Data Analysis and Decision Making-3

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Lecture 29

Welcome back my dear friends, a very good morning, good afternoon and good evening to all of you. And this is the Data Analysis and Decision Making course under the NPTEL (mock) this is DADM-3 under the NPTEL mock series. And as you know this total course duration contact hours is 30 hours and which is split into 60 lectures and each lecture being for half an hour and the total course duration is for 12 weeks.

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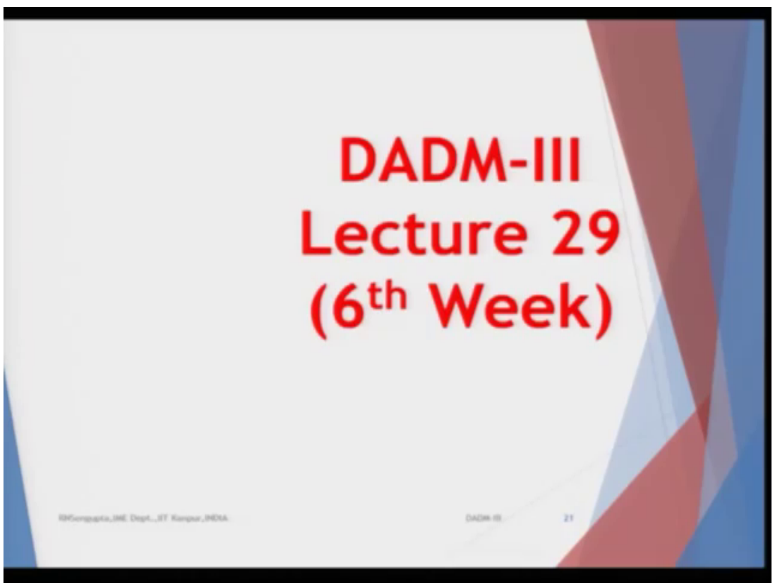

And as you can see from the slide we are in the 6^{th} week 29 lecture and with the 29 and the 30^{th} will we would wrap up the 6^{th} week and basically be halfway through the course. And this after each week where we have 5 lectures of half an hour each, we have an assignment so with the 30^{th} lecture being over we will basically be taking the 6 assignments and after the whole, the whole course is over you will be taking the final examination.

Now if you remember we were discussing about the concept of artificial variables. Artificial means they had been put in to the overall optimization as taking the constraints artificially in order to ensure the basic starting feasible solution is there at one of the corner points. But it may be possible that if all the equality signs are then obviously I said that you can convert the equality signs into greater than, then on less than sign and you can get the solution.

But it may not be possible to have a basic feasible solution to start with for the solution. Hence we basically forcefully put the artificial variables into the system and ensure that the starting feasible solution is formed by those set of artificial variables which have been put and then basically iterate the whole process simplex method in order to find out the optimum solution which is one of the corner points in the basic (feasible) in the feasible region.

And also remember the overall artificial variable, overall concept in the objective function for the minimization it will be maximum value and on maximization would be in the minimum value in the sense that if you are trying to minimize that means you will try to come down and reach the minimum value. So you will basically started a very high value depending on the artificial variables and basically try to eliminate the artificial variable such the value or the overall effect of W which is the weights for the artificial variables slowly vanishes because the artificial variables by themselves will be 0.

Hence they actually is the minimum value of the answer we have. And the maximization case we started at the lowest most bottom as I has possible as negative and then basically eliminate the artificial variables such that the value keeps increasing till ee reach the maximum value. So I will basically solve the problem accordingly.

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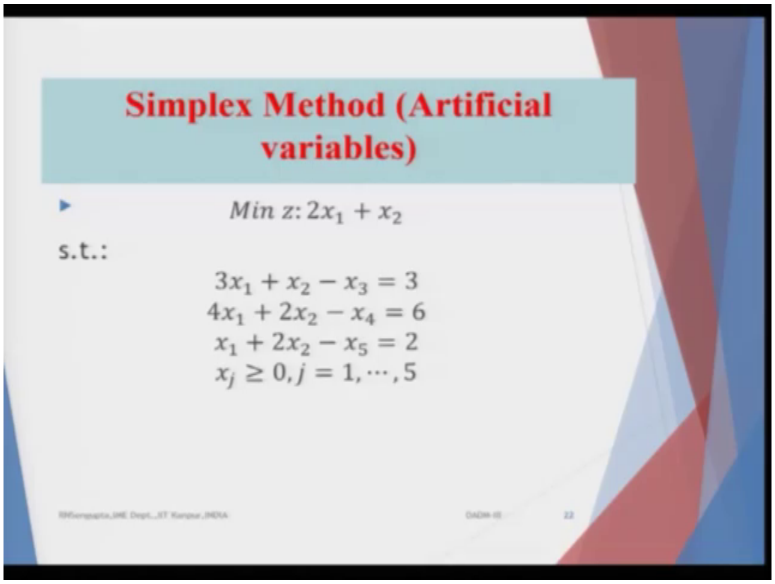

Consider we have the minimization problem so minimization, maximization the concept are all exactly the same. We will just utilize the concept for the (minimum) minimization problem and show how it is done. So in this problem consider that you have three such variables, so the actual problem formulation is 3x1 plus x2 minus x3 is equal to 3 then 4x1 plus x2 minus x4 is equal to 6 and x1 plus 2 x2 minus x5 is equal to 2 and the variables which is basically 1, 2, 5 which are the so-called variables which have been brought into the picture considering the slack and the surplus are such that the overall weightages which are going to come off come out in the objective function for x1 and x2 are 2 and 1 and all the variables, decision variables x1 to x5 are all greater than 0.

