

**Data Analysis and Decision Making - III**  
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**Lecture 32**

A welcome back my dear friends. A very good morning, good afternoon, good evening to all of you and as you know this is the course under the NPTEL MOOC series which is Data Analysis and Decision Making and as you can see from the slide we are in the 32nd lecture which is in the seventh week and this total course duration, total spread over is for 12 weeks which is basically 30 contact hours which when broken up into number of lectures is 60 and as you can see we have finished half of the total set of lectures which is 30 in number which means that we have already covered the the total number of weeks is 6 and you have already taken, 6 assignments after each week where each week.

Where each week we have basically five classes of half-an-hour each and after the end of the 12 weeks where we will be basically taking 12 assignments you will be taking the final examinations also. And, my good name is Raghunandan Sengupta from ME Department IIT Kanpur.

So, if you remember we are considering the Vogel's approximation method, the concept, so it was basically the concept was for any row or column you will basically concentrate that the minimum cost where you can start with, try to concentrate on the minimum cost per unit and try to push the maximum quantum in that particular destination and basically do that allocation in such a way that if some spillovers are left, by the word spillover I mean some total quantum of goods are still to be supplied from the origin or total number of goods are still to be supplied to the destination from or to from the origin or to the destination we will basically use the minimum cost and basically fill up the total allocation based on the values of B's or the A's values which are there on the right most column and the bottom most row. So based on that we are trying to basically do the allocation for the first step.

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### Transportation Problem (Vogel's Approximation Method)

Following the VAM rule we have

- ▶ Transport from factory A to warehouse 05: 800 ✓
- ▶ Transport from factory B to warehouse 01: 400 ✓
- AND
- ▶ Transport from factory B to warehouse 04: 200 ✓
- ▶ Transport from factory C to warehouse 02: 400 ✓
- AND
- ▶ Transport from factory C to warehouse 03: 500 ✓
- AND
- ▶ Transport from factory C to warehouse 04: 200 ✓

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### Transportation Problem (Vogel's Approximation Method)

|   | A            | B            | C            |      |
|---|--------------|--------------|--------------|------|
| 1 |              | 400 $c_{12}$ |              | 400  |
| 2 |              |              | 400 $c_{23}$ | 400  |
| 3 |              |              | 500 $c_{33}$ | 500  |
| 4 |              | 200 $c_{42}$ | 200 $c_{43}$ | 400  |
| 5 | 800 $c_{51}$ |              |              | 800  |
|   | 800          | 600          | 1100         | 2500 |

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So if you remember, so we were transporting if you consider a column, a column you see the total quantum of goods was 800 total needed was at five was 800 which was the minimum cost you push everything there and finish the allocation so A is finished, A is finished means A has been exhausted in in transporting quantum of goods and B has also been fulfilled then you will basically go to B, so now when I consider B it will basically a portion at the particular minimum cost 400 to the destination 1 and 200 to destination 4 and in this way as you go you will basically

from the last, the last allocation which is there from the C origin you will basically give a quantum of 400 to second destination 300 to third destination and 200 to fourth destination.

So fourth destination if you remember the total was basically given as, so this is fourth one was basically and five has already exhausted, five has already exhausted so this is done because this is based on the allocation. I am just going to the next slide to just show you the allocation. So 800 already done. So B was there, I will use a different colour. So B was 400 here, B was 200 here, total is what 600, let us see. 600 is already done. Follow the column and that is based on the minimum cost allocation, the cost matrix which was there.

Then when I go to C, the column, the values so from C, I will basically have 400, 500 and 200, the total one is what 1100, let us see. It is 400, 500 200 which is 1100. And if I basically follow the rows, 5 was already fulfilled by 800 which is here, which is already there I put up black mark So this is done.

So when I am considering the rows. When I go to 2, 2 has 400, so let us see, 2 has already been exhausted, 2 is done. When I go to the next, these are the minimum cost, I will come back to the cost structure again so you will understand it. 3 had 500, so it is already been exhausted. Let us see 300, 500 is done for 3 and the last value was 4, so 4 was 200 plus 200 is 400 which is done. So 4 is 200 and 200 already done and the values of 5 which is already done is taken care of. Now let us see the cost structure, so you will be, you will be convinced that based on the minimum values of the cost we take we basically allocate that any spillover we try to push it to the next higher level of cost, as low as possible.

