Data Analysis and Decision Making - III
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Lecture 28 -

Welcome back, my dear friends. A very good morning, good afternoon, good evening to all of you. And wherever you are in this part of the globe and this is the Data Analysis and Decision Making - III course under the NPTEL MOOC series.

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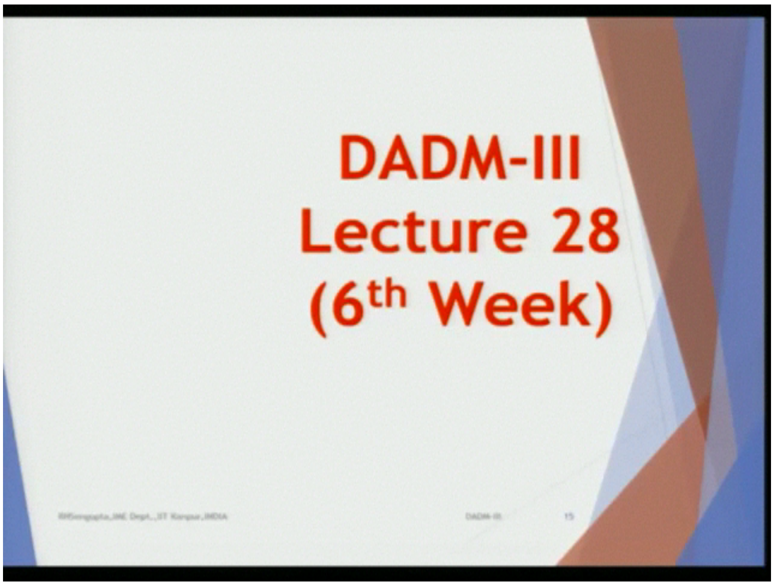

And as you can see from the slide, this is the 28th lecture, which is the 3rd lecture in the 6th week and this total course duration is basically for 30 hours, total contact hours. And the total number of contact lectures is 60 because each lecture is broken now into half an hour or 30 minutes each lecture. And each week and so and the hence the total duration of the total course is 12 weeks and after each week we, after 5 lectures in a week we have an assignment.

So we have already if you can see from the slide if it is 6th week, we have already finished 5 assignments and by the 6th week, and of the 30th lecture you will be facing or taking your 6th assignment. In totality we have 12 assignments and then obviously there is a final examination.

Now in the 27th lecture I just started in the last 3-4 minutes, I was discussing about the artificial variable and why it is there. Because in one of the problem, if you had less than type, you basically or the greater than type, you add the slack or the surplus with the change of sign, it does not matter, you basically can ensure if there is no degeneracy, there is no unboundedness, there is a unique solution so and so forth, constraint all the conditions basically are satisfied.

We can start off at a point where considered as the origin, where the slack or the surpluses are nonnegative and you can basically move in the direction depending on whether you are trying to do maximization or a minimization problem. And move along the corner post till you reach the optimum point, physical point which is either the maxima or the minima. And at that point the variables values which you have would basically dictate what is the decision variables and whether there slack or surplus are present.

But now what? If all the equality signs are true, so obviously we would be tempted to say we will basically break down each equation with a greater than sign, greater than equal to and less than equal sign and solve it, which is fine. But here we will basically add a different methodology and give the concepts such that we will be utilising the concept of artificial variables in a very nice way and give institutive feel that how it can be utilised in trying to solve the problem.

Now solving the problem using the tableau method, which is the pivot column, which is the pivot row, which is the pivot element, how you do the transformation of trying to basically multiply by the pivot row, pivot column, figure out element. And ensure that you at the end of the day, the actual variables which you have on the left inside and the corresponding matrix which is there which should be identity matrix such that the right hand values gives you the unique variable values and the Z value you have already calculated.

So that concept would be everywhere true, here in the case of the artificial variables also. But why we need artificial variables? I will basically go through a very simple conceptual build-up such that you can understand and then a small problem as we generally do. Now consider this very simple case, we minimize, minimization, maximization whatever the concept would not matter. The general concept against, again, I am saying remains the same but mainly the background how we build up.

