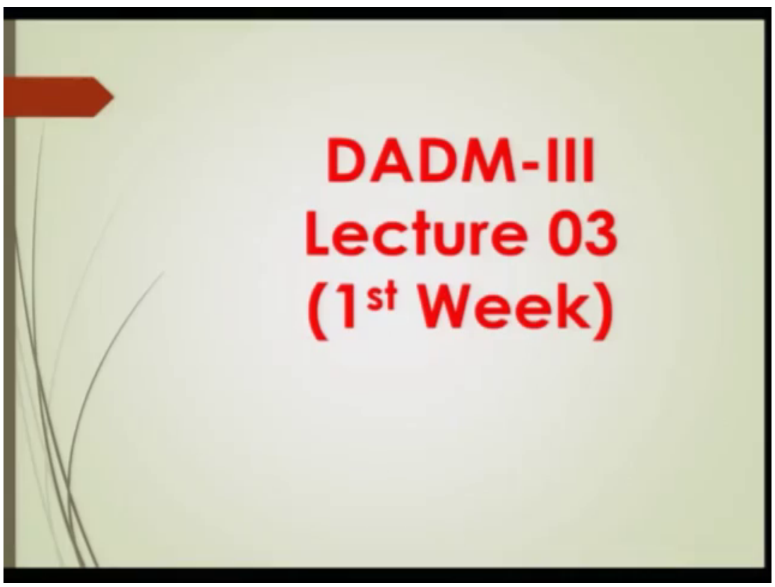
Data Analysis and Decision Making-3
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology Kanpur
Lecture 03

Good morning, good afternoon, good evening to all of you my dear friends. Welcome to this DADM 3 lecture series which is Data Analysis and Decision Making and as you know this is a this course is to do with Optimisation Operation Research and will try to cover the whole gamut of courses which we have discussed and we gave a brief reintroduction in the first 2 lectures.

(Refer Slide Time: 00:42)



So this is the third lecture in the first week and as you know this course is to be run for 12 weeks which is (6) 60 lectures and the total combination would be 30 hours. So each week we will have 5 lectures and after each week of (lec) set of lectures of 5 we will have 1 assignment, so technically we will have 12 assignments which are to be solved by the students and after the end of the course we will basically have the question and answer for the final examination.

And my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur. So yes for the last 2 lectures (ques) lecture 1, lecture 2 we basically give a very-very brief introduction about the syllabus what we will cover and also I mention time and again we may not be able to cover everything but I will give you feel and interest would be build up. Then we discuss few references which were not exhaustive in all the senses that they will cover every all the topics but these are interesting textbooks which are well-well appreciated by the audience in the operation research field.

Then we discuss about the ideas of what we mean by operation research optimisation, we discuss few of the stall words in the historical perspective ideas mention the names and where the links can be found out about this researchers and obviously I did also mention that this is the not the small set of researcher there are a lot of other researchers who have already worked in operation research and related field.

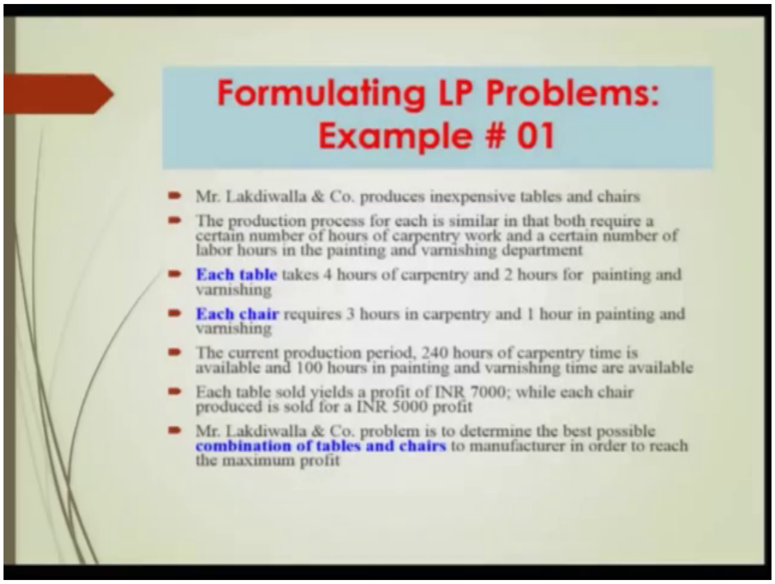
And then I discuss what are very simply what are the assumptions, what are the practical conditions, practicality and what we mean by linear programming and why it is linear, why proportionate is important, why additivity is important, why division of the (out) decision variable are important and I kept mentioning different examples within in the transportation firm or the number of trucks used for the number of distance covered, total cost, litres of paints produce and all these things or several examples you are going to take out materials from square sheet of aluminium and use them to make different components.