So what we will do is this. So I am, I would not fill up the cells values, only if they are filled up I will fill up the values. So I will use the red colour. So this value has a cost structure of C1, this value has a cost structure of C, so I am considering these as 1,2,3,4,so and also, so C12,C23,C33, C42,C43,C51 ,so let us check what are these values are and also even though I am repeating it, please bare with me ,the column wise column, rows adapt the same value which is 2500. Let us see what is C12, C23, C33, C43, C42 and C51?

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**Transportation Problem (Vogel's Approximation Method)**

|   | A   | B   | C    |      |
|---|-----|-----|------|------|
| 1 | 50  | 40  | 80   | 400  |
| 2 | 80  | 70  | 40   | 400  |
| 3 | 60  | 70  | 60   | 500  |
| 4 | 60  | 60  | 60   | 400  |
| 5 | 30  | 50  | 40   | 800  |
|   | 800 | 600 | 1100 | 2500 |

Handwritten annotations:  $C_{12}$  (above B1),  $C_{42}$  (above B4),  $C_{51}$  (above A5). The value 30 in cell (5,A) is circled. The values 400, 400, 500, 400, 800 in the rightmost column and 50, 50, 40 in the bottom row are highlighted in yellow.

**Transportation Problem (Vogel's Approximation Method)**

|   | A   | B   | C    |      |
|---|-----|-----|------|------|
| 1 |     | 400 |      | 400  |
| 2 |     |     | 400  | 400  |
| 3 |     |     | 500  | 500  |
| 4 |     | 200 | 200  | 400  |
| 5 | 800 |     |      | 800  |
|   | 800 | 600 | 1100 | 2500 |

Handwritten annotations:  $C_{12}$  (above B1),  $C_{23}$  (above C2),  $C_{33}$  (above C3),  $C_{42}$  (above B4),  $C_{43}$  (above C4),  $C_{51}$  (above A5).

Now I will be switching from one of from this slide and other. So if I consider the first allocation, I took up A, the minimum cost of 30. So it is 30 here. This is here 30 cost basically, so the total cost incurred for transporting from A to 5 is 30 into 800. Next let me come to B. So I need to have  $C_{12}$ , another value is  $C_{42}$ . So the allocation if it was done for 50, your question would be why not 50? 50 is not possible because already I allocate the total 800 into 5.

So nothing can be done now. So this was not done, not done means not possible, so input the highlight is that this is only done complete, both. So next was 40 was done, then the next level

rather than 50 will be 60, so your allocations would ,40 into C12, not 40, this 400 into 40 and this value is basically 200 into 60. So we will check it and the last values were, the total cost, the total quantum was 400, 500, 200. So the quantum allocations were like this, if I go back.

So this was already allocated, so this is closed so the only po possibilities were these three and the allocations already you have done here so something would be left here at a cost of 60, for this I will take the minimum cost to 60, for this I may take the minimum cost as 40 and that rule was basically, actually intending the fact that we are going in the right direction.

So if I consider the costs, it will be 400 into 40 plus 500, I am just checking the cells C23 into 400, so that was 400 into 40 then 500 into C33, 500 into 60 and the last value, because 200 was still left, 200 into C43 which is 200 into 60. So in this way I have allocated all the quantum. So if I see the cost, again, I will repeat the values.