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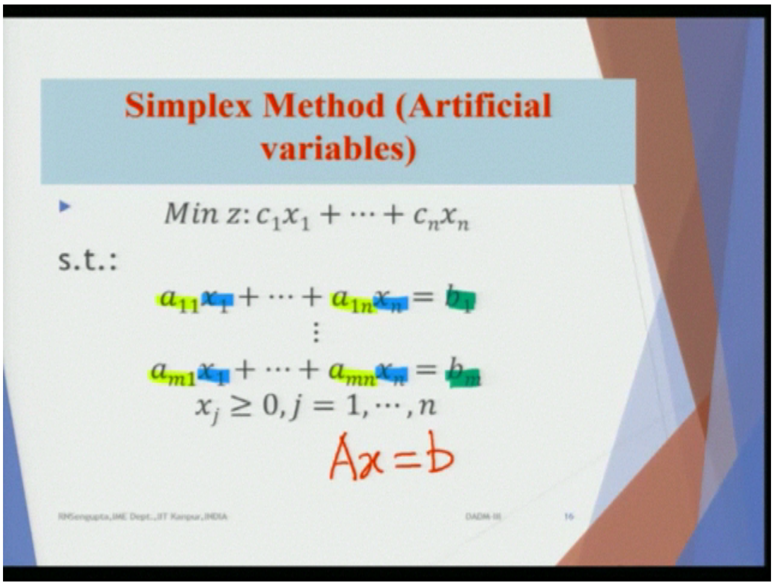

So consider we minimize the cost factors for x1 to and consider again which I would like to repeat that the size of the matrix A is of m cross n, m number of rows and n number of columns. Where m technically would mean number of rows corresponding to number of constraint and the value of N as in Nagpur would be the number of constrains which are specific to the problem which you are solving.

So you want to minimize z and the cost factors are c1 into x1 plus c2 into x2 plus c3 into x3 till the second last term is c suffix n minus 1 into x suffix n minus 1 and the last term would be C n into X n. Now what are the constraints? If I consider the matrix A, then I will repeat and then again come back to the slide. So the matrix A, the first element and I will basically go row by row. First element will be A suffix 11. I would not be using the word suffix, but I will just mention the numbers, so you can understand. So Aa 11, the next number is A 12.

Then the next number would basically be A 13 till the last one would be A 1n. That means we are considering the first row, corresponding the first constraint and the second numbers are corresponding to the variables which you want to consider. Similarly the second row would be A 21, A 22, A 23 till the last element will be A 2n. Similarly, the second last one would be A m minus 11, A m minus 22 till the last element would be A and minus 1n.

And similarly when you come to the last row, it would be A m1 that is the suffixes, the numbers which I am mentioning, A m1, next one is A m2 till the last element is A mn. And what are the…so the values of x element I will just mark with the highlighted being yellow.

So this is the, these are the elements of the A matrix. So there are other values also, I have just made it very short for you to understand.

Now what are the values for the vector B? So B is the right hand side and there is a equality sign. So these vectors are, consider the constraints are equality sign for this problem. These are the values for vector B. And the sizes m cross 1 because there is a column vector. Similarly, if I have the x vector which can be depending on how you have stated the problem can be column or the row. The size is n cross 1 or 1 cross n considering there are n number of decision variables.

So, I should use a different colour, so it is easy for us. So the elements of x vector are these. So actually, you have, so I can use the red color is A x is equal to b which we have been seeing time and again. And the size would basically be done accordingly. So m, a is m cross n, x is n cross 1, b is m cross 1. So obviously that matrix multiplication is valid considering the rows and columns. And here as per the concept as a simplex method linear programming, x j, j is equal to 1 to n are all positive that they are greater than 0.

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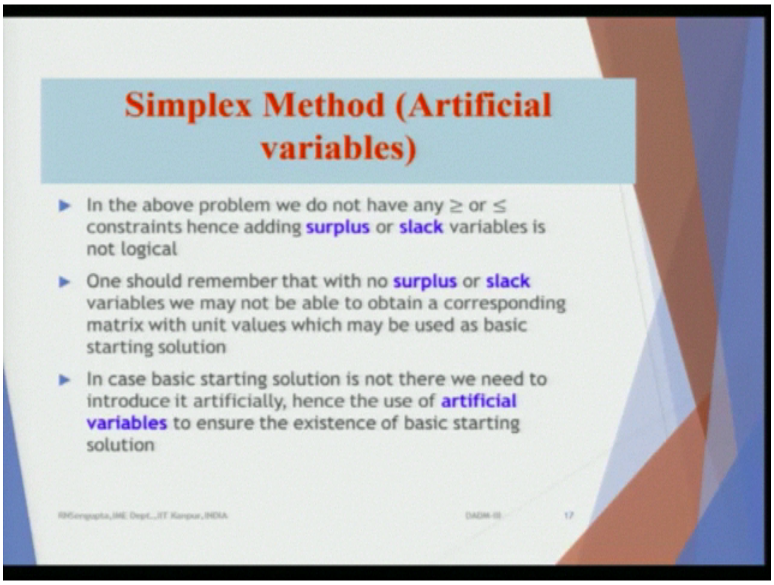

Now here is the background. In the above problem, we do not have any greater than or less than sign constraints. Hence adding surplus or slack variables is not logical, obviously you could have solved the problem which I have not written, we can kind of solved it by greater than equal to, greater than sign or less than sign for each of the constraints. So you would basically now have m added twice each. So you would basically have two m number of constraints. But we are not going to do that.

