Data Analysis and Decision Making - III
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Lecture 26 - Analysis of the linear programming

Very good morning. Good afternoon. Good evening to all of you, to all my dear friends. Welcome back to this DADM - III and wherever you are in this part of the globe once again a warm welcome and this is Data Analysis and Decision Making-III, a course on the NPTEL MOOC series and as you know, this course total duration is for 12 weeks, which when divided into number of contact hours is 30 hours and the total number of lectures, which will be there for this course is 16 number, each course or each lecture being for half an hour.

And as you can see, my good name is Raghu Nandan Sengupta from IME Department IIT Kanpur. Now I am in we are going to start the sixth week. That means we have already completed 25 lectures, that means five such assignments you have already taken, we will be starting the 26th one and each week we have five lectures each being for half an hour and after that obviously, as I said, you take an assignment, you have finished 5, you have still another total of 12 one. That means another 7 one to go. So we end the sixth week.

We are going to do the analysis of the linear programming concept, the revised Simplex method, we will come to come to that and discuss more about the other methodologies in non-linear, linear programming, nonlinear programming and so on and so forth.

Now if you remember, we have been talking about the basic solution, physical basic feasible basic solution, the concept of degeneracy and we have discussed about the concept of unique solution being available for the case when the number of rank based on the fact that the A matrix which was the basically the matrix which has so-called the parameters for the X and X is basically the decision variable vector and on the right hand side you have B which is the right hand side. So X obviously would have both the basic (sol) the actual decision variables plus the slacks and surplus depending on whether the equations are greater than 0, less than 0 and so on and so forth.

Now if you remember we are doing about the sensitive analysis and I did mention sensitive analysis was basically due to the perturbations or the movement in the constraints, that means the right hand side basically w change it one at a time and then you find out that what is the change in the decision variables. Obviously, you can do more than one at a time, two at a time and obviously in that case the whole set of vectors would basically change accordingly.

Now, if you are only concentrating on one of the decision variables, one of the constraints sorry, so obviously we will try to find out the effect of the change of that decision variables on each and one of these x 1, x 2, x 3, till x n considering there are n number of decision variables to give. Now when you consider the concept of see for example changing the right hand side, whether you are increasing decreasing, that is a different question. Trying to find out the change into decision through constraints and what effect it will have on the decision variables it basically is a conglomerative or total effect which will happen on all the x 1 to x n considering both of them are changing.

So what is the individual effect which will have on the decision variables from the constraints that would not be able to, we will not be able to know until and unless you basically do analysis of each one of these variables individually. Like see for example, the right hand side of, if you remember there was a problem where the number of hours being utilized by the machines were eight hours. Say for example, you can only utilize one of them for seven hours and that one extra hours you are going you are like one of the hours you are going to reduce in one of them, can be utilized for the other one.

So obviously in that case one will decrease from 8 to 7. That means less than equal to 7, other can basically increase to 9, so it will be less than equal to 9. So in that case the total effect, which you will have on the machine output on the decision variables, like variable 1, x 1 or x 2, x 3, however, the numbers would be would be found out by individually changing them and then finding on the cumulative effect. So you can do the individual changes like for the first constraint and then consider the second constraint, then combine them to find the result.

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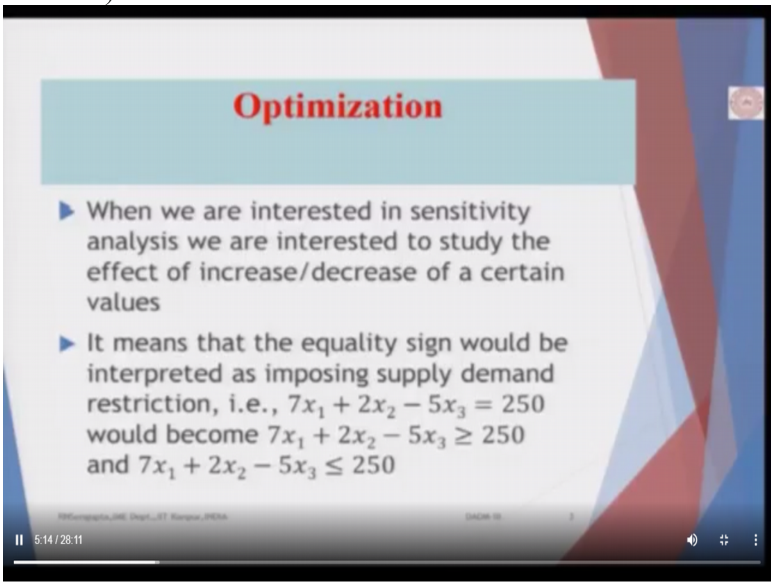