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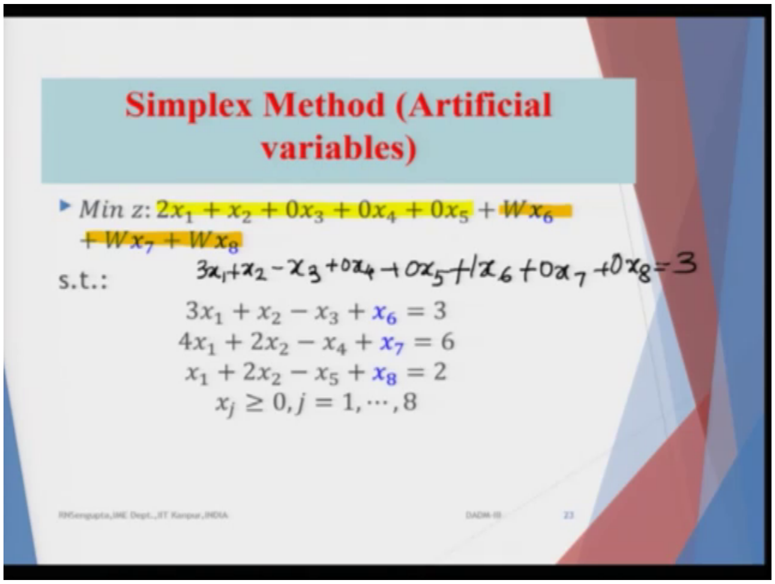

Now remember, the objective function would you have a formulated was basically 2x1 plus x2, now corresponding to the fact we have basically brought these x3, x4, x5 as slack or surplus depending on which are a variable looking and I am considering that they are all negative such that the equality sign is being is there. Now you may ask the question, what if the equality sign was there why did we try to basically bring x3, x4, x5? It is fine, you should not consider that but is basically trying to portrait that how you will tackle the problem when both the slack and surpluses is there plus the artificial variables.

Now slack and surpluses would be the case when depending on the sign being greater than or less than equal to but in this case we have consider the negative signs such that the slack and surpluses have been formulated accordingly. So if you minimize the value of Z, so you basically have 2x1 plus x2 and the slack and surpluses overall quantum of effect on the objective function actually would be 0. So each of these x3, x4, x5 would be multiplied by the factor 0. So the initial part of the objective function has now become 2x1 plus x2 plus 0x3 plus 0x4plus 0x5. Now when you add up these blue coloured variables which I marked in order to make it very simple to understand that they are the artificial variable, so I have basically brought them into the picture as x6 which is the artificial variable corresponding to the first constraint, x7 which is the artificial variable corresponding to the second constraint and xi it is basically the artificial variable corresponding to the third constraint. Now look at and this point, in case x6, x7 and x8 were not there. So obviously were not there your objective function is as this one which is fine technically which means x6, x7, x8 are not there.

So I will basically remove them, now see the interesting part. If you consider the actual equation is 3 x1 plus x2 minus x3 is equal to3 first constraint, 4x1 plus 2 x2 minus x4 is equal to 6, x1 plus 2 x2 minus x5 is equal to 2. Now consider x1 next to as are 0. So if there is 0, what you will have is that x3, x 4, x 5 all will be negative that means they are infeasible basic feasible solution even if we want to start at that point. Hence starting the over overall the simplex method would not be possible. Hence we are being forced to bring this artificial variable in a very intuitive manner such that the actual starting basic feasible solution would have that property where at the origin point whatever it is all other variables would be 0 or greater than 0.

That is what we meant because if x6, x7, x8 are not there x3, x4, x 5 are negative which is not allowable as per the concept of linear programming because we consider all of the set of all the variables to be greater than 0. Now I remove this red this the black mark which was there for x6, x7, x8 and they come into the constraint first, second, third now as I mentioned that if there are the artificial variable and it is a minimization problem will basically have a very high value attached to each of these artificial variable 6, 7, 8. So consider that the high value of positive value for the minimization problem being attached to x6, x7 and x8 is W.

So now your actual problem formulation is I will basically mark the second portion with a saffron colour which means the yellow one was the part where the slack and surplus was there. And once we bring the artificial variables you have an addition part in the objective function which is W x6 plus W x7 plus W x8. Now see how it helps in our problem formulation. Now in this case when the actual formulation has been formulated with this artificial variable plus x3, x4, x5 all these x's J is equal to 1 to 8. As per the concept should we always be greater than equal to 0. So here the actual problem formulation is now this okay I so your I will use the, so the object function is 2x1 plus x2 plus (0x3) 0x4 plus 0x5 plus Wx6 plus Wx7plus Wx8 and the constraints are.

I will write one way one and erase them even though nothing is did not explicitly here and I am basically writing each and every equation pertaining to the rows of the A matrix. So the first row 3x1 plus x2 minus x3, so x4 is not there in the first constraint so it will be plus 0 x 4, x 5 is also not there 0 x5, x6 is there x6 1x6 plus 0x7 plus 0x8 is equal to 3. So the parameters corresponding to the or each of the x vector is 3, 1 minus 1, 0, 0, 1, 0, 0 so this is, so this is actually (0 x8). So this is the first row corresponding to the first constraint. So let us remove it and I will basically now consider the second row.