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**Transportation Problem  
(Vogel's Approximation Method)**

► Hence total cost is  $800 \times 30 + 400 \times 40 + 200 \times 60 + 400 \times 40 + 500 \times 60 + 200 \times 60 = 1,10,000$

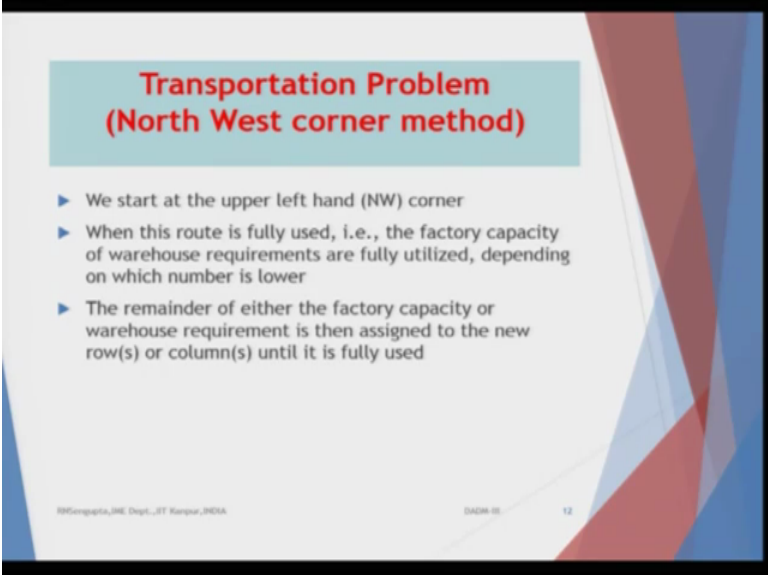
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The values are 800 into 30, the last value in the first column, then it was 400 into 40, 200 into 60, 400 into 40, 500 into 60 and 200 into 60. So all the allocations are done, find out the total cost which is 1,10,000 so this is the total cost which we will basically be, given as an answer for the Vogel's approximation method, so in the approximation is just the starting point, it may be the best possible solution may be the optimum solution we do not know we need to check it later on.

Now we go to the other method which is the North-West corner method. North-West corner method as it means that we will start rather extreme top value, allocate that based on the row or the column once the exhaustion is done, we exhaust, whether the origins or the destinations we just eliminate those columns, eliminate those rows and then go to the next values and go in in in in this manner such that the allocations are done in the minimum possible cost structure. But remember, here also the actual answer we may get is not the best solution it may be the one of the best starting point is based on which we will try to basically arrive at the answer which we want to achieve, that is the minimum cost.

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**Transportation Problem  
(North West corner method)**

- ▶ We start at the upper left hand (NW) corner
- ▶ When this route is fully used, i.e., the factory capacity of warehouse requirements are fully utilized, depending on which number is lower
- ▶ The remainder of either the factory capacity or warehouse requirement is then assigned to the new row(s) or column(s) until it is fully used

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So the process or the algorithm or the overall concept works like this, we start at the upper left corner, when this route is fully used so we will start, check the value we will ask that what is the total quantum of goods which is required at that destination. Is the origin able to meet that? if it is able to meet that, so that is finished.

In case if there is a the equality sign between the origin and the destination then both the origin and the destination are finished, finished in the sense nothing can be done for them, but if there are spillovers in this case the origin has some extra amount of goods to be transported or the destination has some extra goods to be transported, so one is from where you are going to transport is the origin and where you are going to send is basically the destination and if there are

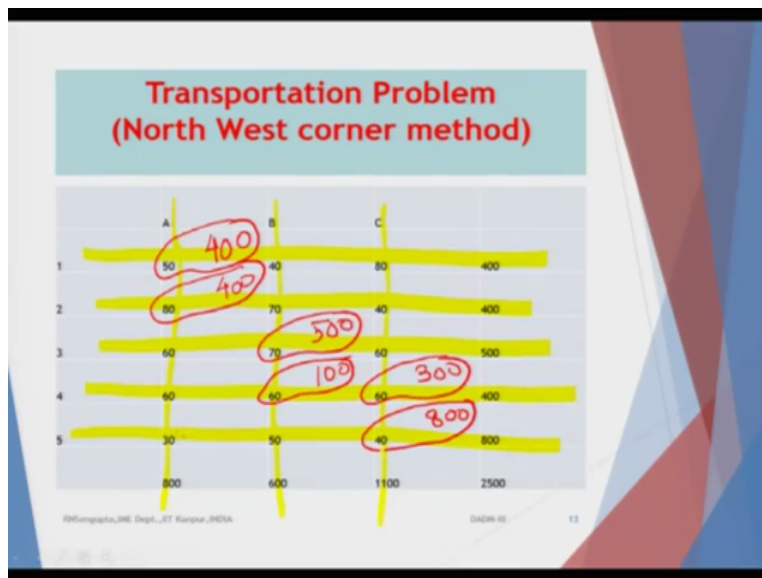
some spillovers left we will basically follow the rules of trying to basically minimize the cost accordingly.