So what we would do is that and what that, what I am going to read in the second bullet point that one should remember that with no surplus or slack variables. This is important, we may not be able to, it is not obviously true that it will always be, we would facing a problem. But we may not able to obtain a corresponding matrix with unit values which may be used as a basic starting point. So if you remember in the problem which I was showing that the constraint was x3, x4, x5, and x6; 1000, 1,500, 1,700 and 500 and 4,800 for the corresponding values of x3, x4, x5, and x6.

So that was possible and that can be utilised as basic feasible solution, but that considering that basic feasible solution to starting point was not possible. What we would do is that what you are going to consider. In case basic starting solution is not there, what we need to do is that we need to introduce artificially by force. The use of some artificial variables to ensure the existence of a basic starting solution such that we can start our the simplex method along the corner point. That means we are forcing them.

Slack and surplus can be some idea which may exist. Because if you are not going to utilise the total amount or demand for a supply, there is some amount or demand which is left or a supply which is left or we are trying to produce say for example, paints and we are using raw materials and we are not able to utilise all the raw materials. There would be some surplus left or the raw materials, which are actually a very practical situation. But these artificial variables are being brought into the picture forcefully in order to basically kick-start the process.

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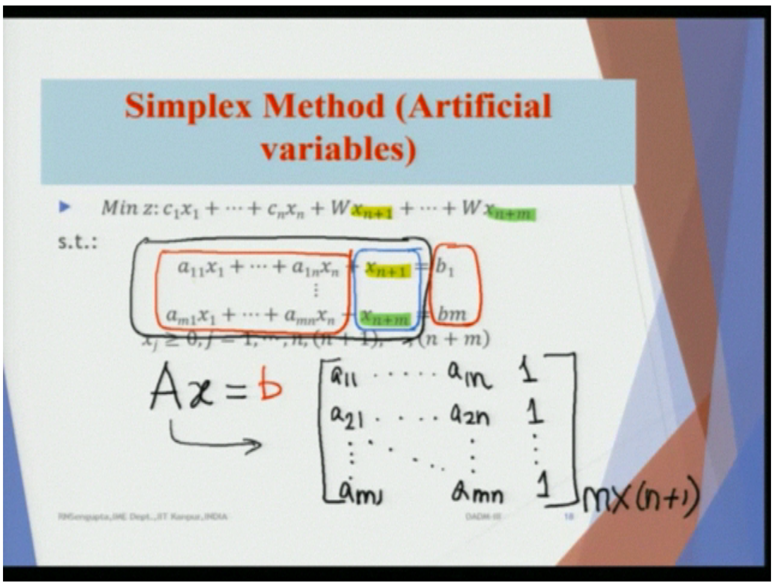

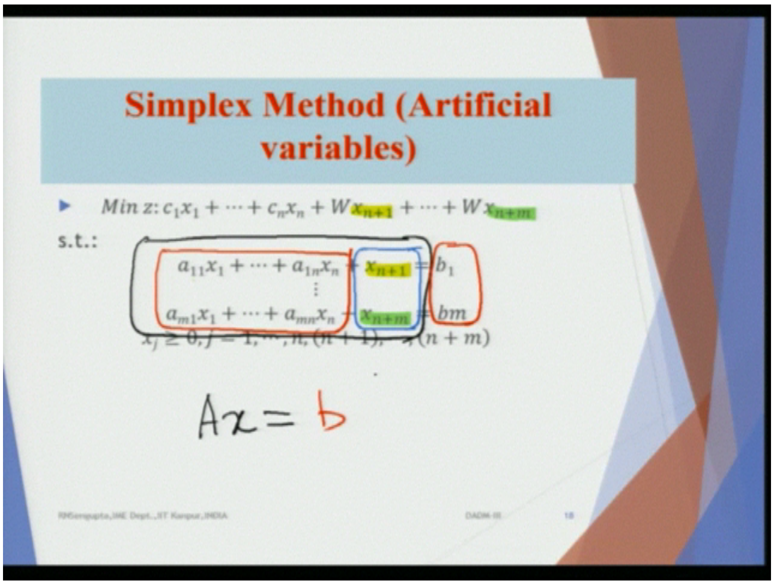

Now let we write the artificial variables in the problem and put the situation in the right perspective. The problem looks very simple with some addition parts. So the initial minimization, so minimization, maximization again, I am repeating the problem solution would be the same. In the minimization problem, the actual formulation was from c1 into x1. So 1, 2, 3, 4, whatever I mentioning are the suffixes, please understand, so I would not be repeating it.

c1 x1 plus x2 x2 till the second last one c n minus 1, x n minus 1 c n. Now we add for each of the constraint we add one artificial variable. So if there are m number of such constraints, we will add m number of such artificial variables. So the artificial variables will now be numbered and I will mark that with the different color. So first I will mark the artificial variables all of them. So these are the artificial variables. So the first artificial variable is x n plus 1, next is x n plus 2 till the last one is x n plus m.