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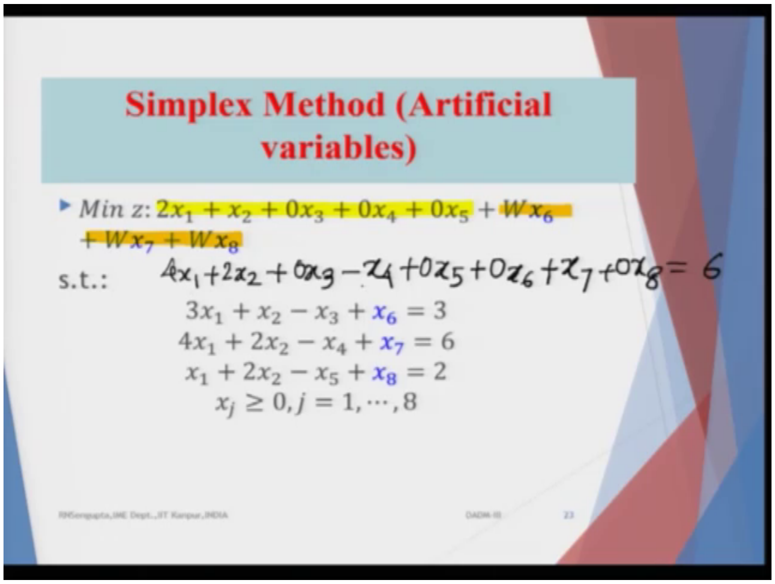

So the second row is this one. So this would be 4x1 plus 2x2 plus 0x3, because 3 was for the first constraint minus x4 which is there plus 0x5 plus 0x6 plus x7 plus 0x8 is equal to 6, so that becomes the second constraint and the values multiplied for each and every vector x is 4, 2, 0, minus 1, 0, 0, 1, 0 so this is the second constraint.

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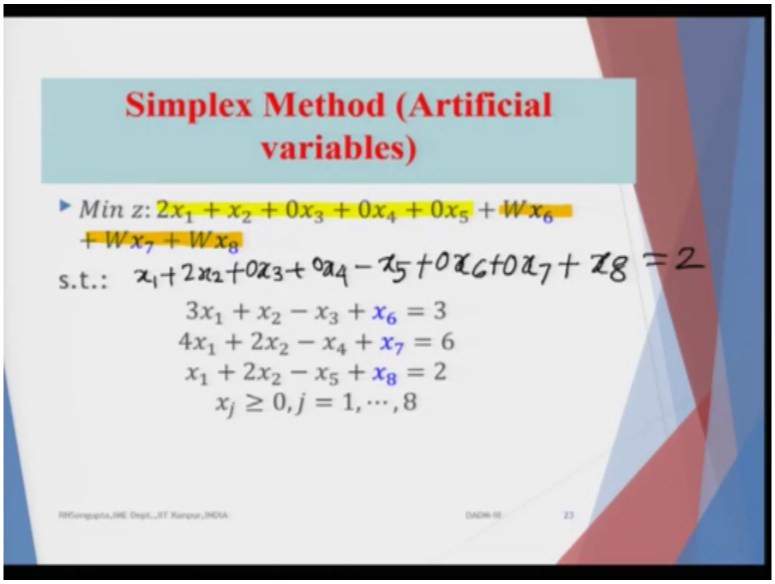

The third constraint is corresponding to the one which would be x1 plus 2x2 plus 0x3 because 3 belongs to the first constraint plus 0x4 because 4 belongs to the (first) second constraint minus x5 it actually belongs to third constraint plus 0x6 it belongs to first as artificial variable plus 0x7 plus as it belongs to the second constraint, artificial variable plus x8 because it belongs to third constraint is equal to 8.

So the parameter values corresponding the third row in the A matrix which will be multiplied for the vector by the vector x and the values are 1, 2, 0, 0, 0, minus 1, 0, 0, 1 and the right hand side B values we know is given as 3, 6, 2. So now I will be switching from this slide and going to the next one but I will keep referring to the slide for the discussion. So now let me write it down and in the top row I will write all the variables x1 to x8.

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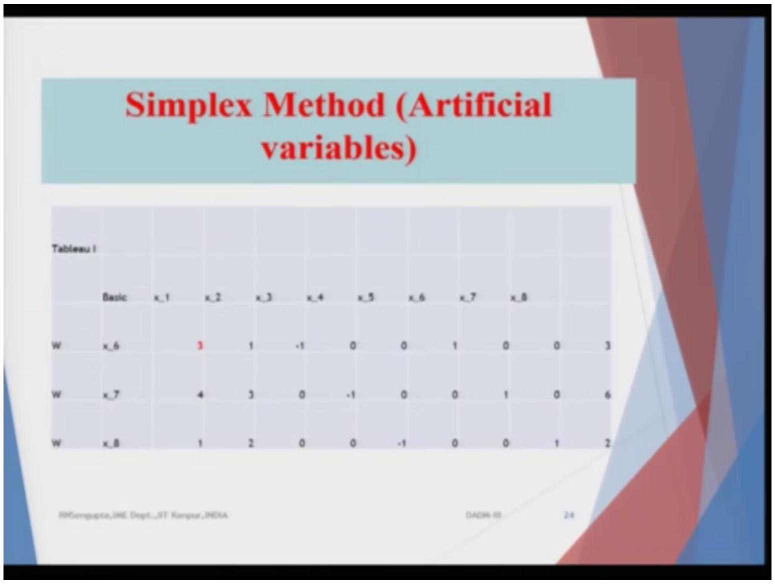

So all the variables are there x1, x2, x3, x4, x5, x6, x7, x8. Now the initial basic feasible solution I said that we will basically have the artificial variable.