When we are interested in the sensitivity analysis, we are interested to study the effect of increase and decrease of a certain values. So certain value is basically if you remember for the case where the perturbations were along the constraints and we are considering the concept of reliability or robustness or sensitive analysis and change of the perturbations was only related to the effect of two constraints considering the two-dimension one and as showed that considering for the normal distribution, you can find out that the overall the center of that circle would basically be the best feasible solution which is inside the feasibility space.

And the boundary of the corner points of those two constraints when they are non-probabilistic and deterministic solution, and as you move more inside into the feasible region, depending on the level of reliability will have for both the constraints. It will basically be a circle of different radius and that radius would basically be given by the concept of beta value or the reliability which you have for the constraints. And later on also if we saw I am just repeating it, please bear with me, we also saw that if the variables were orthogonal to each other the decision variables and if they are normal and if the variance was same, then obviously in that case the circle was aptly describing the overall reliability space or the overall area where we can find out the level of confidence based on which you can tell our results or basically portray our results of the decision variables.

Now in the moment you have see for example, the variances are different, you will basically have an ellipse, the vertical ellipse or a horizontal ellipse considering both are normal orthogonal and as you change the orthogonality of x 1 and x 2 axis, obviously you will have different shapes which would be a symmetric distribution. But in that case trying to find out the overall common area based on which you can say, what is the reliability of the solutions, is still possible.

Then when you go into the higher dimension for the case when it when there are three orthogonal axis x 1, x 2, x 3 and all them of them have the same variance, you will basically have a sphere in a higher dimension as you go, they would basically be hypersphere and obviously the center of the sphere, the center of the hypersphere would give you the best solution in the non-deterministic case and the overall area in and around that point depending on the overall area covered in the sphere or the hypersphere will give you the level of reliability.

And obviously the reliabilities would be different for different constraints. You will basically find it accordingly. You have to basically if you remember if we need to find out the Z transformation in the simple univariate case and solve the problems accordingly.

Now when you have the variances are different for three-dimensional case, obviously you will not have a sphere, it will have ellipsoid depending on which direction you are going. So, if say for example x direction x 1 direction, the variance is very high while x 2 and x 3 are the same which are low, so obviously the elongation will be more along the x 1 direction. Similarly for x 2 or x 3 depending on how the case is.

Now in case if you have an orthogonal surface are not true then trying to find out the overall (cent) so-called in that sphere the center or the in the circle the center or in the hypersphere the center is basically a center of gravity, the point where the overall solution can be found out and the overall area or the volume would give you the level of reliability or the probability. Now if they are non-orthogonal, obviously the shapes would not be a sphere even if the variances are same, they would be basically a class of symmetric distribution.

Now later one, in the diagram, which will, so this I explained using the diagram and if you remember in one of the diagram we mentioned that if in the two dimension case and they are orthogonal the distribution of x 1, marginal distribution x 1 or marginal distribution x 2 are non-normal, then trying to combine them in the orthogonal sense obviously will give you a non-normal surface.

Again it is a two-dimensional surface, but it is not a circle. The center of gravity would be the best optimal solution in the non-deterministic case which will be different from the deterministic case, which is the boundary points. And as you go inside the overall that reliability will increase or decrease depending on the values of beta 1, beta 2, beta 3. That means how many such constraints which you have but trying to find out the overall area how it look like the shape would be difficult when the distributions are themselves non-normal.

And obviously if you go into the higher dimension case, so for the three dimension, four dimension, five dimension if they are orthogonal then trying to combine different type of distribution by itself becomes difficult to visualize also and to find it out and obviously in the non-orthogonal case it will be more like more in-depth analysis to think that how the overall area or the volume look like.

Now having said that in the sensitive analysis what is important is that, that what are the basic solution based on which you can proceed to find out the answers that what are the levels of perturbations or levels which you can have. So consider this in the equation.