So you want to basically minimize the overall loss of aluminium sheet glass. So with this, I will start the third lecture in the first week. So consider very simple problem, now many of you would definitely be very well aware of operation research and optimization tool but I will start slowly. So I will basically discuss a 2 dimensional problem in the sense, there are 2 decision variable and I want to optimize.

Now I also mention that when you are doing optimization, the techniques would be discussed here in general format but how you formulate the problem, what are the conditions, what are the decision variables? Whether decision variables are continuous? Whether they are integer? Whether they are 0-1 type or whether you have quadratic optimization for the objective function? Whatever they are that will basically come from the practicality of the situation based on which you are trying to utilize the problem.

So this practicality of the situation experience feel has to be acquired through experience acquired through more reading. So this has to be done time and again so will try to basically lay stress on that in some manner such that somebody is able to tackle the optimization problem formulation as such while the solution techniques are led straight down rules which we will try to discuss in proceeding lectures for this course or NPTEL book series.

(Refer Slide Time: 04:42)



So consider Mr. Lakdiwala and Company produces inexpensive tables such as, so you produce tables and chairs to sell in the market and Mr. Lakdiwala basically has a company, acquires the wood or plywood whatever it is of different qualities so I am not going to the quality because that will basically bring the concept of what different types of tables and chairs would be utilized. So the production process whichever the production process is, for each is similar to that with both required some numbers of hours of carpentry work, so they have the carpenters work and they produce the general framework of the tables and the chairs.

And after (a) and they use certain number of labour hours so maybe 2 hours, 3 hours, 4 hours some amount of labour hours is required and after the chair is put into place, it has been the (and) the sitting place, the arm rest and all these things have been done, it goes in the painting and varnishing to basically bring a good look, longevity should be increased. Now a general study has been done by Mr. Lakdiwala and Company, the company manager or the workers or the general manager whoever it is, so each table will take 4 hours of carpentry work.

So I am considering 4 hours of carpentry work basically coming from combined efforts, so it need not be one person is working continuously 4 hours, it may be combination of different person working for different components but the total number of hours is 4 and will also consider simplistically that if one person works 4 hours so multiple person also combined together also works for 4 hours. So we would not consider that division of labour is coming out so as the efficiency is increasing, we would not take that.

So each table takes 4 hours of carpentry and 2 hours of painting and varnishing, so in the same way if it takes 2 hours for painting and varnishing will consider that if it is a 1 person he or she takes 2 hours to paint and varnish or if there are multiple person the combined effort which is being put by this multiple person should add up to basically 2 hours. Similarly when we go to the chair so each chair requires 3 hours in the carpentry in the same way when the 1 person is working or the multiple persons are working for the chair it takes 3 hours.

So will consider the 3 hours to be true both multiple division or labour to be true and 1 person to be working and similarly when the chair is finish it needs to be painted, varnished and the total number of hours required to varnish or paint 1 chair is 1 hour again as per the assumptions, as per the norm we will consider that this work which is being done for carpentry or varnishing for the chair or the table is being done on an individual basis or on a the different persons depending on the concept of division or labour but it would not affect the number of hours would not change and they have been stated.

Now remember one thing that we when we are saying each table and each chair we will consider they are of the same type. So any variation in the design, variation of the apples tree and all these things we would not consider that. So that current production period which is totally utilized, so this total utilization would be consider it can be 1 week, it can be so obviously 240 cannot be in 1 week, so it can be a few months or 1 year whatever. It is will consider total collective time which is available for the carpentry and the total collected time which is available for the varnishing and finishing (so or) painting. So it will be 240 hours for the carpentry and it will be basically a 100 hours for the painting and varnishing.