But where do these artificial variables belong to? So let me first remove the color and then basically I will be able to say that. So the first one x n plus 1 belongs to the first constraint. So the first constraint now is A 11, x 1 plus A 12, x 2 till the second last term which is A 1n x n plus x n plus 1 is equal to B1 on the right inside. The second artificial constraint which is not written here, both in the objective function as well as in the set of constraint would be x n plus 2. So it will be coming somewhere here. So the equation actually would be A 12, x 1 plus A, sorry, my mistake is A 21 into x1 plus A 22 into x 2 till the second last term will be A 2n x n plus x n plus 2 is equal to B2.

Similarly I go to the third, the fourth and when I come to the last one, this is a mistake, this should be B suffix m, sorry for that. So the last artificial variable, let me use a different colour, is the green one and it is here. So each colouring scheme would be basically give you, there are only two, so obviously we will understand the…saffron one goes to the first, the green one goes to the last one. So the last equation would be A m1, x 1 plus A m2, x 2 till the second last element is A mn, x n plus x n plus m is equal to B suffix m.

So the initial A matrix was, if I consider the A into x matrix was this one. Now we have added an extra seller column for an element in A and x respectively. So now what is it? The elements of A are now and but remember, very interestingly the right hand side the size of B does not change, it remains as it is m cross 1. So the matrix, I will use the black color here. So this is A, this is a new A, remember that do not the A1 which I have drawn in red. So I am basically now trying to basically draw this or write this whole thing.

So x, x is also different is equal to, b I will use a different colour of red because this was the b which was already there in the first set when they were no artificial variables. But solution does not change in the tableau. So here the size, the elements of A are, I will basically use, so I am trying to basically, I will write down A, then erase it and write down x, erase it and then right down b. So let us go one at a time.

So I am basically going to write A. So it is A 11, I go to the second last one which is A 1n and 1 here corresponding to x n plus 1. The second column is which is not here A 21, A 2n 1. So I am trying to basically write the last constraint corresponding to 1. So what is the size? Size is m cross n plus 1 which is interesting, wait, let me check. Okay, my mistake, my mistake. It is not that because it will change. This is not 1 not always. So I will try to basically do it along this. It is I was trying to basically put it in the same column. So there is a mistake here, I will try to erase it.

And try to basically we would take another fresh slide and mark it. So because that would basically give you the size is not m plus, m into. So this remains same where actually it will be A x is equal to b. So I will now basically write down A n accordingly. So let me, so if we have seen it, then basically let me write it down. So added three blank slides in order to make.

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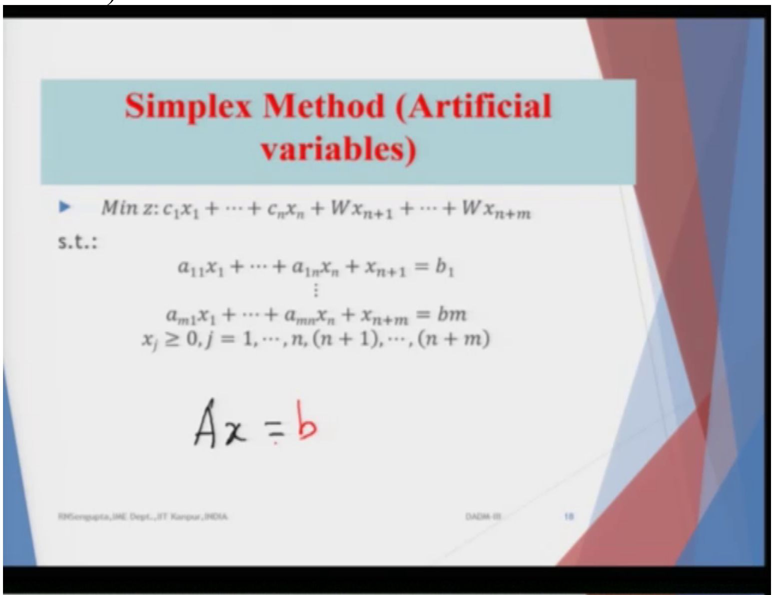