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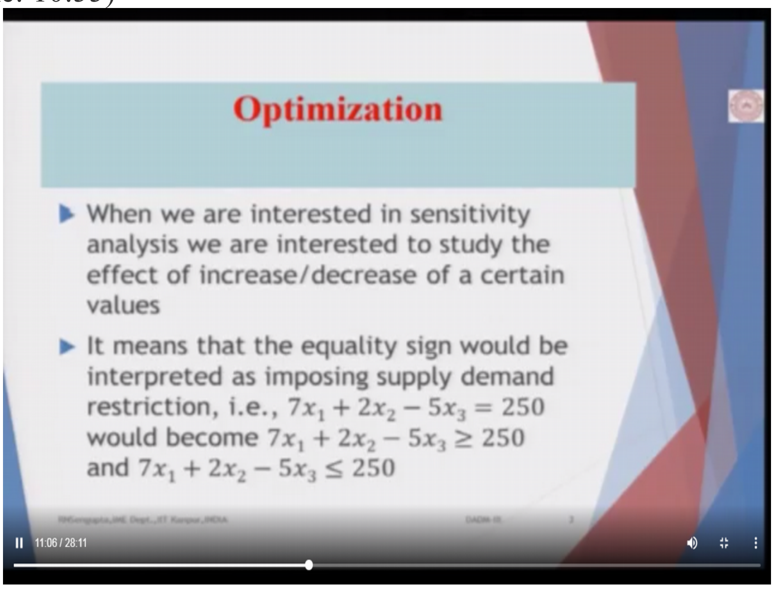

If we have that the equality signs would be when we have less than sign or greater than sign, consider we have this equation where the equation is actually 7 x 1 plus 2 x 2 plus minus 5 x 3 is equal to 250. So obviously that would be a straight line. So if you want to find out and if there are two such straight lines in the two-dimensional case or in this case is 3-dimensional case, there would be a plane, then the corner points obviously would give you the feasible or infeasible point, we do not know but consider there were feasibility points.

And you will basically have a unique solution based on which whether you want to start your optimization problem or not start optimization will depend on the overall basic feasible steps which you have. Now in many of the cases when you are trying to utilize the equality sign, you can either replace by a greater than sign or less than sign such that the greater than sign or the less than sign would have a intuitive meaning.

Consider that I have ten different warehouses and I am trying to basically transport to 20 number of different distributors. So it means that the distribution demand is say for example, 20 million tons or 20 million forget about the units, 20 million and the demand has to be met. So in that case, it would mean that the overall requirement for the distributors have to be at least equal to the overall number of combined number of transportation which is going to take place from the warehouses. So it is like this.

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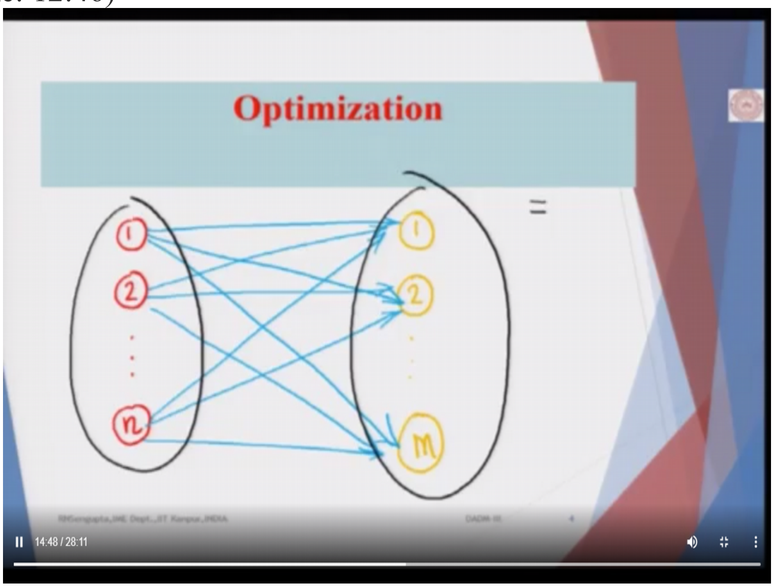

Let me draw. So consider like this. You have 1, 2, n number of warehouses and on the right hand side you have 1, 2, m number of distributors. So the combinations is like this, first one can supply to 1, first one can supply to 2, first one can supply to the mth one also. Similarly the second one can supply to 1, second one supply can supply to 2, second one can supply to m. Similarly the nth one can supply to 1 and nth one can supply to 2, nth one can supply to mth one. So, all the combinations are possible.

So when you are looking at this solution, what you actually need the sum total for 1 till n and the sum total to 1 to m. Say, for example, they have to be exactly equal. So in that case the equality sign will come but now it may be possible that there in the distributors they are inventories and consider the amount of production which is happening at the warehouses or the amount of which is being there at the factories, consider warehouses it is basically where you stuff and the distributors are the lower level.