Again we will consider it is a collective time being taken by different individuals or 1 person working continuously would be basically spending this 240 and 100 hours and it would be same whether collective effort is being put or an individual effort is put. Now obviously once the work is finished Mr. Lakdiwala and Company would like to sell this product in the market and they when they go to sell this market will consider only the profits. Now this is important to understand that we are not going to consider for the time being for the problem we are not going to consider, what are the cost and what are the cost of material and what is the actual production cost of the purchase of raw materials or what is the selling price?

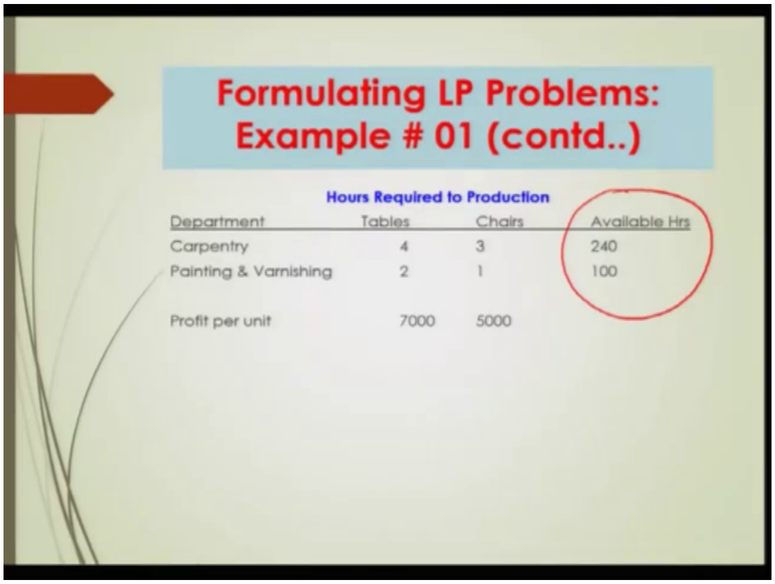
Will only consider the profit which is the price, extra price which you will get from buying and selling this product, buying the raw material and basically selling the finished product. So they are given as Indian rupees 7000 for each table and Indian rupees 5000 would be there for each chair that is the profit which we have. Mr. Lakdiwala and Company is to determine the best possible combination of the chairs and tables, so the best possible combination of chairs and tables it basically means that, what is the number of chairs? Or how many number of chairs or how many number of tables you are going to produce?

Such that you meet some criteria, so obviously this criteria would be dictated depending on what is the business output and what is general business norm based on which the this Mr. Lakdiwala and Company is running the business. So let me read in the last point so Mr. Lakdiwala and Companies is to determine the best possible combination chairs and tables and obviously they would be integers.

If you remember one thing that I mention that for the linear programming problem even though we have consider the divisibility of the decision variables are possible in the theoretical sense, in the practical sense it (mean) may not be possible as you have seeing it that we cannot produce 2.3 number of chairs or 4.7 numbers of tables will consider them to be actually in the case they are integer values but in the for the problems sake will consider them to be continuous values which are greater than 0 and greater than 0 we just does make sense but because we are not going to produce less than 0 number of chairs or 0 number of tables.

So obviously this company's main problem or main issue is to find out the number of chairs and the table it should manufacture and sell in the market such that is able to sell them at the highest profit and such that the total cumulative profit is basically maximize. So your main (ans) query would be what is the number of chairs? What is the number of tables you should produce? Considering the profit which you have for the chairs and tables and the total amount of time to be spent which you have for producing the chair and table, based on which we will basically find that what will be your total profit for the production. So if I consider the total scenario, so if the scenario is being stated here as a matrix.

