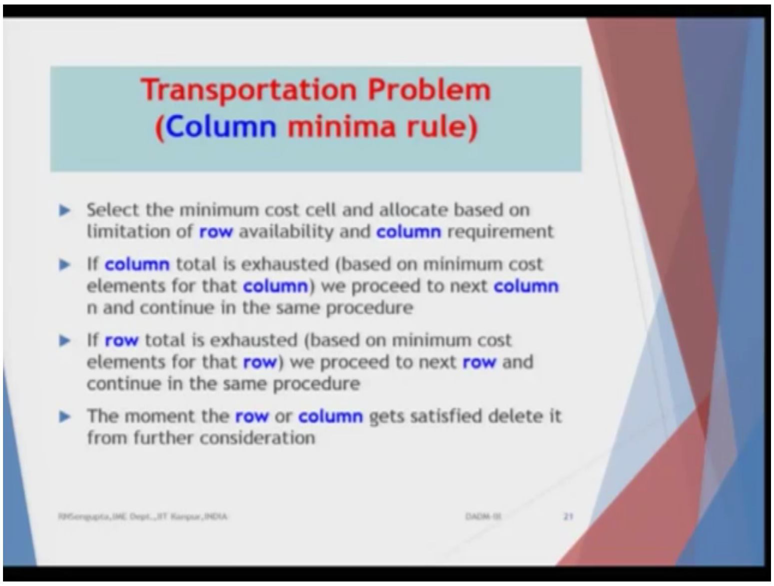


In case there are more than two minima in a row make the decision arbitrarily, choose the one and basically allocate the number such that you are able to get a feasible solution not the optimum feasible solution which will give you a starting point from where you can start your allocation and start your refinement of the problem to reach the optimum point.

In case, there are more than two minima in a column make the decision of choosing the lower one in the row. This is very important to understand because our main concentration are the rows, so if there are any two numbers in the rows are same then you choose arbitrarily. But in the case if there are two minima in the column particular to anyone in the row, you will basically choose the one which is minimum based on the fact that your main concerned is to basically minimize the row or take the concept on minima of a the row. And basically continuing in this fashion.

In case, row and column availability satisfy simultaneously cross of the column and then you will basically proceed in the same direction to meet the allocations as required. Find the least cost element in a row, allocate zero units to the cell and proceed in this manner such that you are able to allocate in the best possible manner. Cross the row and move to the next row such that allocations can be done. So your main concentration is always on the rows, rows, rows and then the secondary ones on the columns, columns, columns.

(Refer Slide Time: 5:18)



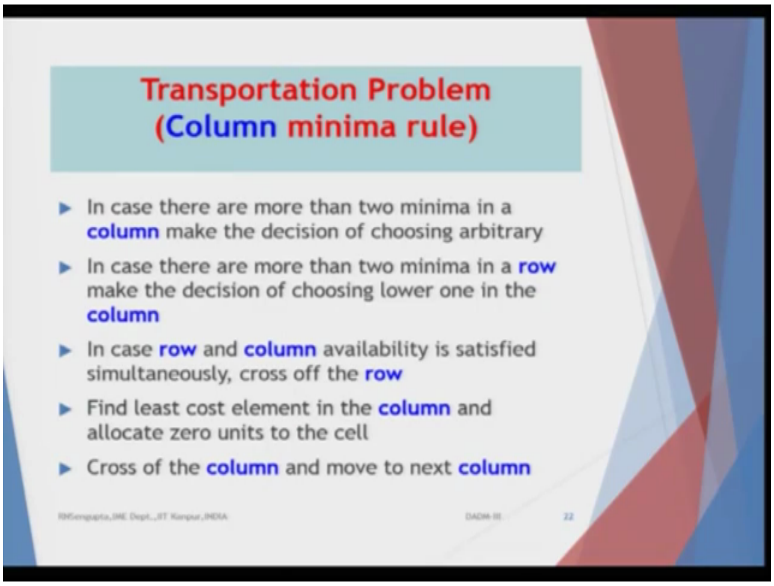
Now when you are going, when we come to the column one the concept remains the same but you are hierarchy of importance now switches from the rows to the columns and the columns becomes the rows. So you will select the minimum cost cell and allocate based on the limitation of the rows and avaibility on the columns requirement. So columns requirements are there which we know A, B, C, rows are one, two, 3, 4 based on the problem.