Now amount of things which is being produced in the factories or being kept in the warehouses which are very big is much more than what the distributor needs. So obviously in that case the total amount or the sum total which you have on the left hand side would always be greater than equal to the amount which is there on the right hand side. So in that case you will have a situation where the inequality signs would be applicable considering the constraints such that greater than signs are possible.

Another can be so in that case, the inventory can be stored in the warehouse. Inventory means that you can keep the goods. Consider other case, the inventories can be stored in the distributor side and nothing can be kept there in the warehouses, in that case the total amount which can be produced on the left hand side would always be equal to less than equal to the total amount which can be stored along with the fact the total amount which is being transported. So, the inequality sign can be both ways, less than time and greater than time based on which you can solve the problem. So in that case, obviously the slack and the surplus will come accordingly.

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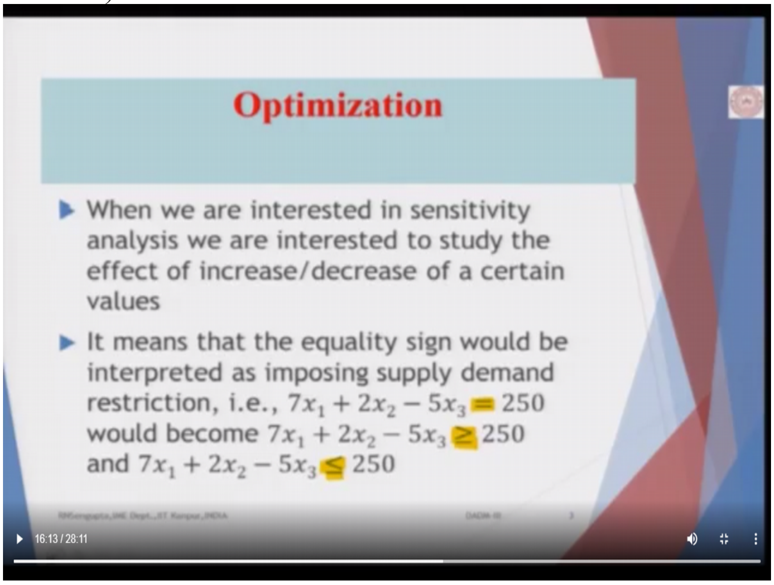

So let me read it again, it means that the equality sign would be interpreted as imposing supply demand restriction, so they can be both supply side and restriction as well as demand side restrictions. Supply means I cannot supply more than this. Demand means I cannot demand more than that. That means that I cannot basically utilize. So in that case if you have 7 x 1 plus 2 x 2 plus 5 x 3 is equal to 250, so in that case if you want to basically do it as a supply and demand one, then equality sign will be replaced by so this equality is replaced by the greater than sign and the less than sign. Now, how intuitively is clear? Because consider this one. Let me go to the next one.

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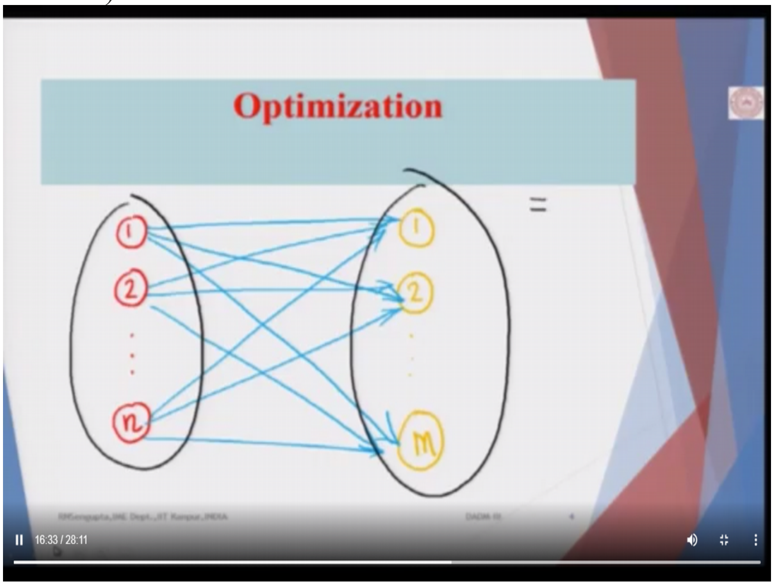

And let me erase it and so this was basically the greater than and less than sign for the demand and supply depending on number of warehouses in the factory, which is there on the left hand side and number of distributors which is there on the right hand side. And remember m and n I have used n here and m here, so it can be interchanged depending on the matrix formulation which you have. If you remember the A matrix which you are talking about m number of rows and n number of columns. So let me delete it first and then go into…..