(Refer Slide Time: 12:01)



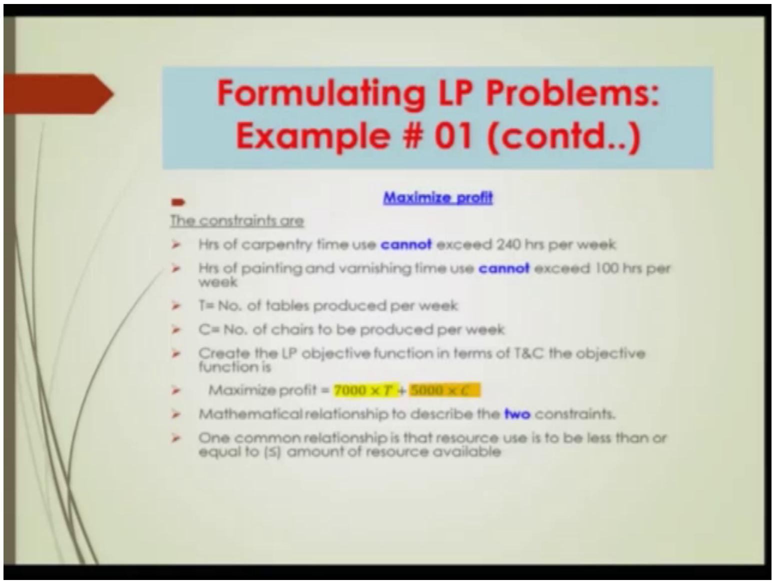
So these are the hours required for two product for the production and remember that when I am mentioning the hours it will be on a per unit basis, so that per unit basis maybe in numbers and maybe in weeks, maybe in years and whatever it is. So if the department is carpentry or the work-shade is the under carpentry so it will basically require 4 hours for each tables and 3 hours for each chair on the carpentry refund.

And if the painting and varnishing work is done per table and per chair the corresponding (prime) timing would be, to required to finish it would be 2 and 1 and again remember I am repeating in please bear with me that whether is individual time or collective time would not matter (between) because will consider them to be same whether collective time is being utilised or whether individual timing is being utilised.

Now if you remember I did mention that total number of hours which is available for carpentry was 240, total number of hours which was available for painting and varnishing 100, so if you check the last column so these basically states the available number of hours in totality in a fixed time spend it does not mean that for infinite time variations, I have basically only 240 hours it may be several example per unit time that may be either 1 year or 6 months whatever it is, we have maximum available is 240 for the carpentry and 100 for the painting and varnishing.

Now we have also mention that the what is the profit, without considering the cost and the selling price, the profits were basically based on Indian rupees was 7000 and 5000 for the tables and the chairs, each table and chair which are being sold in the market.

(Refer Slide Time: 13:39)



Now your actual (point) idea was basically to formulate a problem, formulate the optimization problem such that you want to basically (opt) maximize the profit, so profit is what is the main motivation based on which Mr. Lakdiwalla and Companies working. So the constraints are the hours of carpentry time use cannot exceed 240 per hour per week. So here the time frame of per week is going to come, so whatever number of carpentry work which you do, whatever number of chairs you produce, whatever number of tables which you produce the total amount of time which is there for you for doing the carpentry work is basically 240 hours per week, nothing more.

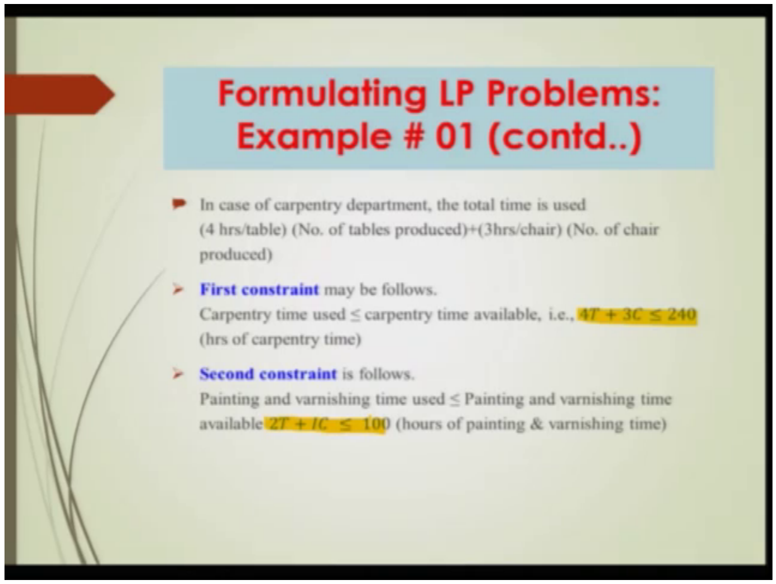
So obviously if you spent 228 number of hours that means you will basically have an excess extra amount of hours per week which cannot be basically utilized that would basically 240 minus the total number of hours which is being actually utilized. On the same way and the hours for painting and varnishing use cannot exceed 100 hours, so in per week so in the same logic will consider that if you are basically trying to utilize the resources to paint and varnish you cannot in a week have more than 100 number of hours for work for this particular work which is painting and varnishing.

