Data Analysis and Decision Making – III
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Lecture – 27

Welcome back my dear friends a very good morning, good afternoon, good evening to all of you to wherever you are in this part of the globe.

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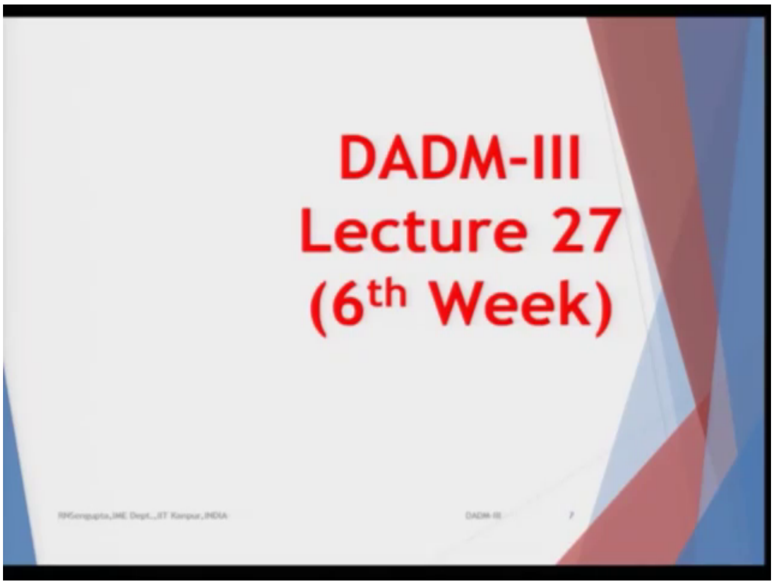

As you know this is the decision analysis and data analysis and decision making 3 course under the NPTEL MOOC series and as you can see this is the 6 week going which is the 27 lecture and this total course duration is you know is total contact hours is 30 which is basic respect and broken into 60 lectures each lecture being for half an hour and each week we have 5 lectures after which we have a 1 assignment.

So we have already completed if we see that means we are in the 6 week that means you have completed 5 weeks 5 assignment and in total will have 12 assignments and after the end of this whole course you will have a final examination paper, which we will generally try to cover the whole of the concepts as you have utilized here

My good name is Raghu Nandan Sengupta from IME department at IIT Kanpur. So if you remember we will still continue with the simplex (dual) duality and primal dual concepts, if you remember we have been talking about slack surplus slack surplus time and again.

Now when we come to the concept of slack and surplus obviously that is been added in order to ensure that we have a basic feasible solution based on the fact that none of the variables are negative because that would be infeasible and as per the concept we consider all of the the slack surplus and the the actual variables decision variables to be non negative.

When we attempt the final solution technically, if nothing is wasted, nothing is used excessively or not wasted, the slack and surplus would be 0 the actual decisions variables would be at the maximum value and will basically have the concept of getting the (opti) optimum value maximum minimum.

Now for the last problem which you are discussing they were they could have been of any assigned greater than or less than or some been equal to but we discuss that if they are been equal to you can (bre) break them up into greater than or less than and basically have demand in the supply constraint depending on the problem where I say that you have different type of ware houses or factories onto the left and you have different types of distributers size been n or m and n and m are not equal all of the nodes in in n are connected to all the of the nodes in m and obviously the demand supply who would basically ensure that is less than type or greater than types such that we can add the slack and surplus accordingly.

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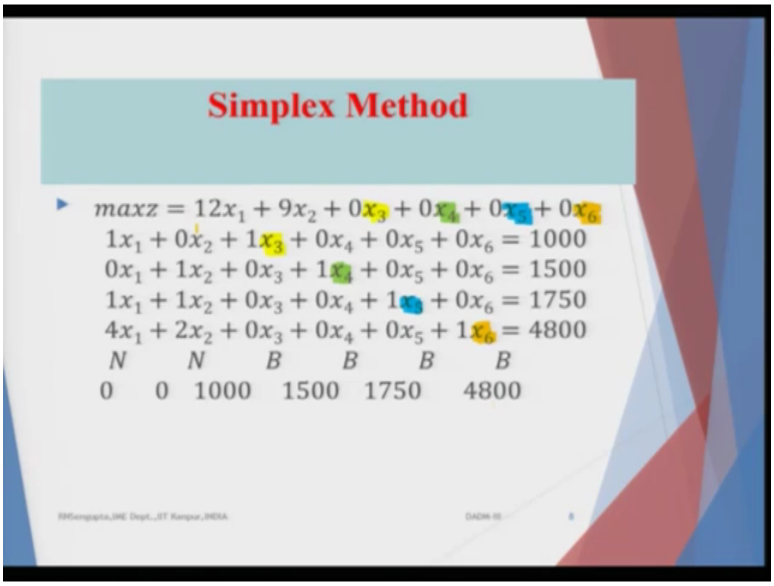

Now when we have the slack and the surpluses and consider these slack and the surpluses are such there are 4 equations we will basically have x3, x4, x5 and x6 so I will basically mark them.

So obviously you can understand but I still them mark them, so x3 is for the first equation, x4 is for the second equation, x5 is for the third equation, x6 is for the fourth equation and obviously their parameter values when they go into the objective function where the maximization or minimization are all 0, so I will not mark it but I am showing x3 is multiplied by 0, x4 multiplied by 0, x5 is multiplied by 0, x6 is multiplied by 0.