If column total is exhausted because your main concentration is on the column not on the rows. If column total is exhausted as it was the row in the first case for the, so mark these things. Here it is blue which is column minimization, there in the two slides it was row minimization or row minima. So your main concentration was on the rows, now it is the column. So if column total is exhausted based on the minimum cost element of that column we would proceed to the next column and continue the same procedure in order to meet the requirements for the columns first.

Now if the column requirements are met then you will basically concentrate on the row. So this is what it says. If row total is exhausted based on minimum cost element for that row, we will proceed to the next row and continue the procedure till the column concept is exhausted. I am repeating the point one time again please bear with me. So column is important concert on the column, so minimum cost allocation done more next to the move move to the next row. Minimum cost allocation done max to the next row depending on the column.

Once the column allocation is totally done then you can go to the next one. So you will basically switch from column take the rows, then column take the rows accordingly. While in the previous case, it was rows consider columns, rows consider columns and proceed accordingly. The moment row or column get satisfied delete it from further consideration as we did. Again I am repeating from the Vogel's approximation method and the north-west corner method because the allocation has done. So those rows and column would not be considered but we will follow the same principle as just mentioned.

(Refer Slide Time: 7:30)



In case, there are more than two minima in a column make the decision of choosing the arbitrarily because your main concentration on the column. So if there are, there are allocations to be done and when you are considering the cost factor, this is minimum cost factor you find out that anyone of them can be minimum you take it arbitrarily.

In case, there are more than two minima in a row, now make the decision of choosing the lower one in the particular column based on which your concentration is on the fact that you are only doing a column minima rule such that all your approximation would be based on the column one.

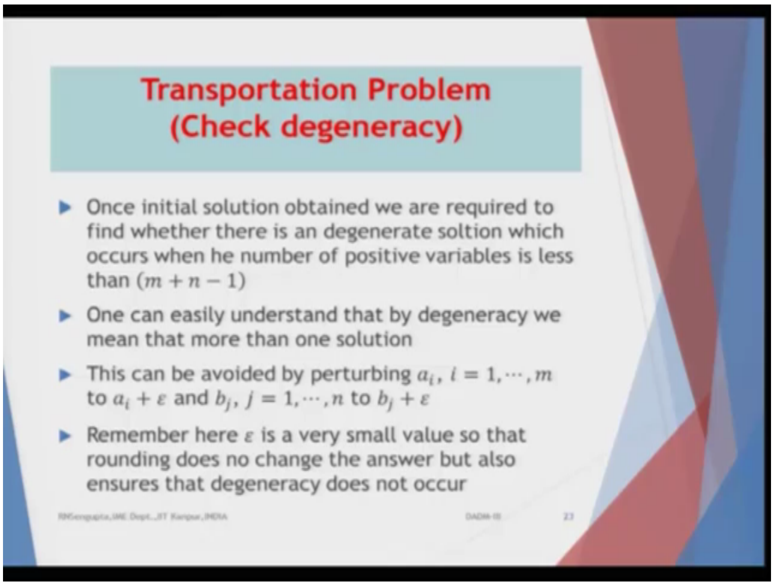
You cannot switch from row to column and column to row depending on what I was trying to explain when you are doing the north-west corner method, you go step-by-step. Suddenly you may be tempted to go to minimum cost but that was the concept which you utilize for the Vogel's approximation method. You would not be utilizing the same concept in the north-west corner method.

In the same way, when you concentrate on the row only concentrate on the row. When you concentrate on the column only concentrate on the column. In case, row and column availability satisfy simultaneously cross of the row because that was the secondary one remove it. Find the least cost element in the column and allocate zero units to the cell and proceed accordingly. And once the column exhaustion is done which was the primary one, cross of the column and move to the next column depending on the co structure which is there.

So you switch your concentration in the first case, I am repeating it from the row then it to the column and in the second the concept when you are considering on the column minima you could basically concentrate on the column and then go to the row. Now these problems, this concept which we considered the Vogel's approximation method, the north-west corner method, the lower minima concept rule, the column minima concept rule, they would give you a good approximation if you get the best answer you are lucky.

As you saw in the Vogel's approximation method and north-west corner method may be I have not checked. Maybe the Vogel's approximation method gave you the best answer which was one lakh ten thousand something. While in the north-west corner method the value was much higher. That means that was one of the corner points where it was feasible but not optimal. So basically we have to search for the optimum solutions.

(Refer Slide Time: 10:01)



So once the initial solution is obtained based on any of these 4 methods. We are required to find whether there is an degenerous solution which occurs when the number of positive variables is less than equal to m plus n minus one. On the concept of degeneracy is like that if you consider the solution which you were doing in the linear programming and if the number of rows or the number of columns based on the rank of the matrix which we considering was such that the no problem no solution or unique solution was possible then obviously it was a degenerate one.