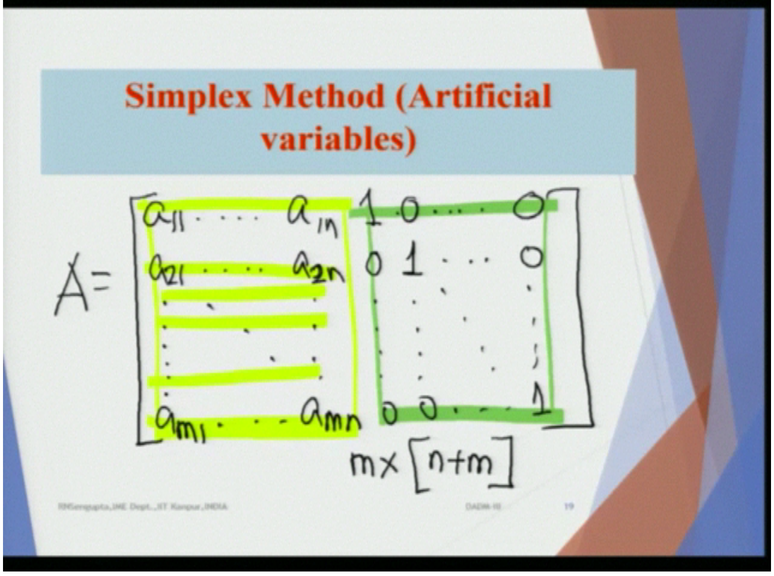

So this was the where we stopped. So it was basically this was where we were and we try to basically proceed. Now what is A? I will use black color and so A is a 11, a n1, 1 all zeros. Where are the zeros going to come? And I am going to come to that. a 21, this is 0, the second element is 1. So there should be 0 here 0, all are not written. So it is m1, mn, then 0, this is 0, this is 1. Now this is interesting. So your number of the size is m cross now you had already n, now we increase to n plus m, so it is n plus m number of columns, so this one. If I consider this, this whole part was already existing and the new one which has come is this one.

Now see the numbers here, it is a principal diagonal is 1 all values are 0, we will come to that later. So that will be multiplied by sum x and let us write the x values. Now remember the x values which we are going to write the technically they would be n plus m. So n plus m would be first n would be all corresponding to the existing decision variables, which is x1 to x n. And the next n plus 1 to n plus m would be the corresponding one to each or the artificial variables which are being added.

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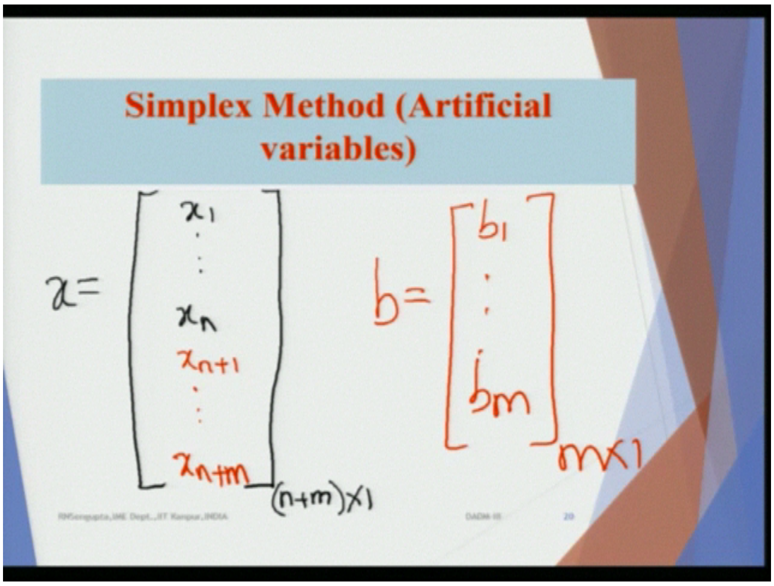