So your now if you look at this this whole tableau I want to spent some time more into the tableau, tableau even I have not drawn but you will understand what we mean by tableau once we go into the next concept of the artificial one because that would be important for us to understand.

So if you if you start with the basic solution which is considered as origin so obviously x1 is 0 x2 is 0 so in this equation x1 goes x2 is obviously multiplied by 0 obviously it will go and x2 also is 0, x4, x5, x6 are not there so x3 would be thousand which is here. So will have the value of x3 as thousand x1 as 0, x2 as 0 when I go to the next equation x1 is 0, x2 is 0, x3 is even though it is thousand is multiplied by 0 so obviously its affect would be 0.

x5, x6 are multiplied by 0 even if there values are non 0, so x4 is 1500 which is shown here and when I come to the third equation x1, x2 are 0 so even if they are multiplied by 1 it become 0, x3 thousand but multiplied by 0 become 0, x4 (multi) even 1500 multiplied by 0 becomes 0, x6 whatever the value is will come to that later on multiplied by 0 it will be 0 so x5 is 1750 so this is 1750 and the last equation x1, x2 are 0 because of the origin so 4, 2 have no affect.

x3, 4, 5 which you are found out as 1000, 1500, 1750 what affect would be 0 because all the parameters are 0 and last is basically x6 which is the only variable left based on the last constant it will be 4800, so when we basically start of the solution we will have x1 0, x2 0, x3 1000, x4 1500, x4 (x4), x5 1750, x6 (480) 4800

So the the basic feasible solution where it start is 00, 1000, 1500, 1750, 4800 and the actual maximum value would be the first (it) the first value x1 is 0 0 so all these value as 0 so obviously you will have a maximum value of 0 then you start moving depending on the maximization and minimization concept which you have.

Now there would be some degeneration condition in linear programming we can have that in this program problem also but first I will show you the the graphical concept why this digenesis occurs

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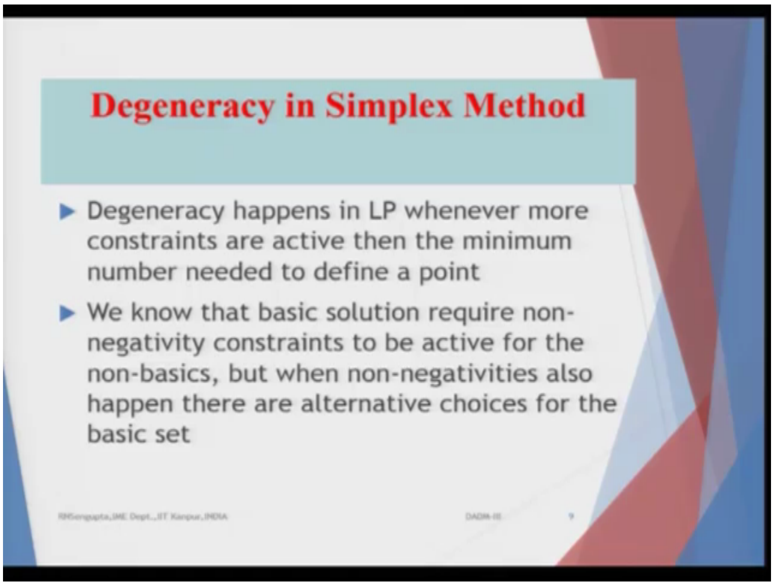

Dysgenesis happens in linear programming whenever more constraints are active and they utilized then the minimum number needed to define a point. So say for example you have in a 2 dimension 2 equations solve them get the 2 points, so they unique solution consider there are 3 question in 2 dimension and trying to solve them basically gives you some redundant equations or some constraints which are active and too many of them are active in that case we will have the degeneracy

Degeneracy points would basically mean that finding out the unique solution would not be possible for those points, I will come to that again as I said with a diagram in a 3 dimensional case because trying to draw a 4 dimension would be difficult so I have tried my level best to give you a 3 dimensional concept.

Now we know in the basic solution requires non negative constraints, so when your basically solving a non negative means because all the points where you start of the basic solution whether the slacks are plus the actual variables all should be non negative.

So we know the basic solution requires non negativity constraints to be active for the non basics but when non-negativities also happens there are alternative choices for the basic set, we can find out so obviously it would mean they that would be degeneracy from where which point to start are basic solution would not be clear to us.

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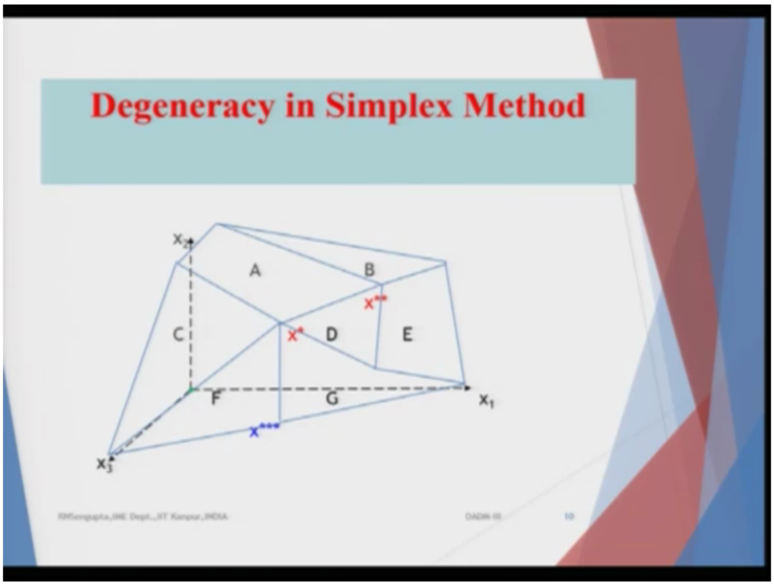

