Data Analysis and Decision Making III
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Lecture – 30

Welcome back my dear students, a very good morning, good afternoon, good evening to all of you wherever you are in the this part of this globe. And this is the Data Analysis and Decision Making III lecture on the NPTEL course, (())(00:26) series. And as you know this total course contact hours is 30 hours, which is split into 60 lectures, each lecture being for half an hour and the total course duration goes for 12 weeks.

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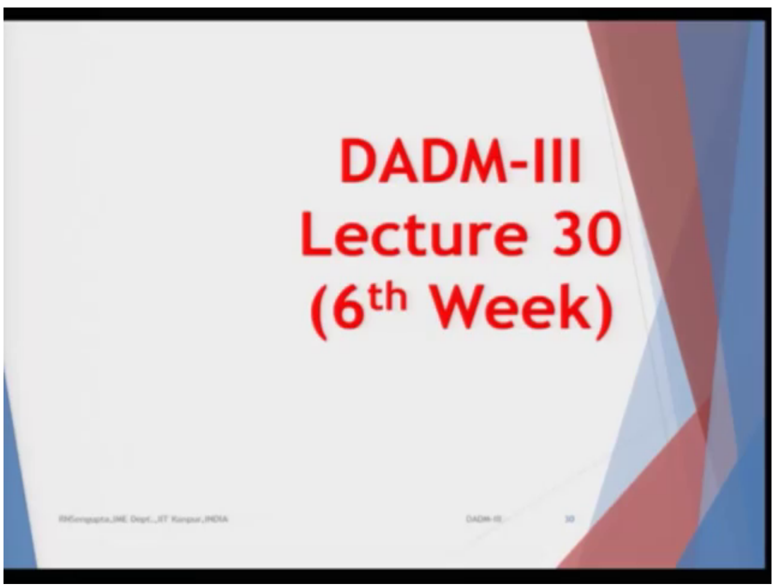

And as you can see from the Slide, we are on the 30th lecture, which is the end of the the 6th week. So each week, we have 5 lectures, each being of half an hour. So we have already taken 5 assignments because after each week we have an assignment by the end of this week, the 30th one, you will basically be taking the 6th week, which is basically the half of the overall course which we over, and obviously, after the total number of assignments will be 12 in number and after the whole course is over, we will be taking the final examination.

So if you remember, we are considering the concept that artificial variables will be important in order to forcefully ensure that there is one basic feasible solution because all of the variables which are 0 and non-zero would definitely will positive which is one of the main conditions of the assumptions for the linear programming problem which we have considered. And based on that, we basically start on the process of the maximization on the minimization problem, depending on which is the entering and the exiting variable that is a different question.

And we also ensure in the objective function the weights which we are going to give for the maximization or minimization problem for the artificial variables would be, for the maximization is as low as possible in negative sense, because we are slowing going to increase our value of the concept of the objective function till we reach maximum. And in the minimization problem we will keep it as high as possible in the positive sense.

So we will basically try start decreasing the values from the high value to the minimum value, which is ensured. Now I also showed that the starting fame basic feasible solution for the considering the artificial variables would be the case where the artificial variables would be non, non-negative some values such that initially we had a, we had a value of 11 by W and basically continuing doing the calculation till we are in the last stage of the problem, which we were basically solving.

Now in this problem formulation, we already have X_1 as decision variable which is non-zero, X_2 is a decision variable which is a non-zero, X_7 is there as a, as a decision variable which is very high, weightages will be there for the objective function because W is very highly positive, because it is in minimization problem and we will try to basically find out any improvement can be done for the, for the, from the third tablo and basically we are able to achieve the goal of trying to minimize the overall problem, okay.