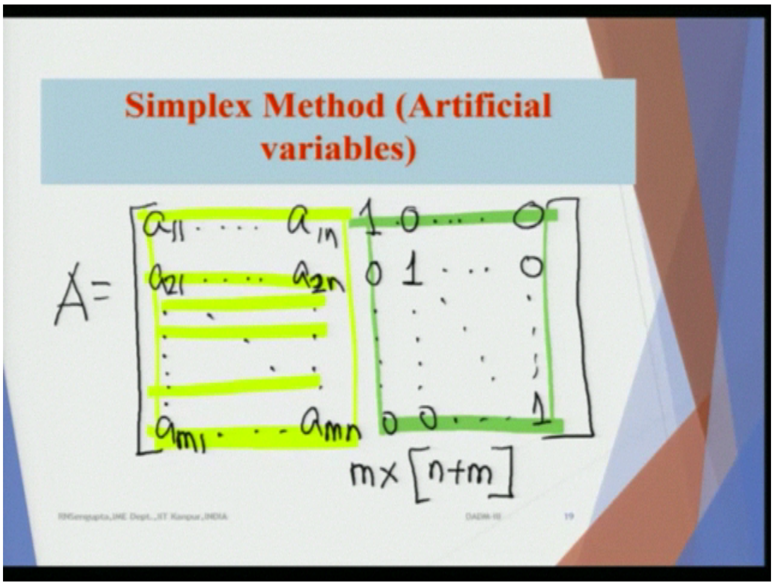

So x would be, so I am going to write x this is a large vector. The first element is x1 go to x n. So it was already there, next element is, if I use the color different for the artificial variables, let me use it n plus 1 and the last one is n plus m. So when I multiply it will be what? a 11 into x 1. Next one is a 12 into x 2, third value is a 13 into x 3, it will go down till the last in that yellow coloured so-called part of that A matrix, new A matrix is a 1n into x n and then what happens?

The next element which you have in this new A is 1. So 1 multiplied by x n plus 1 and all the rest values are 0. So all the corresponding x n plus 2 till x n plus m are all gone, their values would be 0 because the multiplication being there being multiplied by 0. So obviously it would be 0. So if you check this on the right inside what we have? I will use b because that was original one, even though this red color scheme is different. So b was okay before I this, so was size is n plus m cross 1 and b values were all same of size m cross 1.

So once I have A as m cross n plus m into n plus m cross 1, so obviously we will have m cross 1 which is basically b, so your matrix multiplication concept is actually accurate and right.

Now on the right inside as I said, a 11 into x 1 plus a 12 into x 2 dot, dot till the second last element which is a 1n into x n plus 1 into x n plus 1 rest number being 0 is equal to b1. When I come to the next constraint which is the second one, it will be a 21 into x 1 plus a 22 into x 2 dot, dot till the second last term which will be a 2n into x n. The next element would be 0 multiplied by x n plus 1. The next element would be 1 multiplied by x n plus 2 and the rest after x n plus 2 are all 0, onto the right hand side is b2.

Similarly, when I come to the last row which is the mth constraint, it will be x m1 into x 1 till the second last element will be x mn into x n. So why mn? Here, this element and all the variables being multiplied to with x n plus 1 till x n plus m minus 1 are all zero. The last element is 1 into x n plus m is equal to b m. So these x n plus 1 till x n plus m, these are the artificial variables which have added on each of these constraints and they would be utilized for our calculation for the problem.

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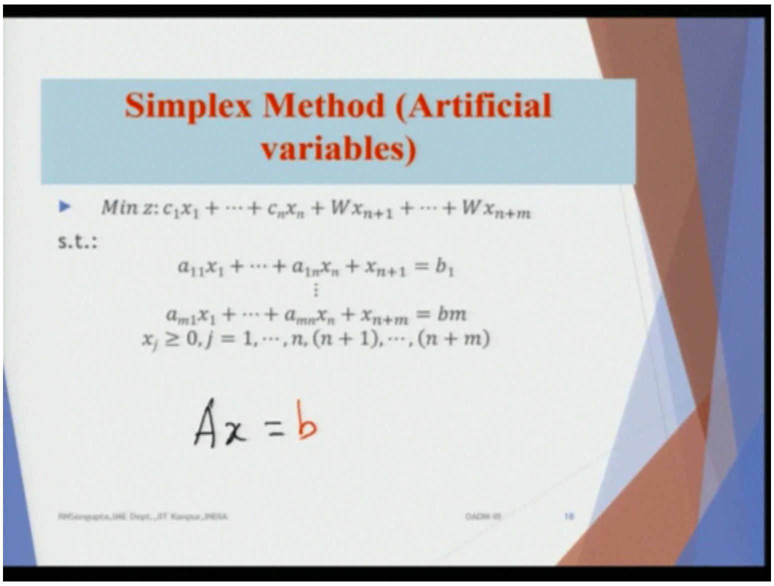

Now if I go to the back to the equation, so this is the artificial variables which I have. Now if you remember one thing these constraints x j, these variables x j, j is equal to 1 to n plus m all would be greater than 0. There are weights if you see in the object function. So in the slack and the surplus they are 0. That means they are not giving me some extra profit or they are, are not trying to bring down my loss in the maximization or minimization problem. These w's are very high values in the positive sense or negative sense depending on the problem which I am going to solve maximization or minimization.