Now let us see this diagram so I am (dra) I have tried drawing into the 3d diagram let us have a look so if you see this slide so x so the point I have not marked too many a points because or else it will become cluttered but I marked some a, b, c, d what are those a, b, c, d I am going to come that within one minute so you have the origin where the highlight is now marking so I will mark it with let me use a other colour because there are red points here.

So let me use the green one so this point is basically the origin what is happening, yes so this is the origin and if I go into the right I have the x1 direction or the variable x1, if I go orthogonal on the top which is y axis generally in the Cartesian coordinates of the graph paper that is x2 and technically the point or the axis which is coming out from the blue dot which is the origin towards me is basically x3, so they are all orthogonal.

Now in a 2 dimension we have a line to basically or or the corner points are basically the intersection of of minimum of 2 lines, now in a 3 dimension there would be intersection of plains I am considering the constraints are linear so they would be just simple plains.

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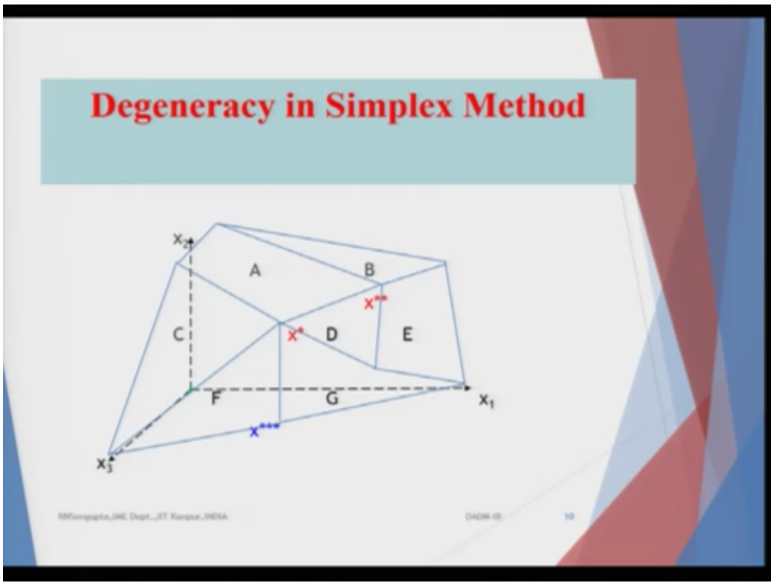

Now the plains I have marked them as A, so if I the plain A is basically a slanted if I consider this room so x1 is going onto the right x2 is going up and x3 is coming here so x A plain is like this slanted like this and you have basically at some angle you have the B plain, you have the E plain.

If you look at the diagram and what I am explaining you have the A plain A plain they have the E plain so the directions maybe different you have the D plains which looks like A triangular one you have the c plain which is just at some angel to A you have the F plain and the G plain.

Now they meet at different points which is immaterial and this overall area inside this plain consider A inside this plains is basically overall feasible region in which the (actua) actual sets of feasible solutions are and as per the concept of corner point solutions the corner points would be given where the plains basically meet each other they can be more than two plains also.

Now let us consider that in this direction you have 1 equation 2 equation for plain a 3rd equation and a 4th equation so x star point which is here is basically over defined that means there are more than needed numbers of constraints which will divine the x star point hence that is a degenerate point that is why it is basically marked as x star in a red colour.

Similarly find out the number of such at any particular point number of edges which are (me) meeting. Now if you consider the point where you have A plain B plain D plain and E plain meeting so that point which is here I should let me use a different colour here, so that point x double star is defined by more than the required number of constraints so hence it will also be degenerate.

When I come to the points which is being defined by the the plains FG and obviously there would be some other thing below so I am considering the number of edges which is meeting in the 3 (dimen) case there are 3 points meeting hence these points which is x, triple star in the blue colour would be a non degenerate point and which is basically can be a feasible solution and will consider that to be feasible and 1 of the contenders for the optimum (sol) optimal solutions in the (())(14:12) set where we when we do the search along the corner points for this overall space which we have which we consider as the feasible region

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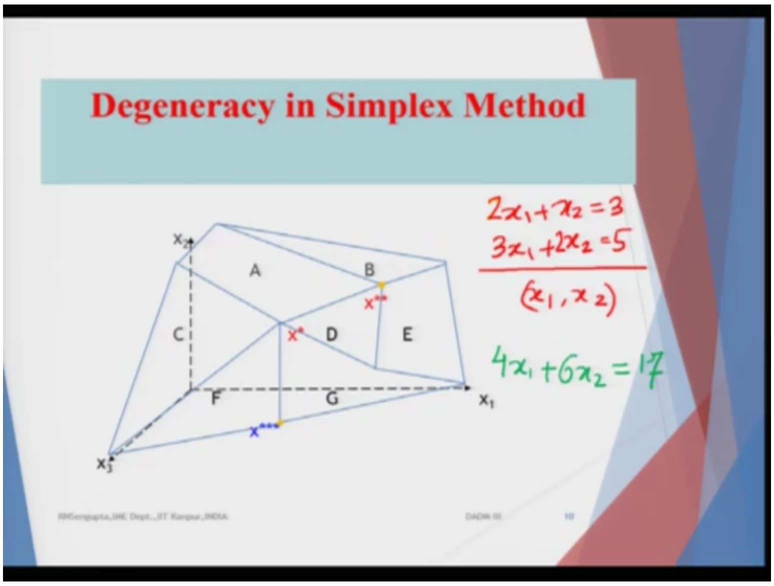