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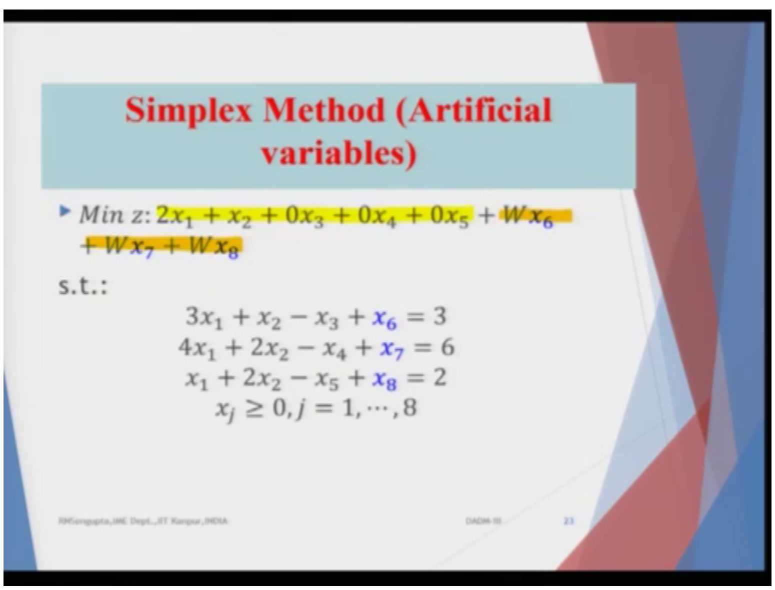

So let us check, because if I put x1, x2, x3, x4, and x5 all 0, technically x6 is equal to 3, x7 is equal to 6, x8 is equal to 2. So which maybe starting basic feasible solution based on which we can start our linear programming. So the parameter corresponding to the row which would basically be the starting basic feasible solution would be on the left hand side (we will) well x6 because we want to find out.

So here is x6 and the corresponding values would be in the tableau would be 3, 1, minus 1, 0, 0, 1, 0, 0 that is 6.

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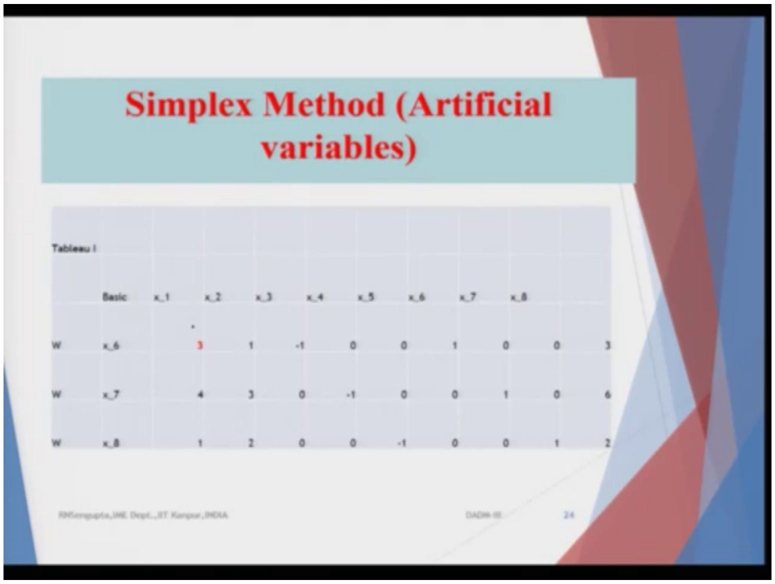

3, 1, minus 1, 0, 0, 1, 0, 0 and what is there on the right hand side?

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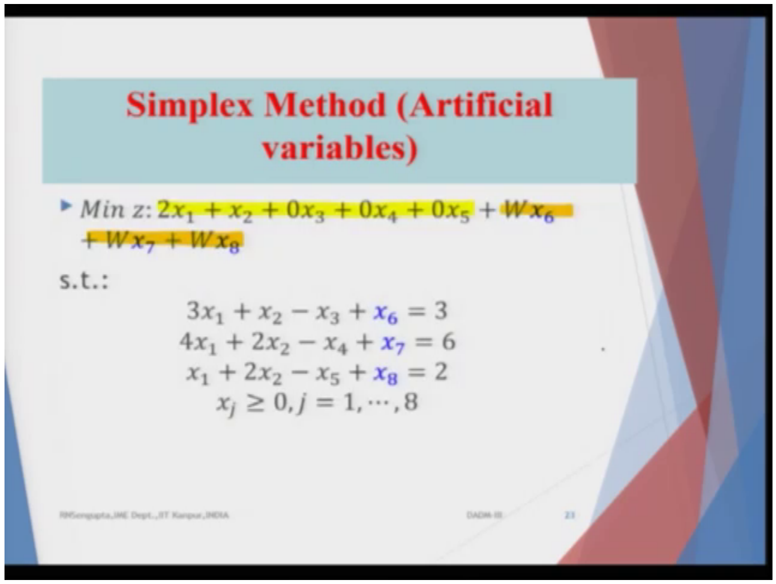

3 as it is. Now what would be the second row in the tableau corresponding to x7? 4, 2, 0, minus 1, 0, 0, 1, 0 and right hand side is 6.