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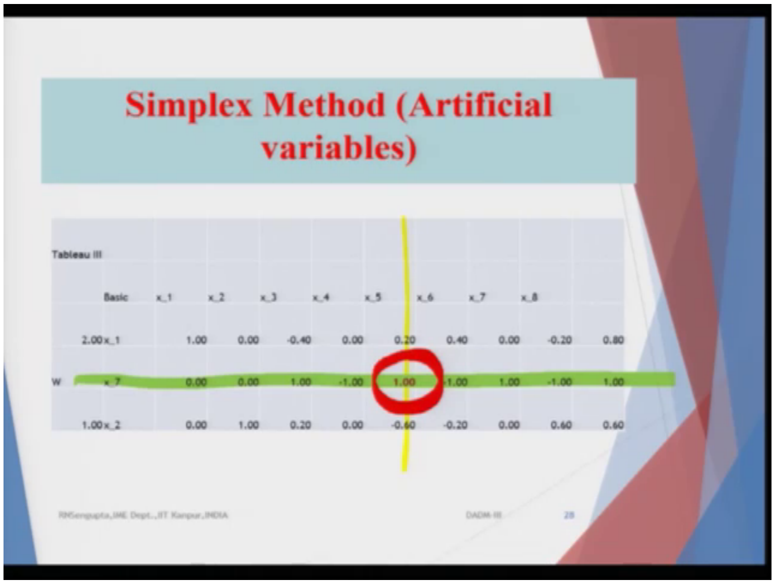

So this, this slide I should basically, yeah. So I should have basically shown this, sorry for that. So, once we basically decide, so which one will be the exiting and the entering one. So, now I consider the, the corresponding pivot row and the pivot column, I will mark them. So this is the pivot column, this would be the pivot row and the pivot element would be this one, which means, so, this is just the general structure of how we do find out the pivot row, in the pivot row column and the pivot element for the mix maximization and the minimization problem.

Now the moment the pivot row and column is decided, it means X, Y will enter, that means one of the artificial variables will enter and X_7 which is basically the, one of the sorry, when the slacks on the surplus we enter and the artificial variable which was still existing in the solution, X_7 will go out from the system.

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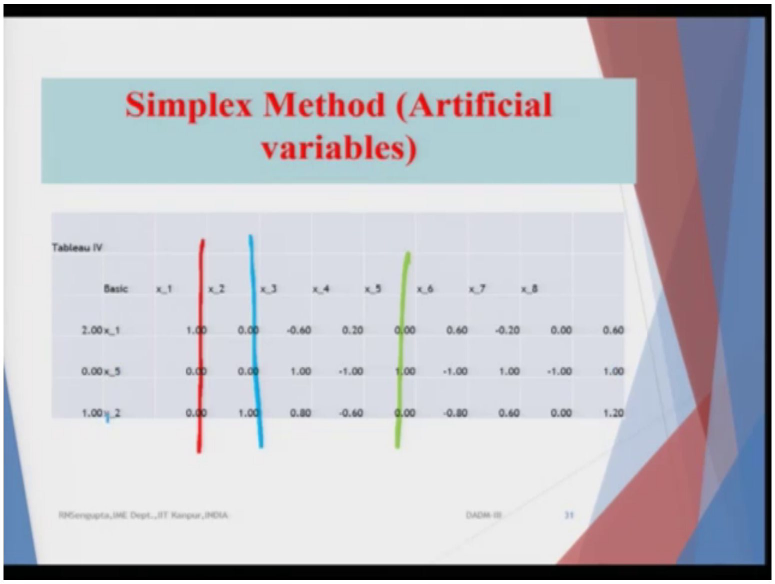

So once we basically do that, our actual solution basically becomes this, which means now I have X_1, X_5 and X_2 as the basic and which are non-zero and the corresponding values of X_3, X_4, X_6, X_7 and X_8 would all be 0 as it should be, as, as this is given. So when I have tried to find our X_1, so the corresponding column wise we will find they are 1, 0, 0.

If when I try to find out X_5, column wise it is 0, 1, 0 as it should be and when I basically try to find out for X_2, it is 0, 0, 1 and how it should be. So when I want to find out the corresponding values X_1 is point 6, X_5 is 1 and X_2 is 1 point 2 and the other values of X_3, X_4, X_6, X_7 and X_8 are 0 as is, as is it.