Now when I go to the 2 dimension one technically if I have equations for example the colour has to be different, consider it is 2, 3 this 3x1, I solve them I get unique solutions so I have some x1 and x2 I solve them and basically proceed accordingly, now consider along with this two equations I have another one

Now in a 2 dimension if we want to basically so this I am given example I am not going to solve them so consider if if we have equations where all of them active and they give us active in the sense, they are giving us some solutions about the feasible region and if you solve them and if you find that that all of them active basically gives me a values of x1, x2 in 2 dimension which is definitely not required as per the norm.

So they would be degenerate in the sense they are not possible and will not be considering those sets of points in order to start our solution for the simplex method or when we are going to consider the basic feasible solution.

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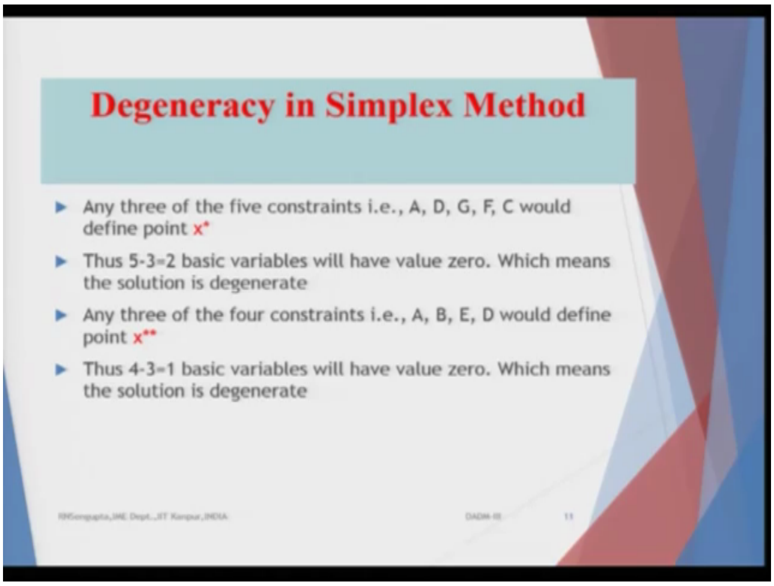

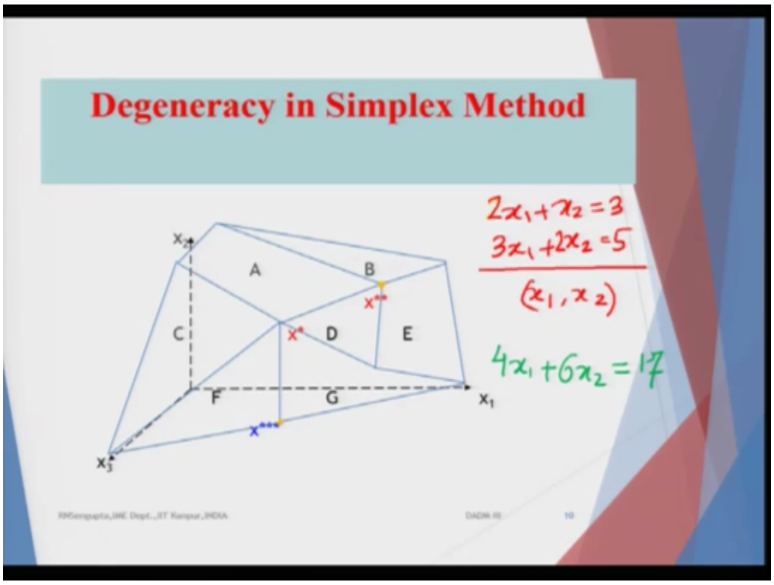

So what are the norms, now here what I have mentioned any 3 of the 5 constraints so here I will even though I have not being able to draw very clearly but I will consider there are 5 constraints which is a, d, g, f and c so these are the 5 constraints in the plain plain concept which is therefore extra, so let us check whether they are these A, D, G, F, C.

So yes you see A, D, G, F, C and the 5 constraints constraining the lines of the plains, so 3 of the 5 constraints would define but we have basically 5 constraints so the points which you will have x star which is red in colour is degenerate. Thus, 5 minus 3 on this 2 basic (varia) variables would have basically value 0 which means the solution degenerate because when we solve the problem technically in the actual solution they would basically be 5 such considering the slack of the surplus what we have in the 5 dimensions and we are able to draw it consider.

So you would not basically have solutions which would be actually possible so it will degenerate to a degenery point which would not be utilized for our solutions. Now when we consider x double star so x double star has A, B, E, D four surfaces and their points is given by x double star so any 3 of the 4 constraints which is A, B, E, D would define x double star.