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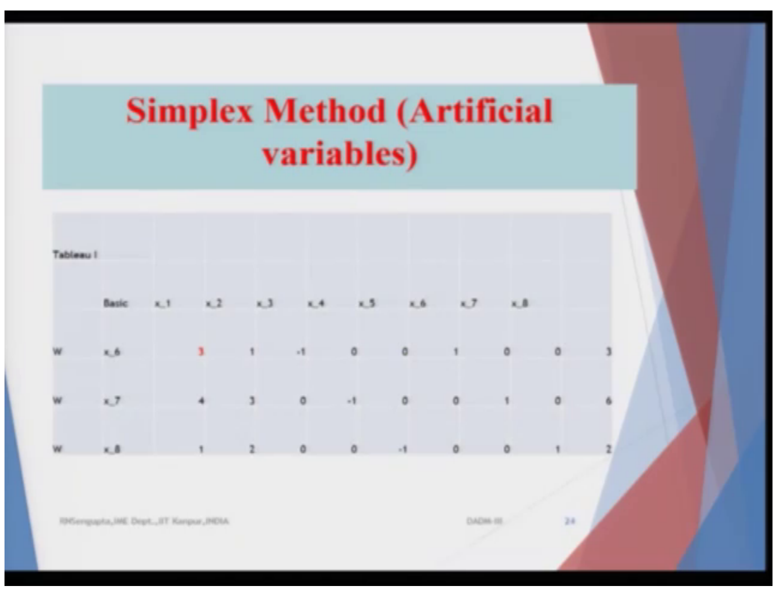

So it will let 4, so it should be there is one mistake here, or should be 2 okay my mistake, it should be 4, 3 it should be 4, 3 then 0, minus 1 so 0, minus 1 then 5 is not there so it will be 0, 0 is 0, 0 then 7 is 1, x7 and x8 is 0. So you see 1, 0 and what is there on the right side?

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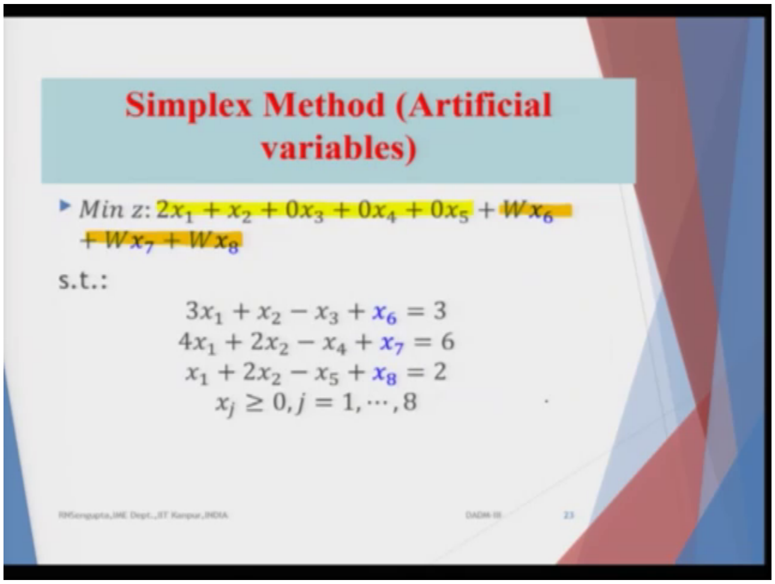

Is 6, the last row would be corresponding to the tableau of the first one which is corresponding to x8, the values would be 1, 2, 0, 0, minus 1, 0, 0, 1, 1, 2, 0, 0, minus 1, 0, 0, 1.

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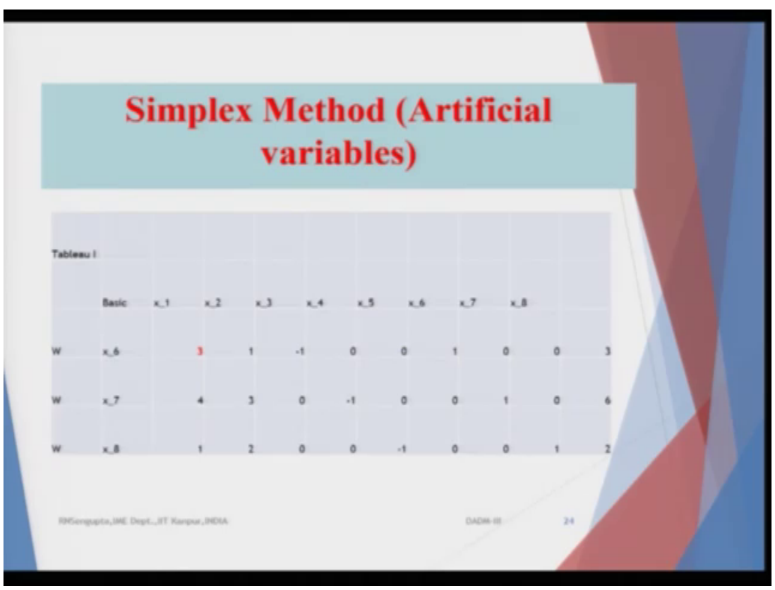

So 1, 2, 0, 0, minus1, 0, 0, 1 and the right hand side is basically 2. Now see the beauty, when I saw so obviously in that case x1, x2, x3, x4, x5 are all 0, so correspondingly they do not appear in the first tableau, tableau 1 but when I solve them technically x6 would have a 1 in its corresponding x6 cell which is 1. And x7, x8 which are non-zero obviously the value would be (0) 0, 0 hence x6 value is 3 as it should be as I just mentioned.

When I go to x7, obviously x2 to x5 are all 0, so it should not be considered. 6, the if you go vertically downs the for x6 it is 0, so it does not matter because it is being multiplied by 0, x7 is 1 x8 is 0 so hence the value of x7 is 6, x8 obviously x1 to x 5 does not matter, the cell values in x6 and x7 are 0, 0 respectively and x8 it is 1, so hence the value of x8 is 2. So technically when I have that equation to solve, it is like this, the objective function and first I will write the overall coordinates of the basic feasible solution or the starting point.