And the number of points we remember the diagram which I drew for trying to basically in a 3-dimension one. Trying to basically define a particular corner point based on the number of planes. If the number of plane is exactly equal to number of equations as the concept was in the Cartesian coordinate, you got unique point it was not if the number of such planes of the equations are more than what is required, obviously you got that degenerate point that is what is being mentioned.

One can easily understand that by degeneracy we mean that more than one solution is possible. So how do you overcome that? We basically do a small perturbation. In the sense, that for each of the a is and b js which you have a is and the b js which you have depending on how the a i and j's have been defined. You add a small perturbation of absolute value that means some small extra amount of value which should tend to 0 and you will try to do the allocation in such a way that the end result would be such the row values adding up would be exactly equal to b values which you have on the right-hand column.

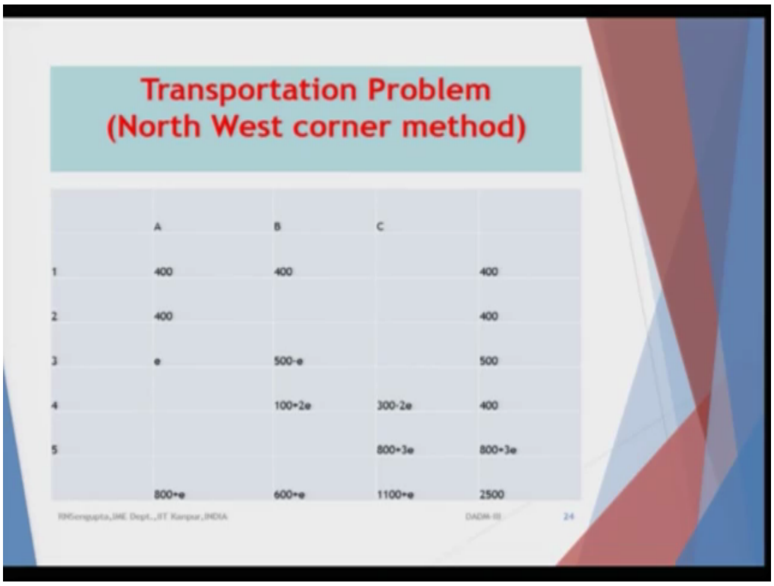
And the column well is when you add up it should be exactly equal to the bottom most row values which you have. But in this process the delta mode of allocation you are going to do would basically shift your total co structure in such a way that you would basically get a feasible solution which can be taken up to basically kick start your process to reach the optimum value. So what you will do is that?

You will basically perturb the values of a is and take value of a i plus Epsilon. So Epsilon is a very small value and you perturb the b js, b js and a is are the, again I am repeating the right most column in the bottom most row values, b js would be done as b js for Epsilon. Remember here Epsilon is a very small value so that rounding does not change the answer of the final value which you are trying to find out.

But it will ensure that degeneracy does not occur and the final values which we will get, so even basically trying to shift if you consider the problem of linear programming degeneracy was that if there were more than one answer and or the corner points was been defined by more than number of equation which was needed depending on the dimension of the problem.

So what you are trying to do if that you are trying to shift some of the constraints such that their actual use in trying to find out the corner points are not required, they can be considered as redundant or not be utilized such that the unique solution is applicable that is what you are trying to do by adding the small perturbations which are there.

(Refer Slide Time: 13:22)