Hence, 4 minus 3 because 4 constraints are there and 3 is basically what is (numb) minimum number required, there would be 1 basic variable which will basically have value 0 which means the solution is degenerate but if we come to point x star so obviously there would be a surface below x triple star here.

So it will be F, G and consider as H so if we have basically 3 constraints or a plains and there solutions basically gives the unique point that would be non-degenerate point and because in that case they would be 0 number of basic feasible solutions which are 0 and we can utilize that to consider as a starting point for our search technique for the simplex method.

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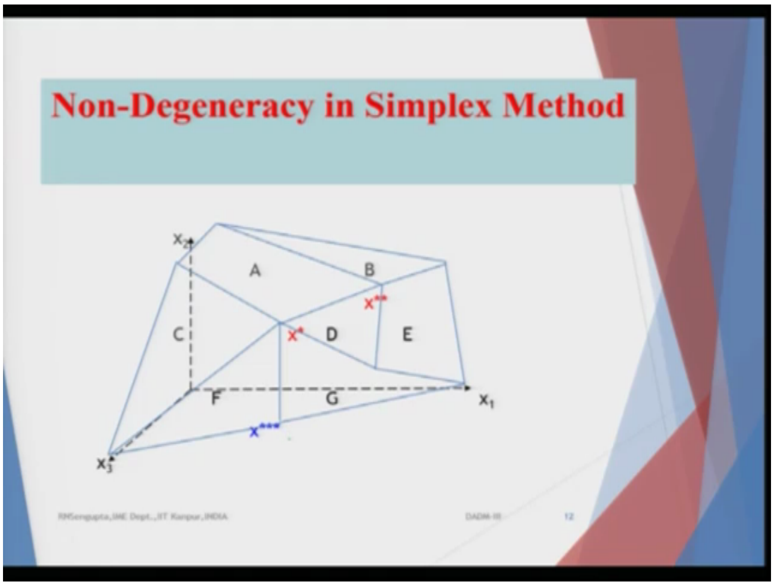

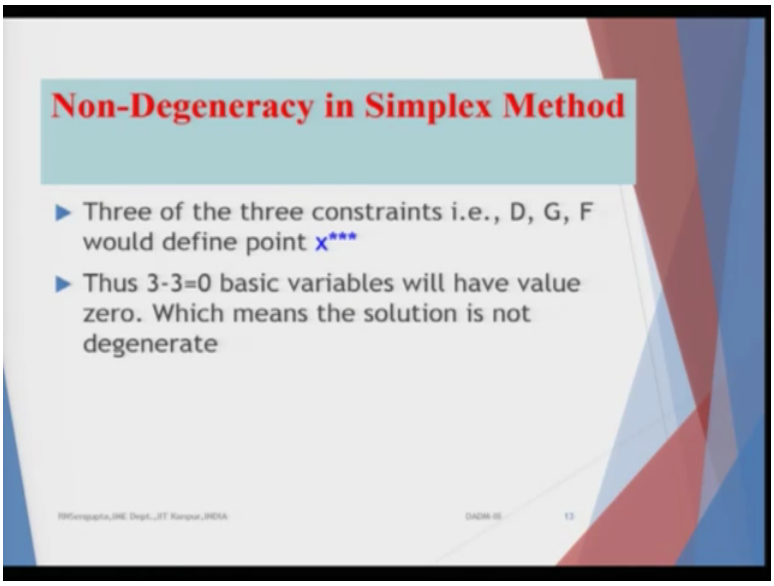

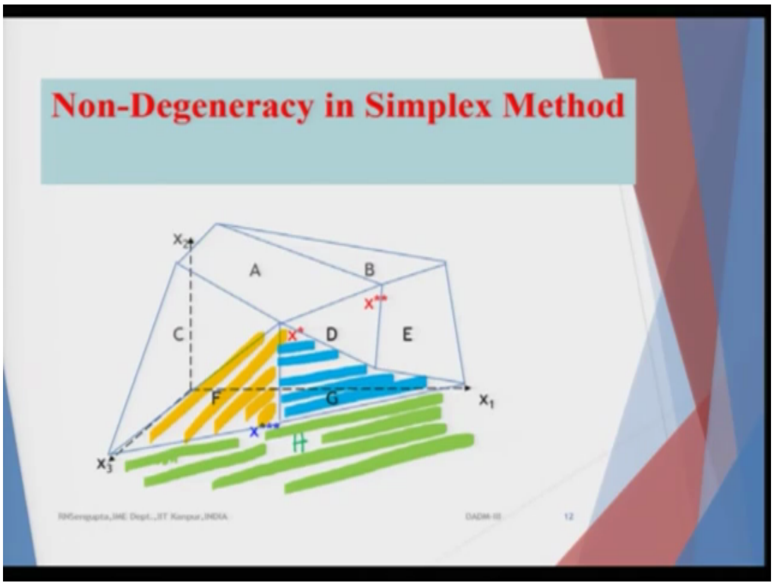

So again I am drawing the diagram but I am not going to (con) concentrate on x triple star which is blue in colour, so the third plain or if it is in the 2 dimension the line has not been shown but the plains are F, G and other one which is H so okay it is mentioned as D okay so D is not right so technically it would be H sorry my mistake here.