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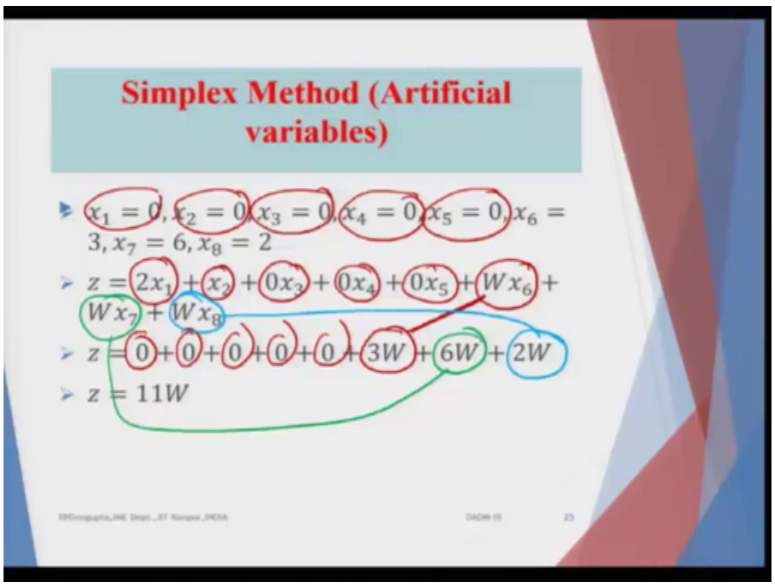

So starting point are this x1 to x 5 are all 0, so x1 is 0, x2 is 0, x3 is 0, x4 is 0, x5 is 0, what was x6? X6 was a right hand side the b value, b1 value was 3. What was x7? X7 was b2 value, the second value in the b vector which was (8) 6. And what was x8? Was the third value in the b vector which was 2. Now with this let us go back to the actual objective function, what was the objective function? Objective function was 2 actually was 2 X1 plus X2. But now we have also the slacks and the surplus, also we have the artificial variable, so formulation actually is 2 x1 plus x2 plus 0x3 plus 0x4 plus 0x5 plus Wx6 plus Wx7 plus Wx8. Now let us put all the values of x1 to x8 correspondingly.

The x1 value is 0, so obviously you have I will say that and keep marking that. so x1 value is 0 here x1 is 0. So when I multiplied this, this value becomes 0 let me highlight it will be better. So x1 is 0, this is multiplied by 2 hence this is 0. So the first value is done, x2 is 0, this is x 2 hence it is 0, x3 is 0, 0 into 0 is 0, x4 is 0, 0 into 0, 4 is 0, x5 is 0, 0 into 0 is 0. Now come to the x6, x6 3, 3 multiplied by W should be 3W.as it should be. So I am only (multi) highlight this, now I will change the colour x7 is 6 and 6 into w is that should be basically 6w, so I let you mark yes it is 6w, so I mark it here, oh so I mark it here. So this is done.

What that going to x8? X8 is 2, so 2 into W is here so the value which comes out is here. So it is done. So the overall minimum value is 3w plus 6w plus 2w is basic l1w and if you remember w is a very high value, so we are basically starting our solution at a corner point where the coordinates are 0, 0, 0, 0, 0, 3, 6, 2 and the value of the objective function is w, 11w and obviously keep decreasing. Now we have to basically fix the thus pivot row and the pivot element.

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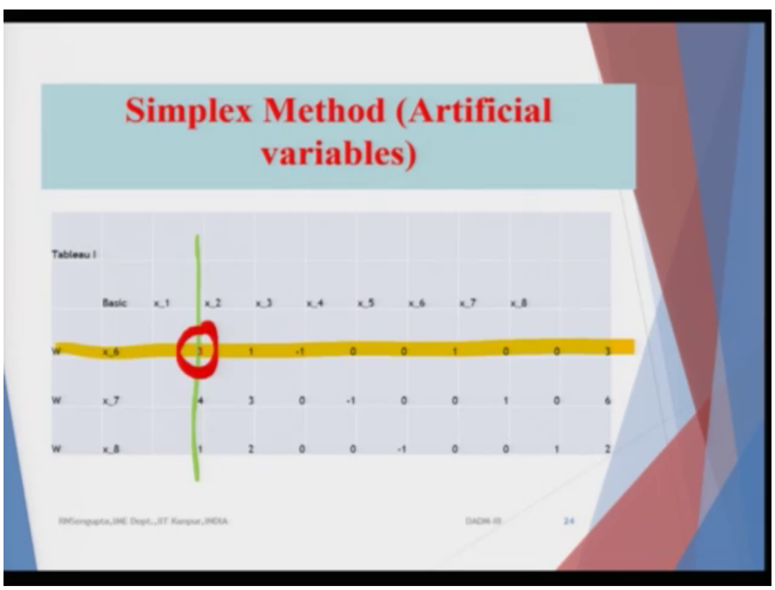

So we follow the same principle. (Like) trying to find out in the maximization and minimization case we take the maximum negative or the minimum positive based on which direction we want to move. And the pivot row and the pivot element would basically give me this. So I will basically mark the pivot row and I will mark the pivot element.

So based on that, now I will mark the pivot element once more, so this is pivot and give that colour, so this is the pivot row and this is the pivot column and this is the pivot element based on that we do the calculations. So 3 is the pivot element based on that we do the calculation which means x1 would be entering and x6 should be going out. So initially when we have the values the actual objective function was as I said was basically 11w which you have already checked.