So it says, it reads, the second point when this route is fully used as the factory capacity of the warehouse requirements are fully utilized depending on which number is lower, you will basically exhaust that and move on. The remainder, as I said the remainder of any of the factory capacity or the warehouse requirement, factory capacity is where you are going to send is the destination and the warehouses are basically the origins. So the remainder of the either of the factory capacity or warehouse requirement is then again assigned to the new rows or columns until it is fully utilized and we move in this direction keeping the minimum cost structure in our mind.

So in this, the main difference between the Vogel's approximation was that, I will just check the minimum cost, allocate their and then move according to the penalties which are there, penalty means because the higher the cost are higher the optimum value will be which is not our main concern, our main concern is basically to decrease the cost as fast, as far as possible. But in North-West corner method, we do not go along the column, we just first concentrate on the corner point allocate it and then move it move accordingly.

So when we go there, so basically I will first give you the the allocations, so let me check, I think best is to basically bring the diagram here. Yes that would be best. So this we are going in the North-West corner, will copy it here, so this would be easier for us to understand. So how the method would work, we save it.

(Refer Slide Time: 14:05)



**Transportation Problem  
(North West corner method)**

Following the NWCM we have

- ▶ Transport from factory A to warehouse 01: 400
- AND
- ▶ Transport from factory A to warehouse 02: 400
- ▶ Transport from factory B to warehouse 03: 500
- AND
- ▶ Transport from factory B to warehouse 04: 100
- ▶ Transport from factory C to warehouse 04: 300
- AND
- ▶ Transport from factory C to warehouse 05: 800

Source: IIM Bangalore, IIM Dept., IIT Kanpur, IIMDA  
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Now you see when you are starting with the North- West corner concentrate on the cell, the first one A1. You will check that what is the total quantum of requirement which is there for 1 which is going to come from A. 1 needs 400; A has 800, allocate all 400 there. So there are spillovers, we will go to the next cells accordingly. So this is how the process will work.

Now let us see how the allocations are done. I will switch over from this slide of the allocation and go to the actual cost structure along with the demand and supply which is there



such that you can understand. So the first point it mentions is Transport from factory A to warehouse 1 400. Let us see whether that is true.

So as A can give 800, it requires 400, so allocate 400 here. So I will if it is done, I will mark with the yellow highlighter. So this is done. The total cost is going to come is 4. So I should not be utilizing the. So total cost is now this. So 400 is still left from A, so let us see what we will do. Now when we see because why I am coming to the answer you will can understand automatically.

Now you will come to the transportation from factory A to warehouse 2 which is 400 now. Now there is a conceptual change. Technically you would have been tempted to transport all the goods to the 5th 400 so that means the cost would have been 400 into 30, that is basically the actual algorithm, logic which was being used for the Vogel's approximation method, but now you are just going to once the North-West points are done you can come to the next point. So the next point is basically, 400 is still left, you do it for 2, 2 is exhausted, is finished, so 400 is done.

Total cost is 400 into 80. So let us see, these are approximation method, remember that, so you may not get the best answer. So transport from factory A to warehouse 2 400. So warehouse 2 is already done, warehouse 1 has had already been done, but now interestingly see you have already allocated 400 400 from A. So A is also done, so A is allocated, so you cannot concentrate on A anymore, you cannot concentrate on 1 anymore, you cannot concentrate on 2 anymore. Now let us come to B.

So the B point would basically mean because the next level will come 1 done, 2 done so B can only concentrate on 3rd destination. So transport from factory B to warehouse 3 which is destination 3. What is total amount? It mentions 500. Let us see, what is the logic? When you coming from B double check what is the total quantum which can be supplied by B it is 600, what is requirement for 3 is 500? So allocate everything there, so you will basically allocate 500 from here. This will complete the requirement of 3, 500 is done.