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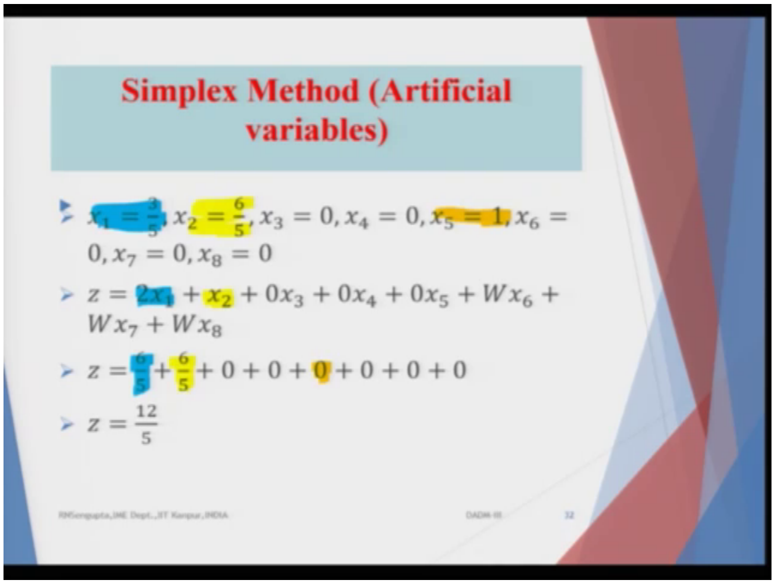

So in the problem formulation when I basically write it down, I have X_1 as 3 by 5, X_2 as 6 by 5, X_3, X_4, X_6, X_7 and X_8 are all 0, X_5 is 1. So when I put it in the objective function, so, X_1 is 3 by 5, so it would be 2 into X_1 which is 6 by 5, as it is. X_2 is 6 by 5, so, this is objective function is X_2 so it is 6 by 5, all other variables X_3, X_4, I am not coming to 5 immediately, X_3, X_4, X_6, X_7, X_8 are all 0 as it is, X_5 unit is 1, it is being multiplied by 0, hence, the total value for that is 1, 2, 3, 4, 5 so, this is the value, so 0.

So the total objective function has now become 12 by 5, that means, I basically been able to minimize the problem to the value of 12 by 5. Now it means that they would at this stage the op, optimum solution, so there would not be any exiting variables and entering variable, so the actual objective function we have ensured that considering the artificial variables in the problem formulation.

We do get a solution which basically guarantees in optimum value in the minimum case, maximization case also we can have the same thing, but it is ensured that the starting basic feasible solution which we started adhered to the properties that if any of these the variables were non-zero they were basically positives. Hence it was possible based on the assumption to start our overall search procedure from that particular point.

Now with this, I will basically go into other area, which is more of again a discussion, but I will give you a background. So consider that you are one of the transporting companies in India and this problem would be considered as the concept of transportation problem. So you are a transporting company, consider you are at the food corporation of India or say for example, you are one of the car manufacturers, maybe Maruti, maybe Tata Motors, may be Mahindra, whatever it is or you are basically one of those retailers which basically supplies goods from one part of the country to the other part.

It can be Flipkart, it can be Amazon, it can be, you can be one of those consumer goods like trying to basically produce fridge like Godrej or it can be Whirlpool, whoever it is, and basically trying to transport at different parts of the country. So what we will consider, there would be some origins from where the products are basically transported and there would be some destination. If you remember, I have discussed that and drawn the diagram, where you have basically a set of M nodes or N nodes as it is on the left hand side and there are M or N number of destinations on the other side. So we will basically say, that they are set of origins. Total number of original points are M in number.

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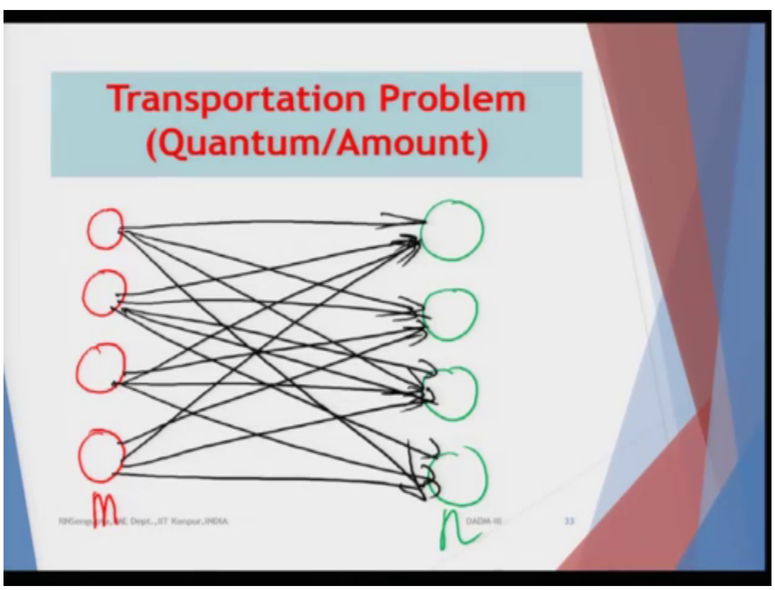