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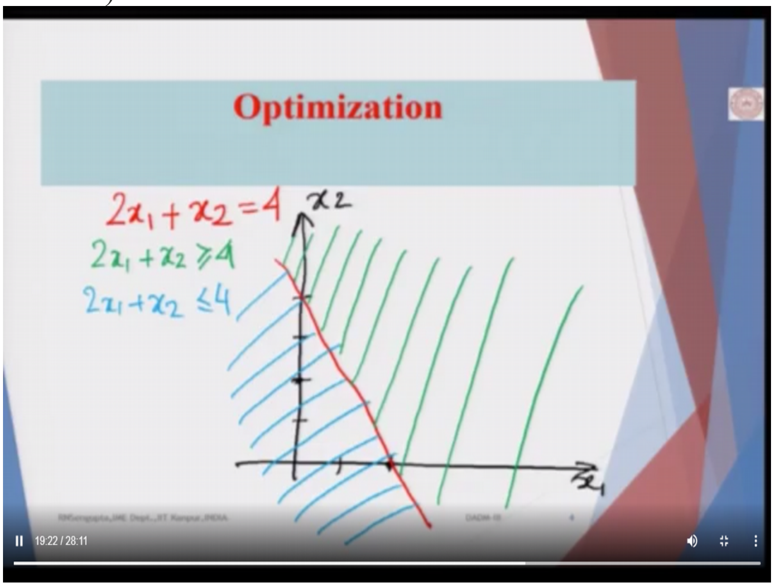

So consider 2 x 1 plus x 2 is equal to 4. So this is the equation. So let me write it down. So this would be applicable, the greater than sign less than sign would immediately come out as we solve the problem for any higher dimension.

So if x 2 is 0 x 1 is 2, so x 1 is 2, so this is one and x 1 is 0, x 2, so my mistake sorry, x 2 is 0, x 1 is 2, so it will obviously come here and if x 1 is 0, x 2 is 4 so obviously it will come here. So your actual line, I will draw the red line first which goes is equality. Red colored. So, I am extending it to the second and the fourth quadrant also, even though it may not be true considering x 1 and x 2 values are non-zero.

Now consider if I put 2 x 1 plus x 2 is greater than equal to 4. So in that case if 2 x 1 plus x 2 is greater than equal to 4, so obviously Z signs would 0 odd coordinates are not possible. So obviously you will have all the areas onto the right hand side including the equality. And when I have 2 x 1, so this will be on the so obviously the second quadrant and the fourth quarter are not applicable even though not and let me draw it in order to make it more clear. So consider this is the area.

So, obviously the common area when you take the integration and so intersection of greater than and less than, that is the red line which comes up. So the demand and supplies are met in the sense that for any equality sign you can put the greater than and less than sign and solve the problem accordingly, similarly for higher dimension it can continue.

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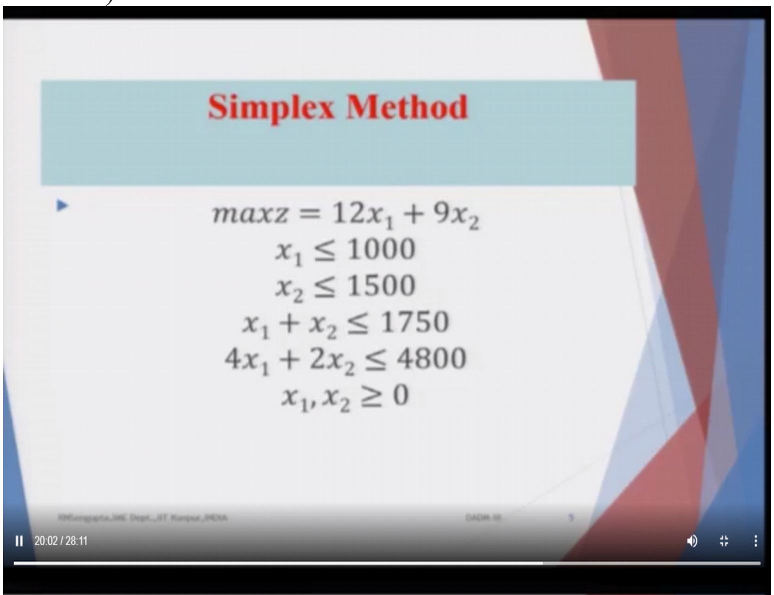