What is the extra amount left? Extra amount left is 100. Now where would you go? Technically, if 100 is still left, you would have gone to 5, but as per the North-West corner method we will go to the next one which is 4. Let us see whether the logic is right. Here, transport from factory B, the left over amount which is 100 to warehouse 4 which is the fourth destination and it is exactly

what I said. I will be basically allocate 100 here, total cost initially was 500 into 70. Now the total cost is 100 into 60.

Now let us see, is 4th met? No, 4th did require 400. We have been able to give 100, so there is still left over, still demand to be met is 300, so we will allocate that, from where.... From the next origin. We will come to that. But let us see, why are we asking the question that we will be allocating it from the next origin? Because B had 600, it has already exhausted 500, trying to supply to 3rd and already exhausted the rest amount of 100 in order to supply to 4. So the whole amount of B is already over

Now if 300 is left, who is to supply to 4? See, when I go to the next point, technically, C would be supplying to 4, 300 of them at what cost, at a cost of 60. Technically would have been best if you have been able to supply at the rate of 40, but North-West corner does not give that, so let us see.

So transport from C to warehouse 4 300, let us see whether it is right. 100 already gone, extra left is 300. So we basically give 300 from C at a cost of 60, so the total cost is 60 into 300. So the total allocation of 4 is done, 4 has been fulfilled. So this is done, so the extra amount left. Let us ask, what is this? From C, C has still huge amount of goods to be transported. So 1100 minus 300 which is 800. So let us see where it can go.

The next level is 5, let us check what is the total requirement for 5 is 800. So you will allocate the total amount 800 to 5. So the total cost is 800 into 40. So 5 has already finished it's all utilization it is met C has already finished, all as allocation met. So the total cost would be 400 into 50. So we are going in the north corner points, move move move like this. 400 into 50 ,400 into 80 ,500 into 70, 100 into 60, 300 into 60, 800 into 40 and obviously it means all the rows of the columns are met.

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| Transportation Problem<br>(North West corner method) |     |     |      |      |
|--|-----|-----|------|------|
|  | A   | B   | C    |      |
| 1  | 400 |     |      | 400  |
| 2  | 400 |     |      | 400  |
| 3  |     | 500 |      | 500  |
| 4  |     | 100 | 300  | 400  |
| 5  |     |     | 800  | 800  |
|  | 800 | 600 | 1100 | 2500 |

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| Transportation Problem<br>(North West corner method) |     |     |      |      |
|--|-----|-----|------|------|
|  | A   | B   | C    |      |
| 1  | 400 |     |      | 400  |
| 2  | 400 |     |      | 400  |
| 3  |     | 500 |      | 500  |
| 4  |     | 100 | 300  | 400  |
| 5  |     |     | 800  | 800  |
|  | 800 | 600 | 1100 | 2500 |

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So if I see the cost, as it is, this is the allocation, when I come to the cost. Cost are exactly this 400, 400, 500, 100, 300, 800, total cost though total quantum have been met along the rows and the columns. So this is the cost structure, I due it but I am just replicating the cost structure and the allocation just for your own own understanding. So this was the allocation which was done, the cost structure which was there, allocation has been done here.

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**Transportation Problem  
(North West corner method)**

► Hence total cost is  $400 \times 50 + 400 \times 80 + 500 \times 70 + 100 \times 60 + 300 \times 60 + 800 \times 40 = 1,43,000$