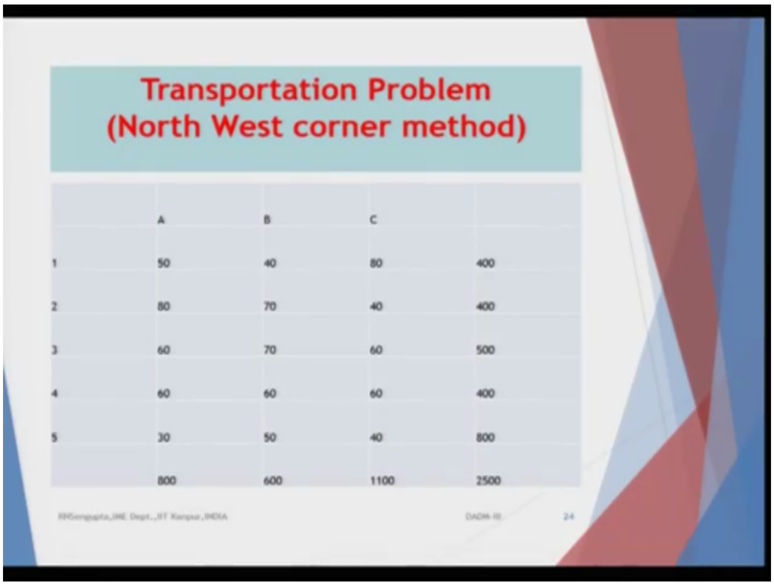


So consider this I will give you an the perturbation concept so that they can be considered. So if you consider the initial problem you at A, B, C as the origins. 1, 2, 3, 4, 5 as the destination. Total quantum of goods required was like this, it was basically 400 for the first, 400 for the second, 500 for the third, 400 for the 4th and 800 for the last one.

Similarly, when I go along the column it is 800 for the A, 600 for B and 1100 for C. Now see this E values which are Epsilon actually, so this Epsilon value which you are adding they are so small in values that the total column wise or the row wise addition always remain the same. So what you will try to do is that you will try to basically shift this value of Epsilon in such a way that they are concentrated on those points where you try to basically get the minimum allocation.

Now this values which you have actually this should not be 400 here. So this values which you have technically would basically increase your allocation in such a way that you will be concentrating on the minimum cost or the lowest costs which you have. So let me copy the co structure is for us to understand, that I am going back to the other slides so sorry for that so see here.

(Refer Slide Time: 14:47)

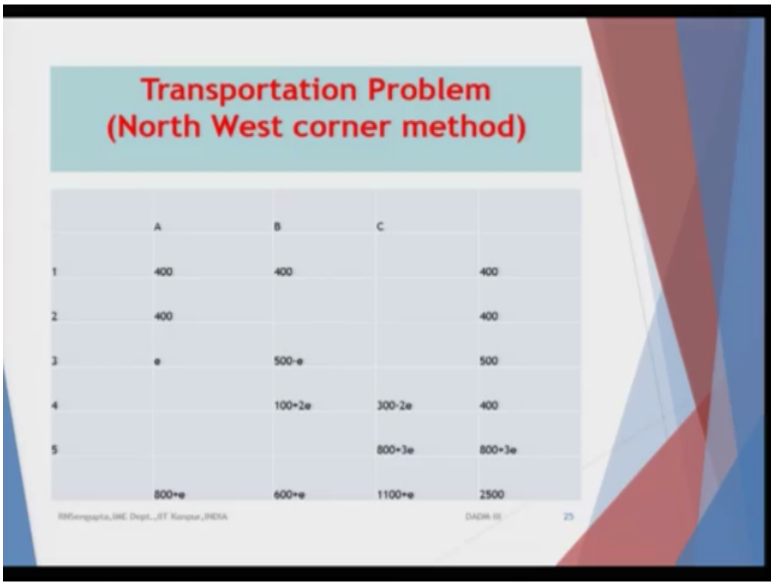


The costs were like this 50, 80, 60, 60, 30 and the corresponding values for b column were 40, 70, 70, 60, 50 and for the last column it was 80, 40, 60, 60 and 40. Now if 400 is allocated there so technically by the north-west corner method remember this is important. 400 was also being done for 80 cost but technically logic says by north-west corner method, I will do that. But is it actually the right one, there are other cost like 60, 60, 30 which are much lower.

So obviously if they are not being meant so in that case we have basically forced our self into a sub optimal point. So in order to overcome that we will basically shift this value. So obviously it could have been shifted, actually this 400 could have shifted to the 30 unit cost per unit. But we have already allocate 400 to the 80 cost, so we will remove some of that value to the 61 and basically go in a cycle such that the balancing is done along the rows and along the columns.

And once it is done we will basically change these values find out the Epsilon values and find out the total quantum of deduction which is to be done such that real locations are done of for the total products which are the x ij in such a way that the total cost basically decreases. So this is what the actual value look I will come to that this problem solution with another problem.

(Refer Slide Time: 16:36)



So rather than 400 I am basically trying to shift some value to the lower value so this E Epsilon value should be such that technically this 400 value which is there in N in the cell A2 should be decreased as far as possible such that amount which is there of 80 unit per unit cost can be utilized as low as possible. So if we are able to reduce by one unit, so it becomes 399, I am able to decrease my cost by 80 rupees but obviously it would mean that I have to basically pass on that one unit of transportation to either cell A3 or A4 or A5 but as the cost are less obviously we will save something.