So now basically let us denote the variables which are the decision variable, so let us denote the decision variables as T and C as the number of tables produced per week because this is important to remember that the time constraint based on which you found out the 240 hours and a 100 hours is based on a weekly basis. So in the same context will try to basically mention the variable T and C which is the decision variables are to be found out for a weekly basis and based on that you can basically find out that what is the total number of production on a yearly basis or on a the semi-annual basis so on and so forth.

So if T and C are the number of tables and chairs produce, so you want to basically find out what is the optimisation problem? So obviously it will come that if each table is being sold for 7000 then the profit which will get for selling T number of tables will be 7000 into T which I will now mark here, so this is 7000 into T. Now when I come to the chairs, it has already been mentioned it will be 5000 per chair sold in the market, so obviously this is also known to us is 5000 into C, so obviously will try to basically find out those values of T and those values of C which will give us the maximum value of the profit based on which this company can operate to find out the number of chairs and the number of tables it will basically manufacture.

So mathematical relationship to describe the 2 constraints would basically now be discussed or basically we will try to find out the answer. So one common relationship is that resources used is to be less than and equal to the amount of resource which is available. So if I want to be basically utilize X amount of money for different type of utilization, so my total number of availability of resources would definitely be greater than the total number of Xs based from which we are trying to basically operate considering them there would not be any other demand for the same product.

(Refer Slide Time: 17:19)



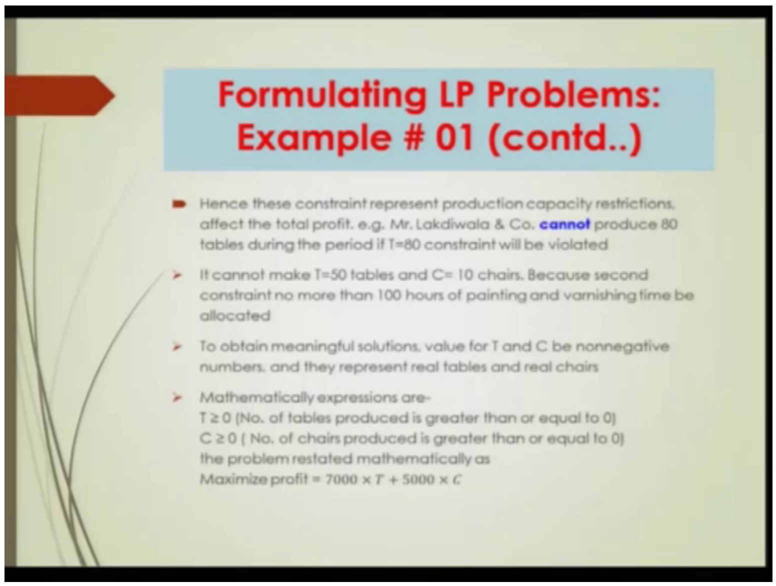
So if I come to the constraints, so these are the constraints which we will discuss in case of carpentry department the total time required is, so now let us go back. So per table we need 4 hours so if the total number of tables is T, so the total number of hours utilized for manufacturing T number of tables is 4T, so that is point 1. See in the similar way per chair utilization or number of hours utilization to produce 1 chair is 3, so if i basically multiply 3 into the number of chairs which are being produced so you basically find out the total time which is being devoted to find out the total production which is T number of chairs and separately will find out the what is the total time required for producing C number of chairs.

Now if that is the case obviously you will try to find out that in the carpentry workshop what was the total requirement of number of hours which we have? So obviously we will immediately say that it is 240 hours, so if that is the case then the total sum of the total number of hours which are being utilized in the carpentry section for separately for the chair and separately for the table when they add up obviously should be less than equal to 240 because any utilization more than that would not basically make any feasible sense.