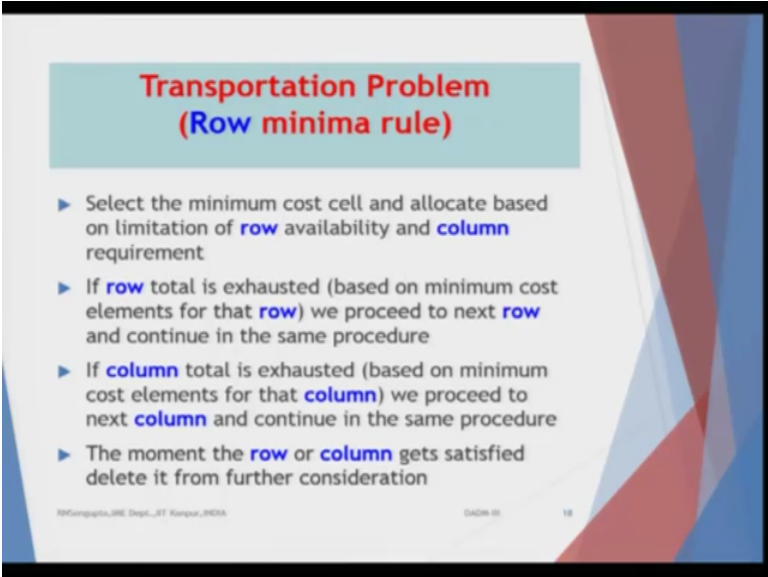
Now when we were doing the North-West corner method, I was mentioning time and again. See the cost this is lower, we were tempted to given allocate there, but that would have broken the logic of trying to basically allocate the total quantum of goods which are basically transporting from any destination to any of the, to any of the origins to the destination, that means from the warehouses to the factory.

Now, in both of these problems Vogel's approximation method and the north-west corner method, we basically found a starting point. So there are other methods also which I mentioned so I will basically go logically and discuss the ideas for the row minima rule and the column minima rule. Now the row minima rule and the column minima rule the concept is similar, but in

the row minima rule you are on the all the concentrations are on the rows and the secondary concentrations are on the columns.

While in the in the column minima rule all your initial primary concentrations are on the column and the secondary concentrations are on the rows. So you will try to basically fulfill the demands supplying and the allocation based on the cost as well as the allocation which are there for the rows first, in the row minima rule and then only go into the column to meet their demand, that means for the column demand. And while for the column minima rule your main concentration would be to minimize and optimize the columns, and then any spill over's basically goes to the row. I am repeating it, but it will become very simple when you understand.

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**Transportation Problem  
(Row minima rule)**

- ▶ Select the minimum cost cell and allocate based on limitation of **row** availability and **column** requirement
- ▶ If **row** total is exhausted (based on minimum cost elements for that **row**) we proceed to next **row** and continue in the same procedure
- ▶ If **column** total is exhausted (based on minimum cost elements for that **column**) we proceed to next **column** and continue in the same procedure
- ▶ The moment the **row** or **column** gets satisfied delete it from further consideration

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CHAPTER 10

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So when you go to the row minima rule the work goes like this, you select the minimum cost cell and allocate based on limitation of rows availability and column requirement. If you go through the rows, I will come to the problem later on because first I want to basically finish the concept of the actual problem solution after the the concept of Vogel's and Approximation and the North-West corner concepts are done for starting the feasible point as a best start starting point to basically reach the optimum.

So, in the row minima rule you will basically start of the rows availability and the column requirement, if the row total is first you concentration is the row, that means the the in this case it was 1,2,3,4. So if the rows are exhausted the requirements are done for the first one depending

on the minimum cost, based on the minimum cost as I said we will proceed to the next row, based on the cost and proceed accordingly. So we will check the row where is the minimum cost allocate everything.

If there is some spillover go to the next allocation if the column is still unfulfilled and if the column is done then you go to the next row and basically row column, the primary secondary the row column, primary secondary and you go accordingly. If column total is exhausted, based on the minimum cost element of the column we proceed to the next column if they are left because the rows are important for us and you continue in the same procedure doing the row column allocation in the primary secondary manner.

The moment the row or the column get satisfied you delete them from their further consideration because if the origins have supplied everything, or the warehouse have supplied everything you remove them, or they have been met with all their demands, remove them from concentration you will go that in this way concentrating on the row column and proceed accordingly.

With this I will close this second class for the seventh week and continue discussion the row minima rule and the corner column minima rule that is easy for us basically go further actual problem solving. Once we have we are done with the approximation solution to start with we will go into the actual optimum solution in the later class. Have a nice day and thank you very much.