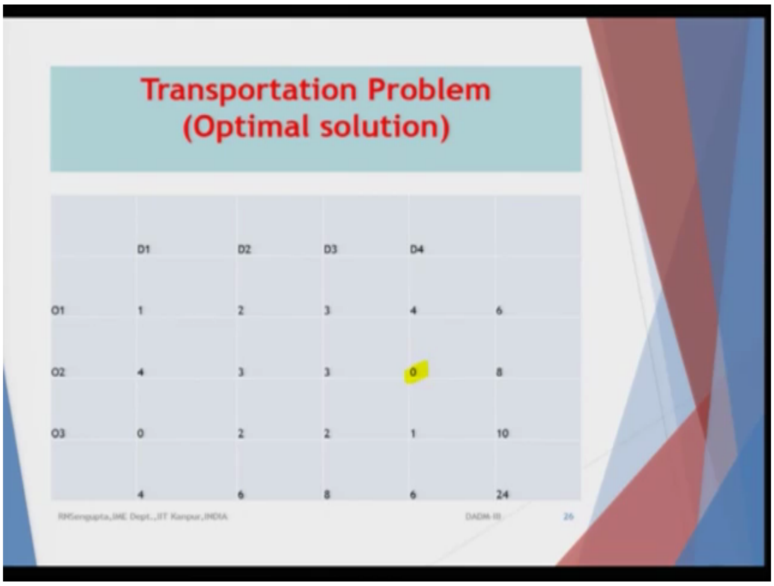
In the same logic if we go so the total co-structure which will be there for the b column is that we will be trying to utilize some extra amount of goods to be transported between b and 4. That means it becomes 100 plus 2 Epsilon and correspondingly the value decrease in b 3 would be by some Epsilon value 500 minus Epsilon such that the total cost if you consider along the rows and along the columns are increasing or are decreasing that is not the problem.

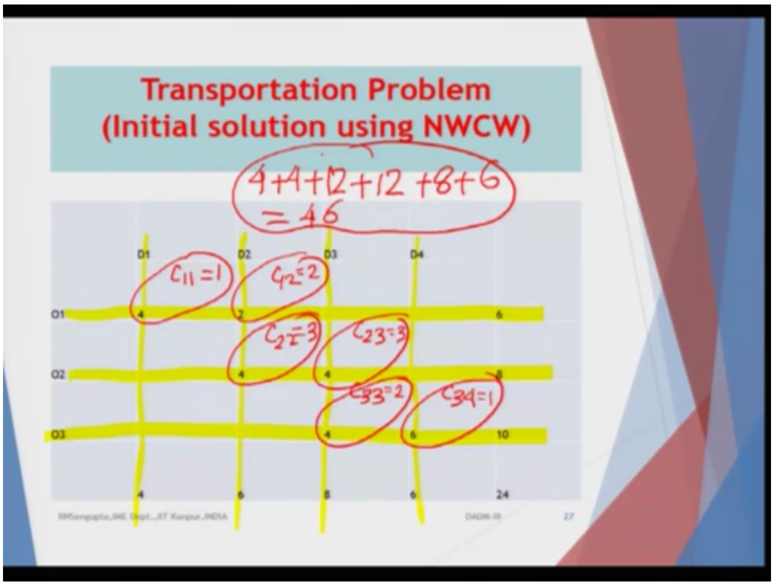
What we need lookout is the total value of transportation the total combine A and B always is 2500 such that any value change in Epsilon which is very small value would not basically effect our answer. So similarly the values which we have for the C column are 300 minus 2 Epsilon and 800 plus 3 Epsilon such that the total value which is in the bottommost cell in column number one this C is 1100.

Actually it should be 1100, is 1100 per epsilon and if I consider the row wise there are 400, 400, 500, 400 and 800 plus 3 Epsilon. The epsilon value being very low so obviously the total transportation values for all the rows and the columns add up to what it was actually given.

Now let us consider the concept of the optimum solution and I will be utilizing this concept of degeneracy later on. So you have the destinations D1, D2, D3, D4 and you have the origins of O1, O2, O3 so the total destinations are depending on how you basically consider the M and N.

(Refer Slide Time: 19:13)





It is 3 4 or 4 3 whatever it is. The cost structures are given, the cost structures are from D1 to O1 is 1 D2 to O1 is 2 I am reading in the rows D3 to O1 is 3 and D4 to O1 is 4 and the total quantum of goods required or to be transported or required at O1 is 6. So 6 units are required and whatever the costs are. Similarly, when I consider the second row, so for origin two the total concept of the cost structure is 4, 3, 3, 0. So 0 means that if I transport any goods from D4 to D4 and O O2 in that cell so let me highlight it these value zero.