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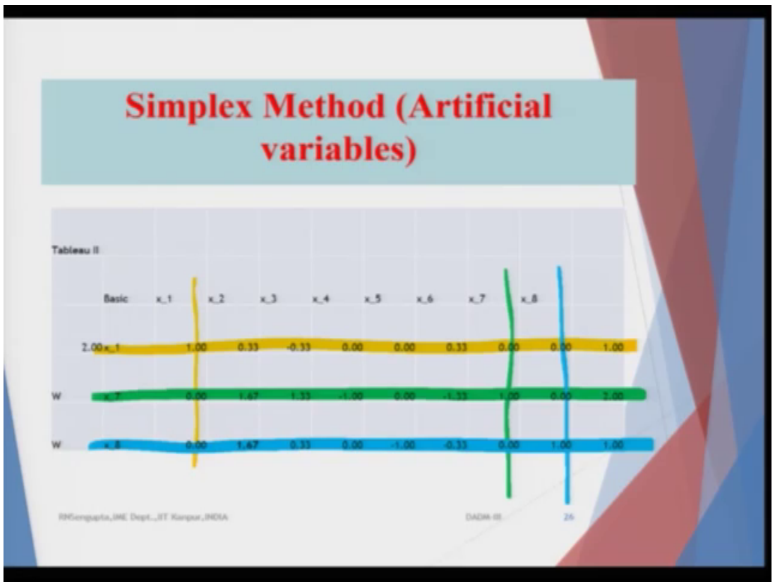

Now as x1 enters and x6 exits, so the new tableau like this, so now I have x1 enters and x6 goes so my new tableau is this. So x1 comes and now the corresponding changed new rows as per the concept of how we convert the utilize the old row and find out the new row based on the pivot element the values comes out of this. Now it is interesting, the x1 value would be non-zero now x2, x3, x4, x5 and x6 are all 0, x7 would be non-zero, x8 would be non-zero. So let us check the corresponding columns for this x1, x7, x8 and find out they are a unit matrix corresponding to those corresponding elements which are there.

So x1 has 1 here, x7 is 0, x8 has 0 which is right. So hence the value of once we do the multiplication (x) x1 value would be 1, when we go to so let me highlight it, which is done. When I come to x7 so check the value of x7 in that cell is 1 while the corresponding value of x1 the x8 are 0. So obviously the x7 value is 2 I have done the calculations and when we come to sorry for that yes when I come to x8, x8 is here and x8 if I consider here, the corresponding column values are 0, 0, 1 so hence the value of x8 is 1.

So now we need to find out, what is the co-ordinate and what is the actual value of the objective function? So let us write down the coordinates, so coordinates as I remember x1 would be 1, x2, x3, x4, x5, x6 are all 0, x 7 to x8, 1.

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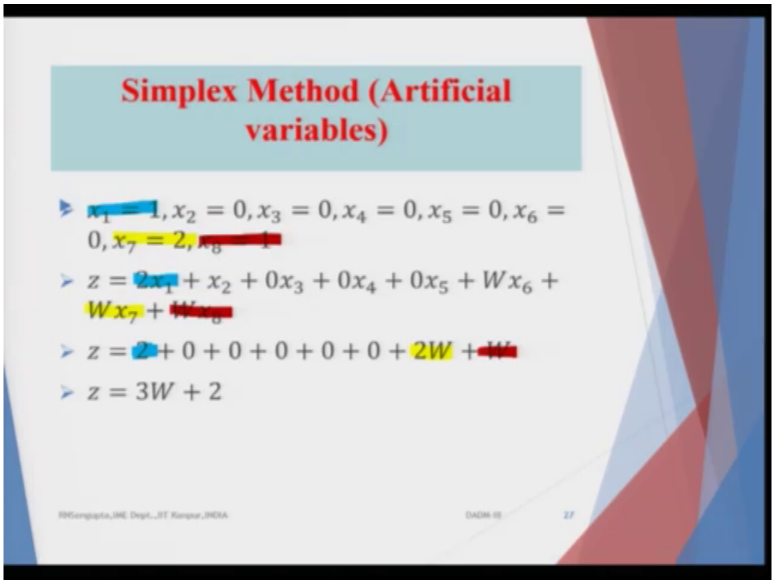

So let us see, so here it is x1 0 x2, x3, x4, x5, x6 are all 0, x7 is 2, x8 is 1. So let us put those values in the objective function, so the objective function was 2x1 plus 2x2 plus 0x3 plus 0x4 plus 0x5 plus wx6 plus wx7 plus wx8, now when I put them I will just mark the values which are non-zero. So obviously this was non-zero, so this is a non-zero hence the value of 2, 2, 3, 4, 5, 6 are all 0, so 2, 3, 4, 5, 6 are all 0 so they vanish it does not matter. X7 is 2, so this is 2, his is W so it will be 2w, so it is 2w as it should be, x8 is 1.

So, w into x1 is 1, 1, 1 W which is W, which is W. So the total value of your objective function now initially it was 11. Now it has become 3W plus 2 that means it has started decreasing as should be because W is a very large value it was first initially 11W, now it has basically decrease to 3W plus 2. So now the new coordinates are as I am again repeating 1, 0, 0, 0, 0, 0, 2, 1 and the actual objective function value is 3W by 2.