So if I am allow to draw so basically I will try to draw it , it will be easy for, so I have different type of, okay, I won’t mark it, I will, so you have two, three, four this are the origins, you have the destination one, two, three, four. So the context are happening like this, and third one. So it looks scattered, but generally this is how the network system is being drawn, and obviously, can be, they can be intermediate positions also.

Like one of the sets of destination or distributers was not the retailers, but we will consider them as it is. So the total number here is M, total number here is N. Now as you are transporting the goods, there are two important things to remember, number one is, what is the cost structure for transporting from the ith one in the M set of, of origins to the to the jth one in the set of M J number, J being the suf, the suffix corresponding to the N number of destination, point one.

So that means, what is the cost structure? Now which would be given. Point two is that which is more important, we want to find out that what is the total quantum of products which have been transferred from the ith one to the jth one, where I is equal to 1 to M on the left hand side and J is equal to 1 to N on the right hand side. Now you will basically have two such different matrices in, it should be remember that the cost structure would be given to you, while the values of the transportation, now the total quantum of the products which you will transfer from the mth origin M, M number of origins to the N number of destinations, you have to basically find out based on some pricing, principles. So what is that principle? We want to come to that within two minutes.

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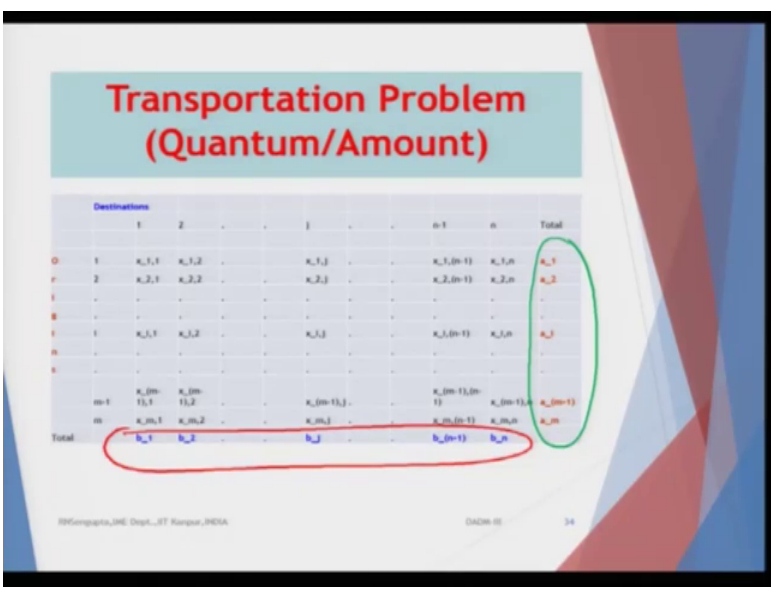

So here is how I have drawn the, the overall metrics. The metric is like this. So on the left hand side we have the origins, so the origins are marked 1 to till ith one, then it goes to M minus 1 into M and the destinations are marked as 1 to J1, till N minus 1 into N so, you, as, as the diagram showed M number on the left hand side, N number on the right hand side from my side. Now the total quantum of goods which has been transported from the first origin to the first destination is given by X suffix 11.

So this dash 1 comma 1 is the suffix, so that means I am transporting X11 on quantum of goods from the origin one to destination one. Similarly, I am going to transport X12 amount of goods from origin one to basically destination two. Similarly, I am going to transport X1j from origin one to jth destination. The second number is, second last number is X1, N minus 1 is basically the total quantum of goods which I am going to transport from the first origin to the nth minus, N minus 1 destination and to the final nth destination, the total quantum of goods manufacturing, supplied is X1, N.