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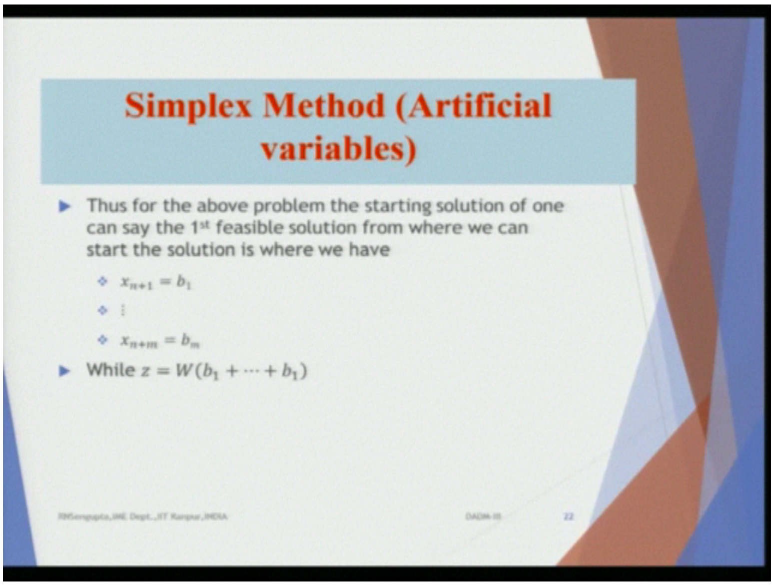

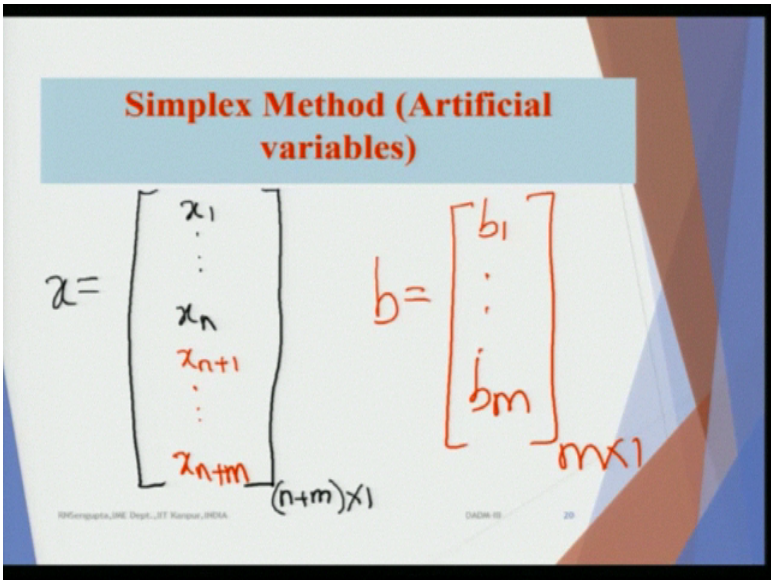

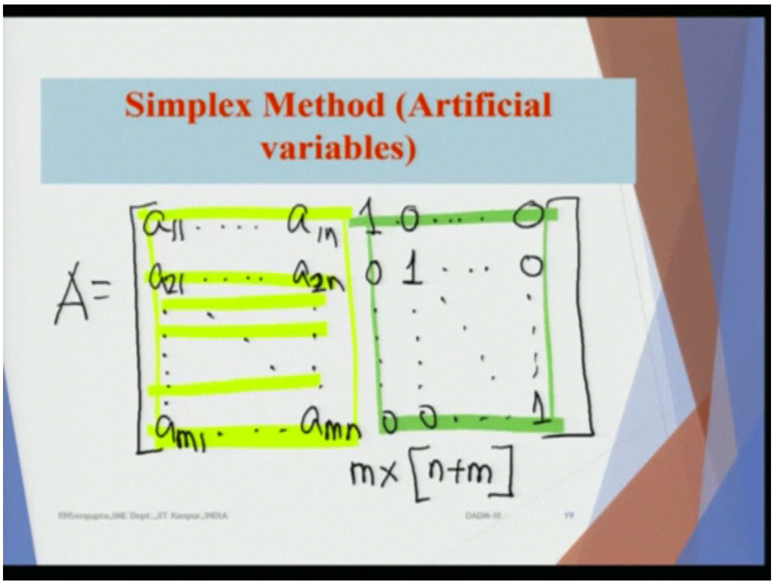

So thus for the above problem the starting solution of 1 which we can say, why we can say the starting solution is now available is due to this reason. I created one extra blank sheet, please bear with me. So once I come here, so consider the original point is origin is taken. So the technically in a dimension higher than 3, so all the variables x 1 to x n are not there, we do not consider, consider they are the origin.