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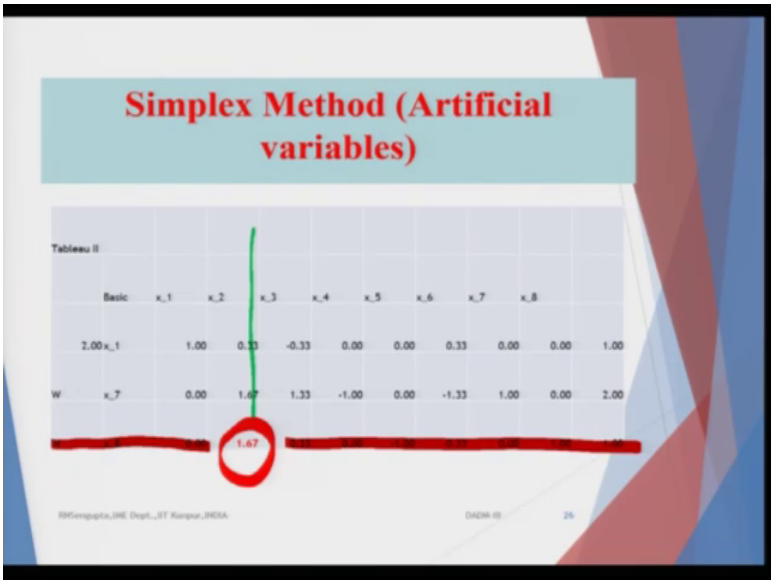

So when I basically have the corresponding (I will) erase it, so in order to give you an idea that what is the, this was the second tableau so when I, once I find out the pivot row and the pivot column, so corresponding I have the pivot row, I am not marking this value because this will be get dark so it will not be able to make out, so and the pivot column is this one hence the pivot element is 1.67 based on this so the concept with the logic is very simple. We consider the pivot row, pivot column which means x2 will now enter and x8 would be going out. So once we do the matrix multiplication or the pivot row, in pivot column based on the pivot element.

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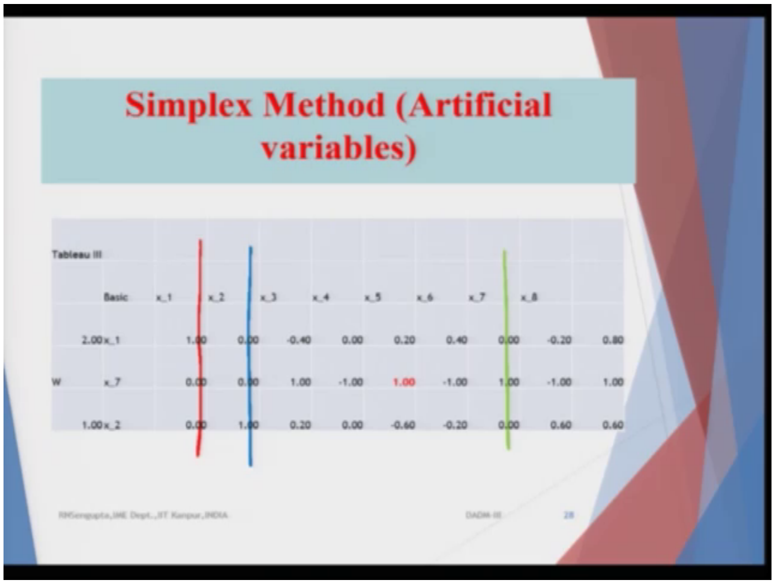

The actual value comes out to be 6 which means see here x8 has gone x2 has come. Now let us check the corresponding columns for x1, x 7 and x2. So x1 if I check it is 1, 0, 0, 0 hence the value of x1 is 0.8 as it should be. If I check the value of x7, so where is x7? So what are the column values? 0, 1, 0 as it should be so x7 is 1 and x2 which was not there not has entered so if I consider the corresponding column is 0, 0, 1 as it should be hence x2 is 6. So the values now will have is x1 is 0.8, x2 is 0.6, x3, 4, 5, 6 are all 0, x7 is 1, x8 is 0. But let us write it down.

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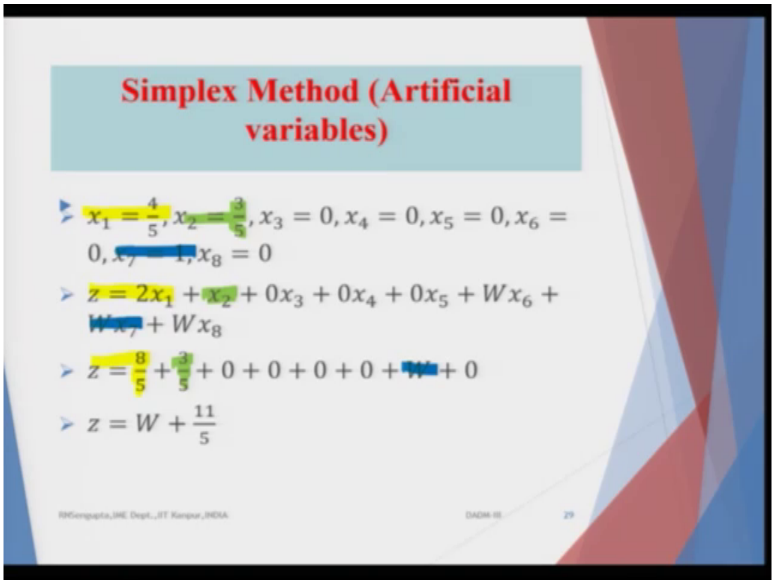

So here it is 0.8 is basically 4 by 5. So x1 is 4 by 5, x2 is 3 by 5 which is 0.6 as it is and the values of x3, 4, 5, 6 are all 0, x7 is 1, x8 is 0. So if I put it in the objective function it is 2x1. So obviously that that would remain oh its too darker colour let me change it. So this is 4 by 5 as it should be so the values 8 by 5, x2 is 3 by 5, x2 is here so is 3 by 5, x3, 4, 5, 6 are all 0, x7 is there so is 1, is 1 into W so is w. So the actual value now becomes W plus 11 by 5. That means it has now initially it was 11W then it was 3W plus 2 so is decreasing.

Now next values is W by 11 by 5. We will continue doing this example and basically check the answer that it as we reach the minimum value. So with this I will close this 29th lecture and start discussing of the last stages of this problem for the artificial variables in the 30th lecture. Have a nice day and thank you very much.