And then on the right hand side I write the value of A1, which means, the total number such goods which is being supplied from the first origin, if I add them up that is X11 plus X12 till, till X1j and then continue till X1 N minus 1 and X1 N is equal to A1, that is point one. Similarly, if I see the ith origin, from ith origin I am transporting to the first destination XI1 to the second destination XI2 to the jth destination X, I, J to the N minus 1 at the destination XIN minus 1 to the nth destination X, I, N.

And the sum of all these that means XI1 plus XI2 dot dot till X, I, J then dot dot till XI N minus 1 and X, I, N is equal to A, AI, AI A suffix I. When I come to the, the M minus 1 origin, the total quantum of goods being multiplied, I am going a little bit slow I understand, but let me explain. So the total amount of goods being transport from the M minus 1 to the first destination is X, M minus 1, 1 to the second is X M minus 1, 2 till the jth one is X M minus 1, J to the N minus 1th destination is X M minus 1, N minus 1 and the last one would basically be X M minus 1, N, if I add up all these values starting from X M minus 11, XM minus 22 till the last one, second last one, which is basically XM minus 1, N minus 1, plus XM minus 1 N basically gives me the value which is on the right hand side is A suffix M minus 1.

Now if I go to the last origin, the total quantum of goods being supplied I, I would not called up all the destinations is XM1, XM2, X, M, J, XM N minus 1, X, M, N and the total value if I add up is basically A suffix M. Now if I add up all these values, this is the total amount of goods which is being supplied starting from the first to the mth origin and the total sum would be A suffix 1 plus A suffix 2 till, till the second last one was A suffix M minus 1 and the last one is A suffix M.

So the total amount is what I mean transporting from all the origins which is they're on the left hand side. Now let us check the, the concept of the eqi of the values when I am looking from the destination side. So when I am looking on the destination side, destination one is getting goods from all the M number of origins. So the first quantum which gets is X, 1, 1; X, 2, 1; X, I, 1; the second last one is XM minus 11 and the last one is X M1 with that means for the first destination, the total concept of, of the total addition which I get, if I add them all up, this one is B suffix one.

So total destination one needs B suffix 1 quantum of goods, which is being supplied by all the origins. Similarly, origin 1 or origin 2 is basically supplying to all the destination. So there is a mutual understanding between supplying and, and for the destination and the origins. Similarly for the second destination, the quantum of goods which is been supplied from the first to the mth destinations are basically X, 1, 2; X, 2, 2; X, I, 2; X M minus 1 to and X M2.

So the total sum if I add them, add up for destination 2, basically comes out to be B suffix 2. Similarly for the jth one, the values are X, 1, J; X, 2, J; X, I, J; X M minus 1J, X, M, J and the total value if I add up basically comes to BJ. Similarly, when I came to nth minus 1, the values are X1 N minus 1, X2 N minus 1, XI N minus 1, till the second last one which is X M minus 1 N minus 1 and the last one is XM N minus 1 addition of all these thing basically comes out to B suffix M minus 1.

And the last basically destination which is nth one, the quantum of supply which is happening for each and every origin from 1 to M are correspondingly like this X1N, X2N, XIN, X M minus 1 and XMN and the total value if I add up basically comes out to be the value of BN. Now a very interestingly, the total value which is being send from the destination is basically B1 plus B2 till the last one which is BN. Now if there is no extra amount being transported from either of the origins and no extra amount is basically being demanded by the destinations, if there is an exact balanced equation.

Like, in a bucket, water is entering, water is going out. And if we consider the amount of water spill over is 0, obviously, it mean the input and output balances out. In this case, the sum of all the A’s should be exactly equal to the sum of all the B’s and that will be utilized as one of the main constrains on mere condition based on which we basically try to optimize our problem.