And for each of these constraints, we have only one artificial variable being equal to the right hand side, which is feasible point is applicable such that x n plus 1 is equal to b 1, x n plus 2 is equal to b 2, x n plus 3 is equal to b 3 till the last one, x n plus m is equal to b m and we basically start our whole analysis based on the feasible point. That is what I just mentioned few seconds back.

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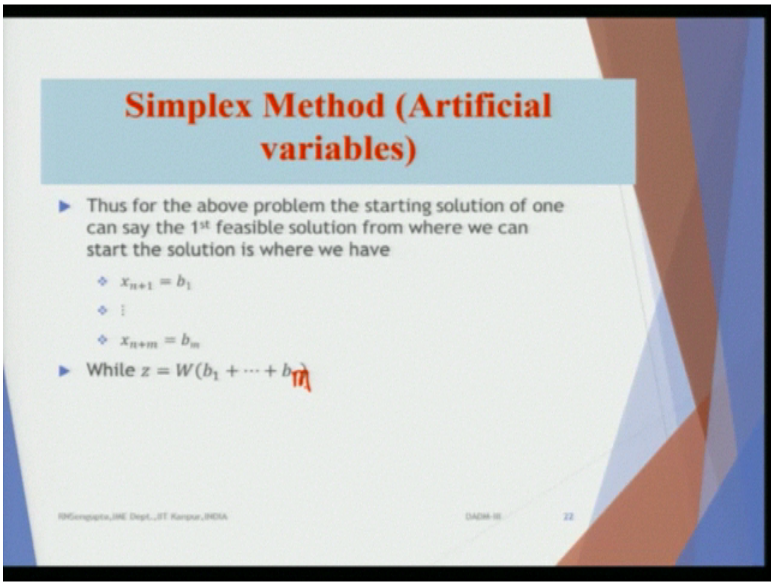

Thus for the above problem the starting solution of 1, can say that at least the first feasible solution is present from where we can start the solution and the solution is like this, x 1 till x n are all zeros. X n plus 1 is equal to b1, x n plus 2 is equal to b2 till x n plus m is equal to b m. And while the z value would be x 1 to x n are all zeros, so they would definitely will be not there in objective function. But we have also have artificial variables multiplied by w.

So technically in the minimization problem, in the maximization problem, we are at extreme points with a very high or a very low value, based on which we will try to basically reduce or increase depending on the optimization problem. So hence the value of w would be important in order for the conceptualisation stage. So the actual value of z would be w in the bracket multiplied from b1 plus b2 till b m, this would be b m. So b1 to b m are the corresponding values of the artificial variables.

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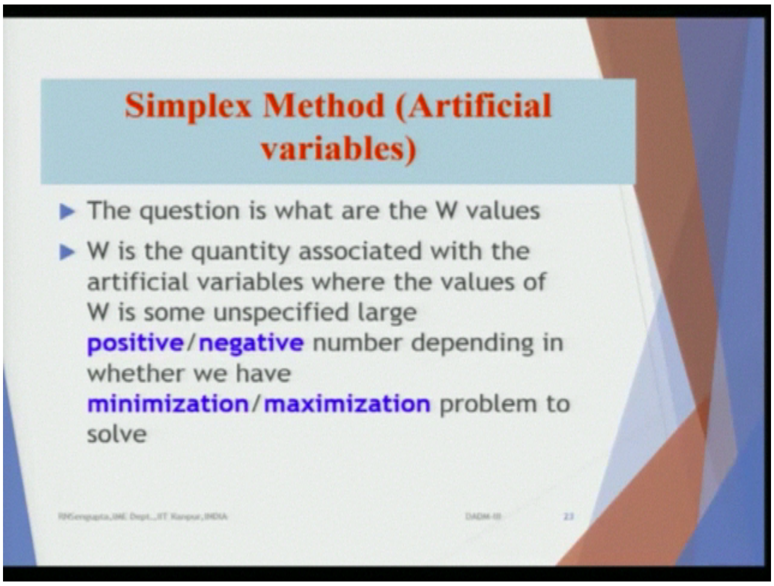

Now here, I will basically say something. So the question is that what are the values of w? So W is a quantity associated with the artificial variables themselves, where the values of w is some unspecified very large positive or very large negative values depending on whether you are trying to do a minimization problem or the maximization problem. In a sense that as you move along the corner points we will ensure that movement would always be in that direction where it will basically for the object function if it is the maximization problem, so you see negative being maximization means it will keep increasing and try to reach that optimum value point where the objective function by itself would be increasing.

In the minimization case, it is a very high point, hence it is positive. So we will keep decreasing depending on the gradient in which direction we are moving and we will try to reach the minimization. So with this, I will end this lecture and basically discuss this concept with the simple problem solution using the artificial variables in a very simple simplex problem. Thank you very much and have a nice day.