So because we were considering that d if it comes out and meets the surface at this point but it has not has been drawn so I will basically change it to surface H which is below so surface H is this one see if I marked as a as a… while f would be this one and g would be, they look in the 2 dimension but they are just slanted plains straight obviously but slanted plains.

It is like this corner points here and the surface here so the corner point which is (her) at this point is x triple star.

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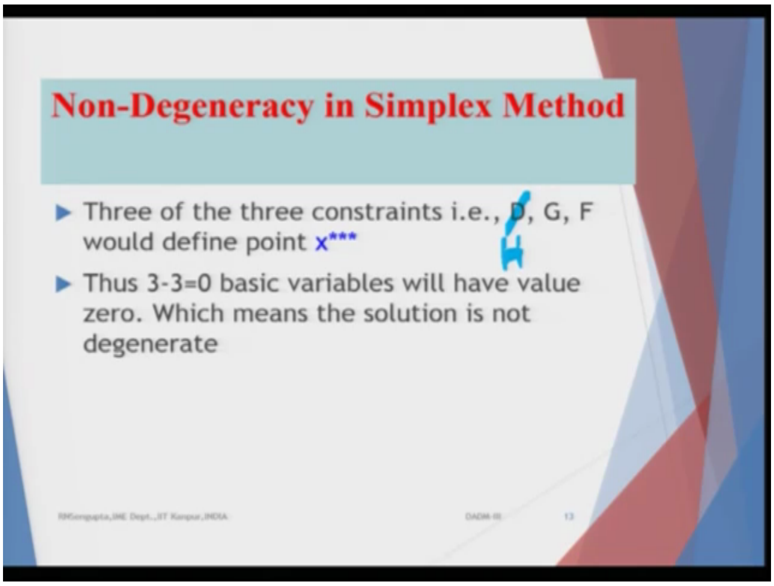

So three of the constraints D, G not D this will go this will be H, G and F would define the point x triple star. Thus, three minus c because there are 3, 3 constraints there are there are variables which are required x3 so 3 minus 3 0 variables will have value 0 which means 0 number (basi) basic variables have value 0 which means the solution is not degenerate and they can be considerate for our concept of as one of the contenders in the basic solution.

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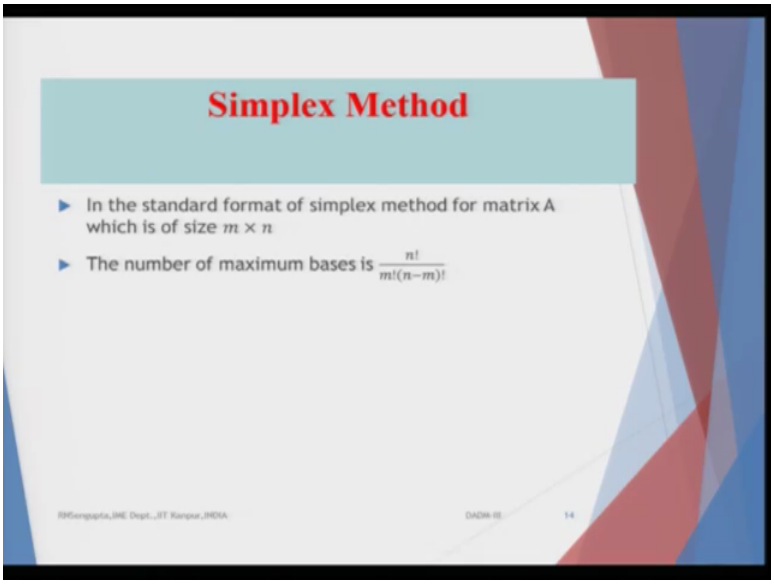

Now in the standard format of the simplex method which is of size m cross n and we will consider the m being the rows and n being the columns which if when you consider from the constraint point of view the solution so m would be number of constraints n would be the number of variables including the actual basic variables, slacks, surplus and also later on we will see the artificial variables.

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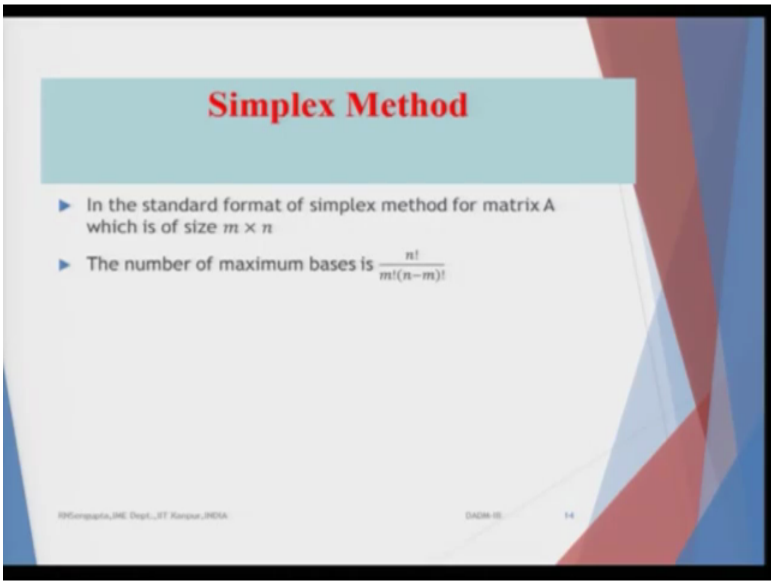

And obviously if you and it does not mention anything about the the numbers of m and the numbers of n it only defines what is m and what is n, so for the matrix a the number of rows is m as in mangoes, n as in nose or m as in Mumbai or as in Nagpur.