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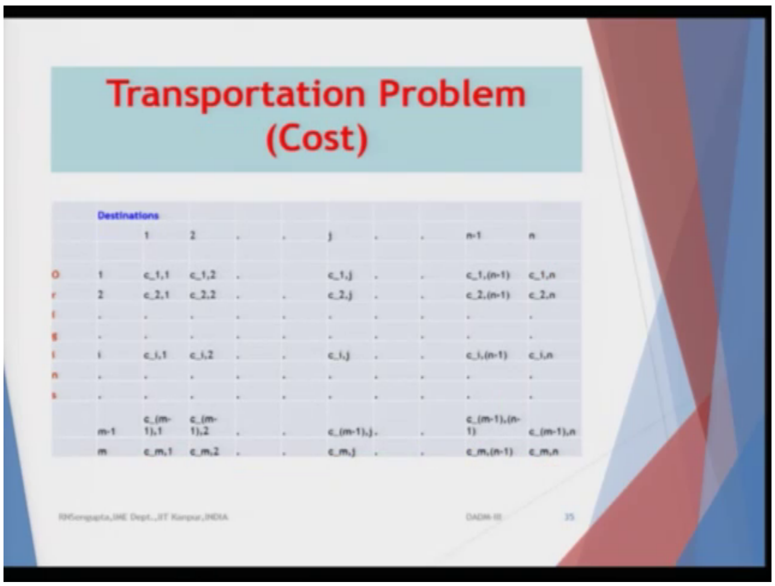

So now, this is basically the quantum of amount, now comes basically the cost structure. So the cost structures I have, again, it is in the same way. The cost structure being of transporting from one origin to one destination is C, 1, 1 from one origin to the second one is C, 1, 2 till the last one which is from one origin to nth and destination is C, 1, N. Similarly for the second origin, the cost structures I will only repeat the first and the last one values is C, 2, 1 and the last value is C, 2, N.

Similarly for ith origin, the values are, I only repeat the first and last cost structure is C, I, 1 and C, I, N. And similarly, if I consider the mth origin corresponding to the destinations one to N, the cost structure is C, M, 1. This, all these values I am reading out after the X's or the C's are all in the suffixes. So it is C, M, 1 till C, M, N. Now these cost structures are known to me based on which we will basically formulized the problem.

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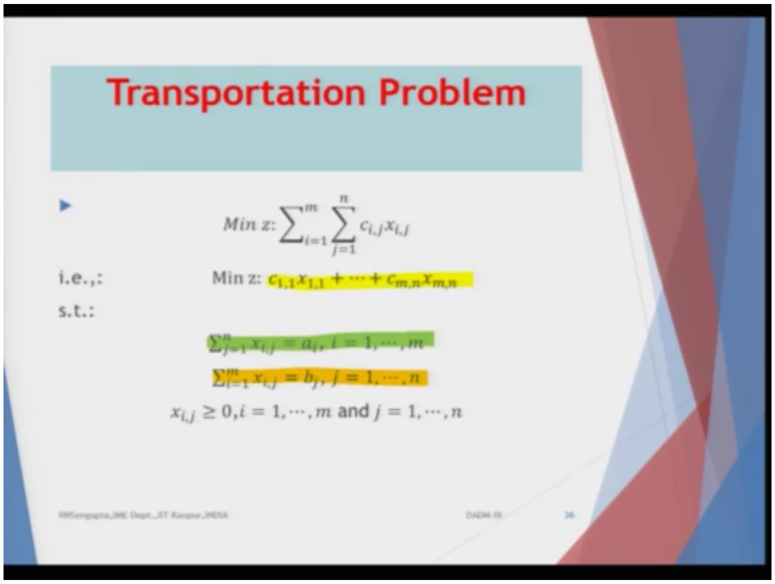

So what is the problem we want to solve? Our main problem formulation would basically be to minimize the total transportation cost based on the fact that X, I, J is the amount which is being transferred from the ith origin to the jth destination multiplied by C, I, J, where C, I, J is the cost of transporting one unit of product from the ith origin to jth destination, where I is equal to 1 to M, J is equal to 1 to N.

So this is the problem formulation. Minimization, double summation, I is equal to 1 to M, J is equal to 1 to N, C, I, J, X, I, J and I want to basically minimize. So remember it was basically a, a, a, a linear programming problem which we have done. So hence, the minimization problem as given is C11, X1 till C, M, N into X, M, N. Remember M and N need not be equal, in 1 case it can be M is greater than N on another case it can be basically N is greater than M.

And obviously, if M is equal to N, then in that case, you will basically have a symmetric metrics. Now what are the constraints? As I mentioned, the sum of each of the rows corresponding the origin one all the amount which you need to transported to all the destination add ups, add ups to adds up to A1. Similarly for the second row, addition of all the amount which had been transported from the origin two to all the N number of destinations would basically be A suffix 2.