So is 0 and which practically is impossible that means I can transport any amount of goods between D4 and O2 provided that allocation is being met till that level the total cost would be always 0 you know. When I go to O3 the corresponding cost structure value as 0, 2, 2, 1 and by the way I missed it the total quantum of goods required by O2 is 8 and the total quantum of goods required by O3 is 10.

And if I consider the destinations D1, D2, D3, D4 the destinations actually would be requiring or if you consider the total quantum of goods which are to be supplied by those destinations are 4, 6, 8, 6 so if I add up the columns of the rows which are the addition of the a's or the b values it comes out to be 24.

Now I follow this principle let us go back to there. So I consider the value of D1, so I am basically considering the north-west corner method. I have 4 units the total concept is 6, so what I do is that? I allot 4 quantum at O1, D1 cell, so D1 is met let me to mark it. So D1 is met. So which I cannot do any more from D1 so but O1 is still to be met so I go to D2. D2 as 6 units so out of the 6, 6 is this 6 units and O1 needs 2 more so 2 is done. So 4 plus 2 is 6 is done, so the total allocation for O1 is done.

Now D2 has still for units it gives 4 units to this, so 4 plus 2, 6, 6 is done. Then I come to so that means both O1, D1, D2 are all complete and exhausted. Then I come to O2, O2 has already consumed 4 units it needs total 8, so it needs an extra 4. So it asked from D3, D3 gives 4 so O2 basically exhaust its allocation of 8.

Now D3 has still for units because 8 minus 4 is 4 so now it will look in the next origin, the origin requirement is total 10. So D3 basically has extra 4 ,so 4 is gone and then the 6 amount of left so D3 is gone.

So when I come to O 3, O3 has total 10, 4 is met, 6 is met by D4 so the total allocation for O3 is also done and D4 for 6 is also done. So the total quantum requirement is 24 is allocated. So initial solution we use the north-west corner method remember. So if you had basically gone into the concept of Vogel's approximation method it would be like this let me go back.

So I will check the minimum cost, so 4 unit is required, I immediately go to the zero value and give it there okay. Before I go for that so if I am basically considering this north-west corner method allocation so it would be let me write the values in order to make our understanding so this is C 11, this is C 12, C 22, this is C 23, this is C33, this is C 34 and this is the costs and this is not, not in required so this is not there. So what are the cost let me write it down. So this is one what is C 12 is 2 and 3, this is 2, this is 3 so another cost is 2 and 3 then is 3 and 2, is equal to 3, is equal to 2 and the last value is 1 so this is 1.

So the total cost would be 4 into 1, 2 into 2, 3 into 4, 3 into 4, 2 into 4, 1 into 6. This 4 plus 4 plus 12 plus 12 plus 8 plus 6 the answer to it 4, 4 plus 8, 8 plus 12 is 20, 20 plus 12 is 32, 32 plus 8 is 40, 40 plus 6 is 46. Now let us go to the Vogel's concept. A Vogel's was given what you will do? You will check the minimum row, so 0 cost you will put everything in in zero so 4 would be met.

Then once this D1 is met based on the fact that 4 has been utilized, you will check that how much is O1 willing to give. It was willing to give 6 so obviously 2 is still left then when I go to the next concept of the cost. So now it can give to technically O1 can give to 2, 3, 4 at this cost but it will come at the cost of 2. It will allocate that amount into of 2 units at a cost of two.

So in that case O1 will be exhausted, D1 be exhausted then when we come to O2, O2 will immediately see that there is a cost of 0 so we will try to basically give the total amount of requirement for D4 which is quantum is 6 it will exhaust 6 out of the 8 at a 0 cost and basically have the 2 units left. If the 2 units left then it can go into any because D1 is already gone, D2, D3 is left so it can do arbitrarily at a cost of 3 and then basically proceed.

So if 3 is done so obviously it will go to the next level so D1 is exhausted, D4 is exhausted, O1 is exhausted, O2 is exhausted and the rest quantum at a cost of 2 would basically allocated to O3. So obviously you will get a different answer accordingly so with this I will I will I will close the third lecture. In the seventh week which is a thirty third lecture and continue discussing about the actual approximation actual problem solution in the transmission problem and then further go into the other areas of nonlinear programming. Have a nice day and thank you very much.