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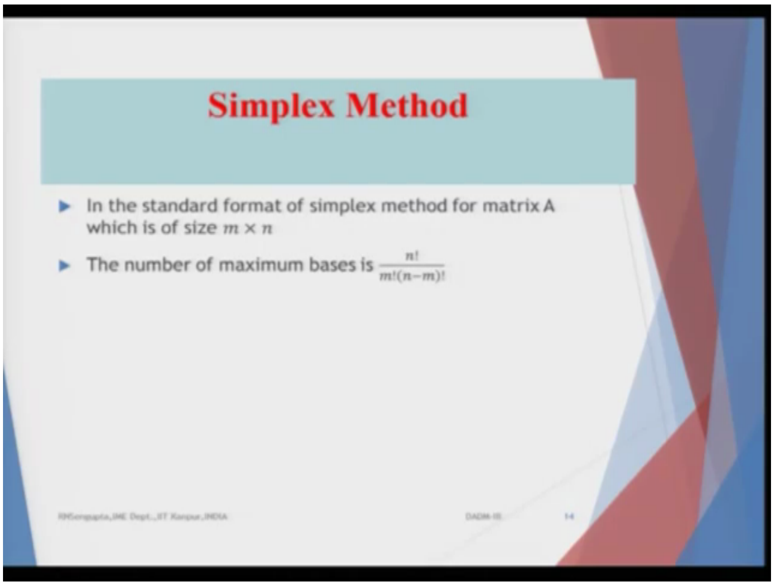

So the number of maximum basis which is possible would be given by factorial n divide by factorial m into n minus m and based on that you can basically proceed to how to how you can find, it is like this how you divide the whole whole set so you can basically divide the whole set into m number and then left would basically be n minus m, you are trying to basically consider the total number of combination which is there.

Now as I mentioned as we have already considered the slack and surplus so we will basically go into the history of a little bit about the artificial one.

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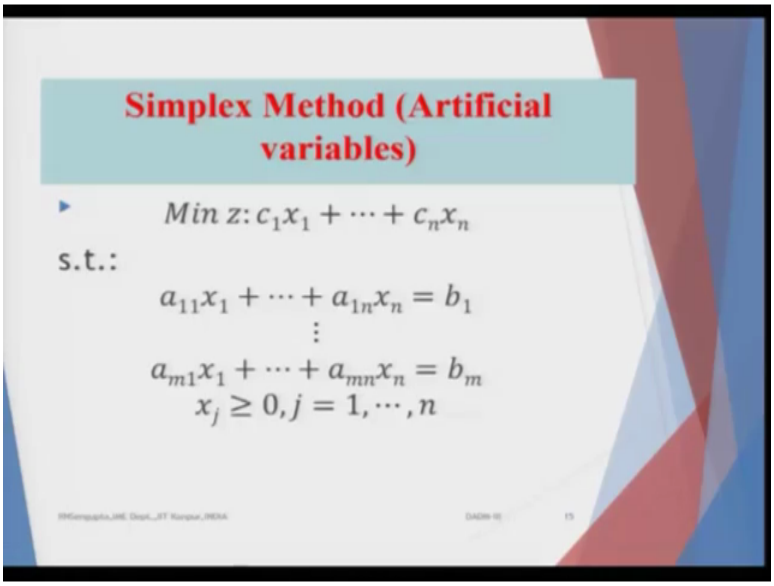

Now we have been talking about the artificial variables time and again but the point is what are the (arti) artificial variables and why, now if you if you remember the background for the slacks and surplus it was basically the constraints being of the type greater than and less than and they were basically related to the production of the supply.

If you remember the n number of distributers n numbers of warehouses or the factories on the left hand side and m number of distributers on the right hand side and all being connected, so whether connection is at all available that is a different question but will make connection available between each of these n nodes and each of these m nodes.

Now whenever you are, we start considering the slack and surplus we intuitively assume that if I consider the the problem where the actual variables are 0 that is that the origin the slack and the surplus would obviously come out automatically to be positive and they can be consider as the basic feasible solution from where you can kick start your simplex method.

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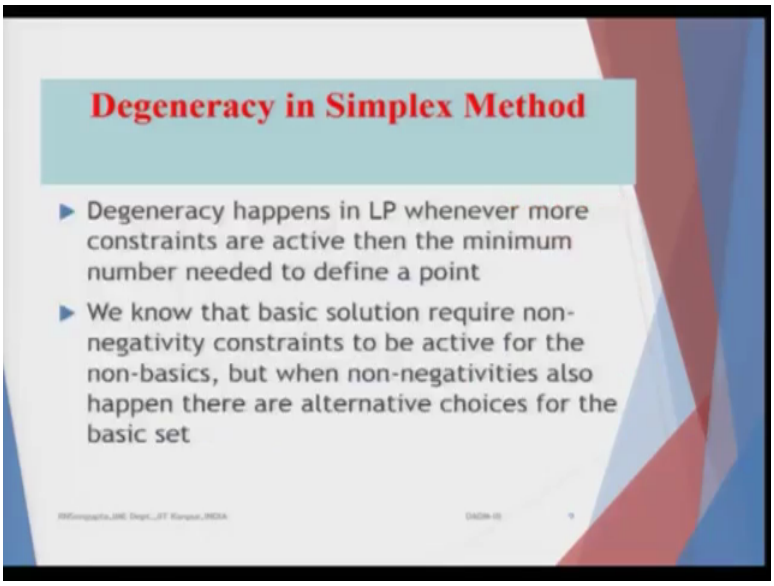

So if I consider I am going back to the solution generally I do not scroll up but I will just give you that… so if you consider this problem they were either, they were of the less than types so obviously you had the plus values of x3, x4, x5, x6 if they were greater than type that minus sign would just be taken initially and then you can reverse the minus sign to a plus sign considering that how the change of the signs can be done on the right hand side and the left side and the left hand side of the equation.