Similarly for the last row which will have is basically which is being the total amount which has been transported from the mth origin to all of this destination one to N, if I add up is basically becomes exact equal to AM. So here I have the equations corresponding to the rows. And how many such equations there? There are M number of equations corresponding with the M number of origins that means there are M such constrains in the first set.

Now let us go to the column wise. So column wise is basically of the destinations starting from one to N. So the summations would basically be X, I, J, but here I is equal to 1 to M, but I am basically trying to count what for all this things. So, X, I, J, and I is equal to 1 to M would basically be equal to BJ which is the value, which is given at the bottom most row. So it is B1 to basically BN and that values should always balance. So I would basically have summation X, I, J, I is equal to 1 to M is equal to BJ, J is equal to 1 to N.

So now I have in the objective function there is only one objective, constraints are M and N that is M plus N and based on that we have to basically solve the problem. Now remember here, equality sign is there, obviously, we need to have basically add artificial variables if required. So the total number of artificial variables now added would be M in the first, N in the second. So that will be M plus N.

In case if the, the, the constraint were such that the total amount was less than equal to or greater than equal to, then in that case we will basically have the slack and surplus and formula the problem accordingly and then basically solved both in the artificial case, variable case as well as in the slack and surplus case the problems correspondingly. Remember here X, I, J's, are all greater than 0, I is equal to 1 to M and J is equal to 1 to N.

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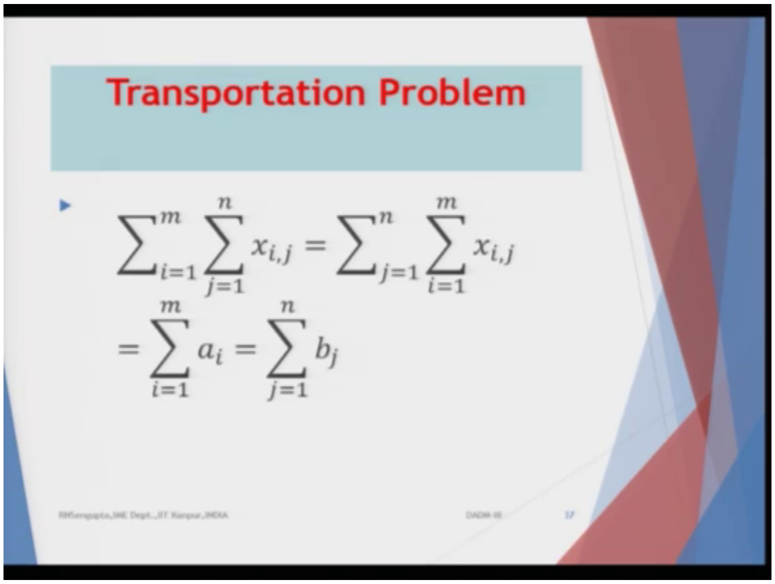

Now here the actual problem formulation is, we have one, the left hand, okay, another thing, which is very important, if I add up all the values in all the cells for the, the metrics which basically denotes the total quantum of transportation which is going to take place from each of these origins to the destination which is basically X, I, J, and you are basically going to sum up I is equal to 1 to M, J is equal to 1N that should also be equal to the concept that if I sum up all these X, I, J.

But I do first the summation from I is equal to M and 1 to M and then I sum them up from J is equal to 1 to N, the values should exactly be equal to the either the sum of all the A's or the sum of all the B's .So how is that? If the balance is there, there no spill over, there is no loss, there is no demand which is unmet or there is no supply which is basically extra supply, then the summation of A1 to AM dot, dot till AM will be exactly equal to B1 to BN and basically adding up all these values accordingly.