So if basically highlight that so 4T plus 3C is less than equal to 240 depending on the carpentry time. So in the similar way when I come to basically the painting and varnishing one, so the total (number) amount of hours required for this painting and varnishing one would be 2 into T plus 1 into C, so this 2 is basically the total number of hours which is required for painting and varnishing by 1 table and if I consider trying to basically find out a varnishing and painting for 1 chair it would basically 1 hour, so this is the value of one which you have.

So again if I go back to the problem, I have the total number of hours which is available for us is 100. So hence the total combined amount of hours which will have for us would basically be the first equation is 4T plus 3C is less than equal to 240 and the second one is basically 2T plus 1C is less than equal to 100.

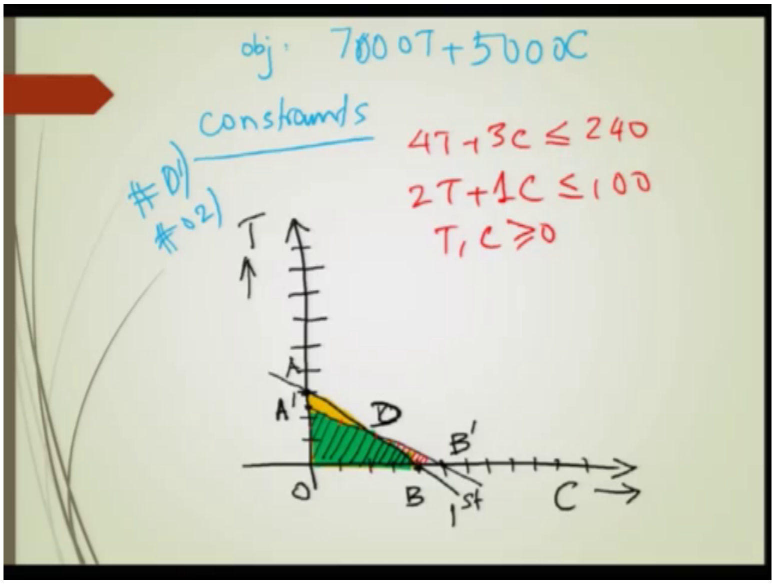
(Refer Slide Time: 19:46)



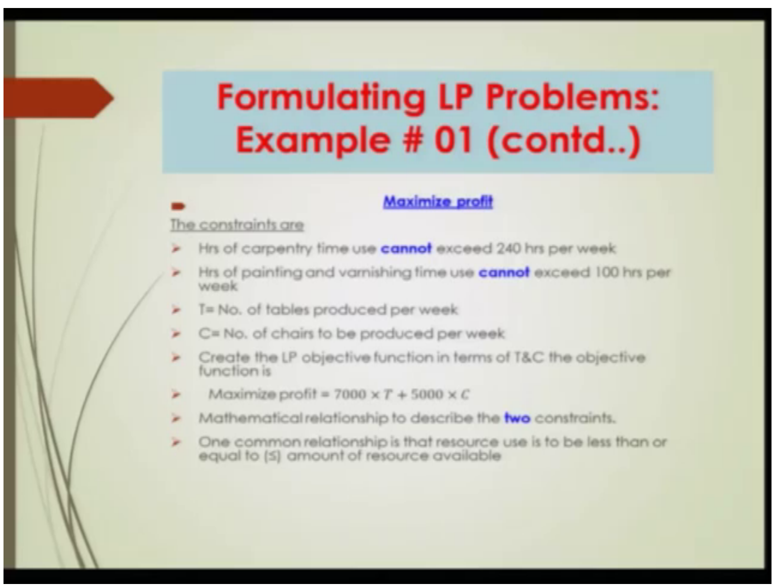
So continuing with the problem hence this constraints presented, present production capacity restriction which affects the total profit so Mr. Lakdiwala and Company cannot produce 80 tables during the period if T is equal to constraint is basically to be violated. So what will try to basically (())(20:07) understand is that depending on the output which you have we will try to basically find out how the solutions can be found out.

So this solution basically I will try to basically go it slowly and try to do it graphically in order to make you understand because the graphical concept will give you an idea that how does the algorithm works for the search techniques of the linear programming concept or the simplex method which we will try to solve, ok let us do it here which will be easy for us to do. So what are the constraints so let me write down the objective function.