Now when I am talking about the basic solution which I did mentioned but I when I was considering this problem so the tableau when you have the a matrix they would be set of identity matrix inside that a which will give the initial values corresponding to the basic solution from where it going to start.

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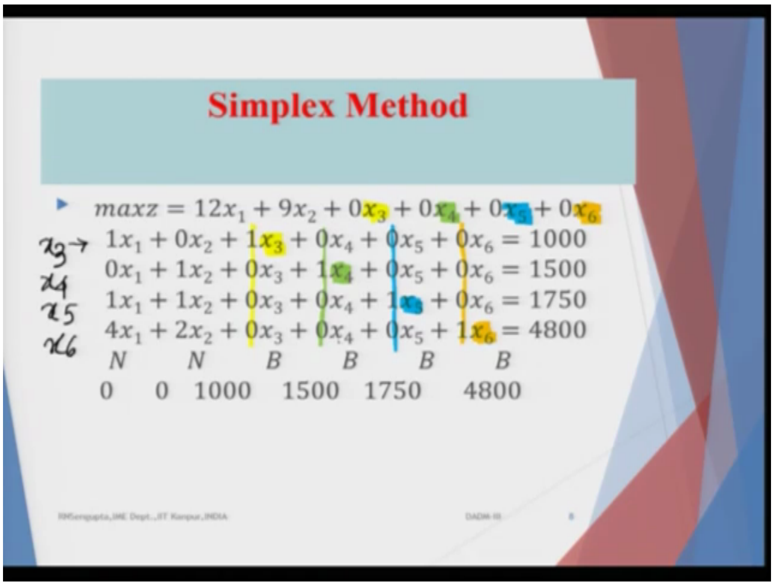

So initially this would be, if you remember this is 100 so on the left hand side you have x3 then the next point in the tableau and the next row you have x4 which are actual solution are basic feasible solution at that point when you going to start, the third variable would be x5 and the fourth one would be x6 and x1 and x2 are not there.

So the next column is 0100 so that is the vector, so 1000, 0100 the third one is 0010 and the fourth one is 0001. So let us see what we actually mean, I will use red colour because red colour is not there so it would not be confusing.

So I have here x3 let me use wait… so this is x3 can you read it, it is very difficult to read so I will erase it and use the black one. So this is x3 which is corresponding to the 1st constraint, x4, x5, x6 so when I solved it the matrix when you multiply so on the (lef) this x3, x4, x5, x6 are on the right hand side which is basically x vector and the a values are if I go along the row 101000 so when I multiply obviously the n and obviously in the values of x1, x2 technically they are actually 0, they are forced to be zero.

So it will be 1 into 0 plus 0 into 0 plus 1 into x3 plus 0 into x4 plus 0 into x5 plus 0 into x6 is thousand that is why I am getting the value of x3 as 1000. Next when I multiply x1, x2 are at the corner point which is there in 0. So it will be 0 into 0 plus 1 into 0 plus 0 into x3 because x3 whatever the value is we have found 1000 it does not have any affect.

So 0 into x3 plus 1 into x4 plus 0 into x5 plus 0 into x6 is equal to 1500. Hence, we get x is 4 is equal to 1500.

Similarly in the 3rd constraint it will be 1 into x, x1 is zero so it goes 1 into 0 plus 1 into 0 plus 0 into x3 so it vanishes, vanishes means it does not have a effect 0 into x4 obviously it does not have any affect 1 into x5 plus 0 into x6 is equal to 1750 so you get value of x5 as 1750.

Last equation being 4 into 0 obviously value is not there 2 into 0 value not there 0 being there for x3 does not have any affect that means it is not there 0 into x4 does not have any affect 0 into x5 does not have any affect 1 into x5 gives you 4800 so the overall value of x6 is 4800 now this may not be possible and we basically giving the concept of artificial variables.

Why this is not possible let me basically go through an example in the next class with me with this I will end this lecture which is the second lecture in the 6 week and I'll basically start of discussing the concept of artificial variable.

How they help us in trying to get a basic feasible solution in the in the discussion, now you may be asking that why do we need a artificial variables if there are equality constraints on obviously technically no slacks and surplus have to be added obviously you can add it by a greater than sign lesser than sign that is a different question but we do not have any slack and surplus.

That means we do not have any set of variables like as we saw it here which can put into non 0 greater than 0 that is non negative and can kick start your own simplex process if you do not have that situation that luxury for us we actually need the artificial variables and what is the actual affect in objective function will also see that later.

With this I will end this class and discuss the artificial variable concept in the later class, thank you very much and have a nice day.