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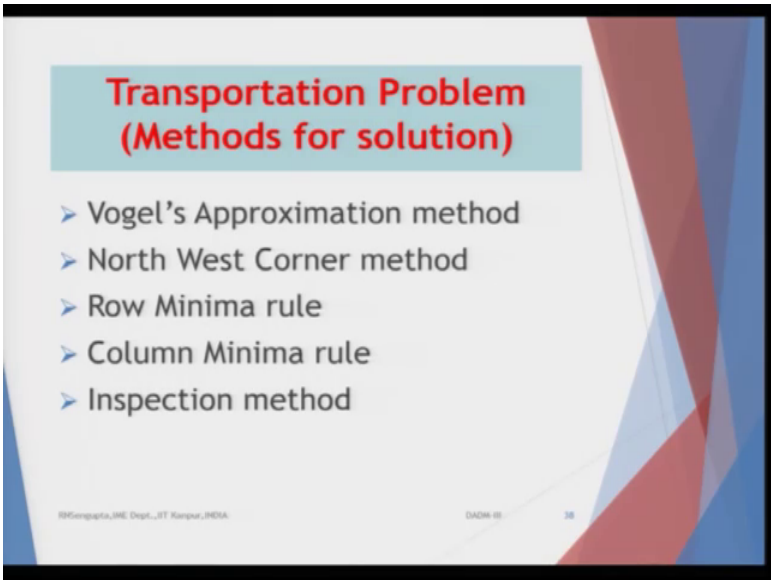

Now what is important to note down is here. To solve this problem, there are different methods. And I will only discuss these methods very simply. These are the simplest method that we could have use it, but these methodologies give us much better starting and very good approximation in the beginning. So one, one of them is basically the Vogel's Approximation method, the second one is the North West Corner method, that means you go to the North West Corner method and basically balances accordingly.

Other is the Row Minimization rule, that means basically consider on each of these rows, rows means the, the origins which you are going to consider, one is basically, the column concept which where you basically concentrate on the, the concept of the, the destinations and one is the basically the base rule which is a thumb rule which is on the Inspection methodology based on which we do. Now, what is the general Vogel's Approximation, I will come to that later on, but let me give you a basic preview.

So what you want to do is that, consider that you are looking at any of this origins. So origins you want to transport. Once what you want to transport what will you look at? Your main concern would be to look at any one of the origins where the demand has to be made the fastest, point one. Point number two is that, when we meet the demands, we also want to meet the demands at the faster at the destination, based on the fact, the cost is also the lowest.

So you will basically try to choose the vect, the vector corresponding the cost, the sale value which is the least and also try to basically pick, pick up that particular row for which the, the demand has to be meet the fastest, because if you are not able to meet the demand there would be unfulfilled demand. So I, and an as this is a minimization cost remember. So as this is a minimization cost, I will basically choose the one for which the cost structure is the least and where the demand utilization would be the maximum and the fast, but there is a catch.

The catch is that, consider the first origin basically has 400 number of units to transport and consider the, an it basically supplies, it should supply to say for example either the destination one, two, three. Now destination two considers the cost is the least. So you obviously will basically supply all the goods from, from origin one to destination two. But now, becau, because for the cost structure, but now consider the whole of the goods are not met at the destination, which was that immediately we will supply all the goods from origin one and the extra one which is still left will basically try to supply from the next minimum origin such that the cost structure we try to basically increase it.

The cost structure obviously will increase, but we will increase in the least quantum jumps because CI's are the unit cost which you are going to basically add up for additional amount of transportation of one unit of a element from one, one of the origins to one of the destinations. So if it is C, I, J, so C, I, J, basically means, I am basically trying, trying to transport one unit from the ith origin to the jth destination.

Now if, if your cost structure, if you could basically look at the other way also, that means rather than considering only the rows, we can only look at the columns, that means basically I will try to minimize my overall concept from the columns wise such that I will consider that the cost structure which is minimum from the, from the column wise and then I will basically try to meet and get all the demands from the origin which is basically the minimum cost.

So in this way, what I am trying to do is that, either I look at the destination point of view or I look at the origin point of view based on the fact I only consider the cost as the main predicative based on which I am trying to minimize, but also remember that, if there is some unfulfilled demand of there, there are excess demand both at the destination and the origins, we will try to basically consider that in such a way that the balancing would be done.

So the Vogel's Approximation, North West Corner method or the Row Minimization and the Column Minimization would basically consider this concept in order to basically optimize considering that optimization fact is that we want to basically minimize the cost. So with these, I will close this 30th lecture and basically start of the Vogel's Approximation, North West Corner, Row Minima and the Column Minima method and the Inspection method in the seventh week starting on the 31st lecture. Have a nice day and thank you very much.