(Refer Slide Time: 20:46)

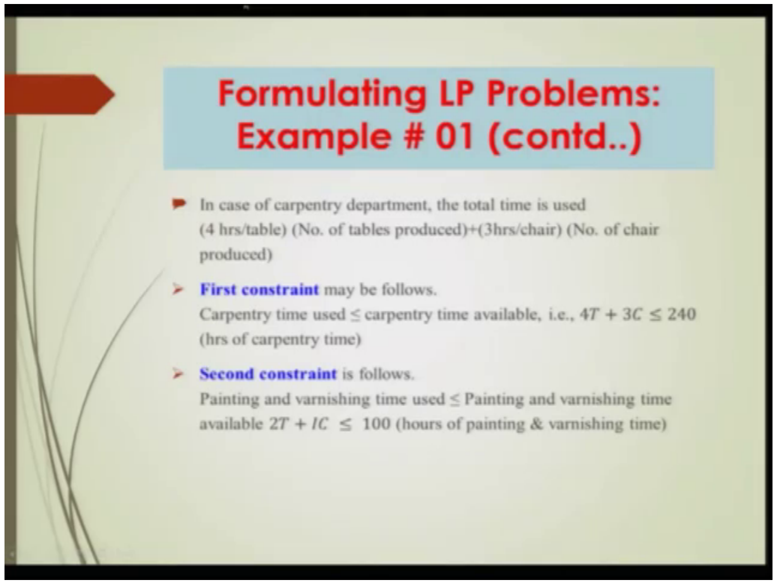


(Refer Slide Time: 20:59)



So I will use, what is the objective function? 7000 T plus 5000 C and the constraints are, so the constraints will basically come from the carpentry and the varnishing.

(Refer Slide Time: 21:31)



So carpentry and varnishing 4T plus 3C is less than equal to 240, 4T plus 3C is less than equal to 240 (and) 2T plus 1C is less than equal to 100, so this is second constraint and this would be and through now this we will try to solve it and remember T and C as I already discussed are is basically greater than 0 which is the non-negative part. So let us try to basically formulate the problem. So obviously non-negative so any values of T and C which are in either the second quadrant or the third quadrant or the fourth quadrant would basically we note, so will only consider the emphasis being there on the first coordinate, first quadrant only.

Now consider the values as, so I will basically draw so if I consider the first equation it sees that T and C if they are 0 it is less than 240, so obviously 4T is about 60, so I will be ok basically I will mark T and C here, so 4T is 60 and C is 0, 10, 20, 30, 40, 50, 60. So this are considered as 20, 40, 60 this is the fun and then if you consider C it with basically be T as 0, it will basically be T so 10, 20, 40, 60, 80 so this is the point, so if I now 0 basically affects it so the portion, so this is the first part, so this is the feasible part for the first.

So I will write it down, the second one is T if you put C as 0, T would be 50 so T is 50 is here, here I put C value with T is equal to 0 I will basically get 120(6) 40, 60, 80 is just 100. Again 0 value is basically be met. So if I consider now for this person this is the feasible set but if I consider the joint one, so it will basically be of different colour, so your total area, this is A, this is A prime, this is O, this is B, B prime.

Now look at this problem, technically the triangle not the triangle, the figure A dash O, B and ok let me mention this is C also. So this C would not do because C is a so let me put it as D, so D is basically the figure A dash O,B and D is basically the actual feasible region, where you can find the solution. But the question is that if ask you that you are as a (())(27:32) and I ask you that what is the best solution? So technically how many such feasible solutions are? So obviously will say that whole set starting from A dash O, B and D is feasible which means there are infinite such points.

Now step back, if you remember I did mention about 1 person when we were discussing the second class by the name of Karmakaar, so he basically proposed the interior point algorithm where the searches basically now shrinks to only the corner files which are O, B, D and A dash, that means infinite searches have basically now come down to only 4. Now what you need to do technically, you basically need to find out each and every points so what is the coordinates of A star, A dash O, B and D? Find out what is T and C, plug them in the objective function and solve them to find out that what is the objective function value then rank them from highest to the lowest and take the decision accordingly.

So this is the (())(28:36) L1 framework, I will discuss the problems more so in the fourth lecture depending and how the progress is, have a nice day and thank you very much.