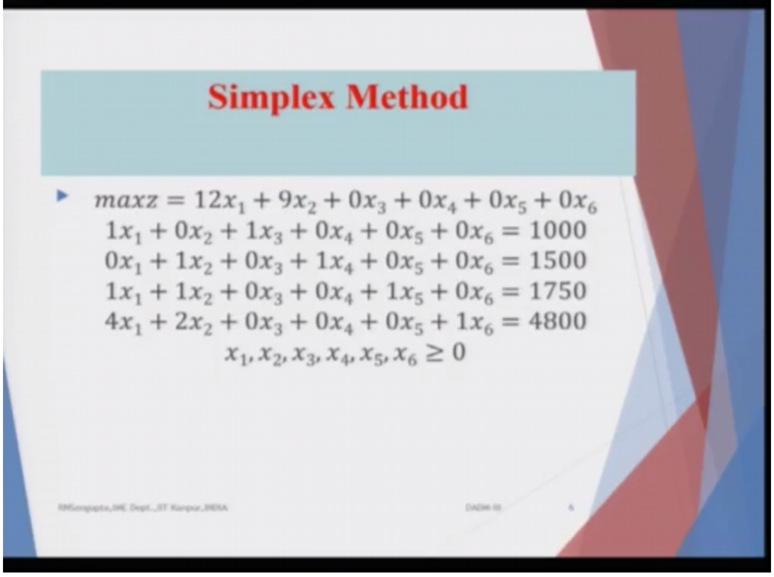

Now consider this equation. So your max 12 x 1 plus x 2 you want to maximize it to simple two-dimension one. You have x 1 is less than equal to 1000, x 2 is less than equal to 1500, x 1 plus x 2 less than equal to 1750, and 4 x 1 plus 2 x 2 is less than equal to 4800. So obviously in this case, so let us go one by one. In this case, if you have the less than type, greater than type considering they are all less than type, so your problem formulation is very simple. I put slacks on the surplus. So I have Max 12 x 1 plus 9 x 2 plus 0 x 3 which is the corresponding slack or surplus which is coming for the first constraint plus 0 x 4 which is the sack or surplus.

I am using the word slack and surplus depending on the greater than or less than sign. So I am not differentiating the less than sign or greater than sign, you can basically find it out accordingly. So 0 into x 4 which is the sack or surplus for the second constraint plus 0 into x 5 which is the slack or surplus for the third constraint and 0 into x 5 which is the x 6 which is sack or surplus for the fourth constraint. So obviously this 0 factor means their overall effect on the objective function is 0.

So obviously when it comes to the constraint, it will be the first constraint was obviously x 1 was actually was x 1 is less than equal to 1000, so, in that case it will be 1 x 1 plus 0 x 2 because x 2 was not there in the first constraint. But x 3 would be coming with factor of 1 because x 3 is basically the slack surplus corresponding to the first constraint plus 0 x 4 plus 0 x 5 plus 0 x 6 is going to 1000.

Similarly when I go to the second constraint, the second constraint actually x 2 is less than equal to 1500, so in that case x 4 slash surplus will come. So the equation becomes 0 x 1 plus 1 x 2 plus 0 x 3 plus 1 x 4 plus 0 x 5 plus 0 x 6 is equal to 1500. But third constraint was in the sense x 1 plus x 2 is less than equal to 1750. So in that case x 3 and x 4 are already consumed as slack slash surplus in the first and second constraint. The slack and surplus for this equation will be x 5. So it will be 1 x 1 plus 1 x 2 plus 0 x 3 plus 0 x 4 plus 1 x 5 plus 0 x 6 is equal to 1750. And finally in the last equation it was 4 x 1 plus 2 x 2 is less than equal to 4800. So in that case it will be I am adding the slacks slash surplus which is say x 6, it will be 4 x 1 plus 2 x 2 plus 0 x 3 plus 0 x 4 plus 0 x 5 plus 1 x 6 is equal to 4800.

Now watch here, so obviously it will become x 1, x 2 were already greater than zero and we will also ensure that x 3, x 4, x 5, x 6 are greater than 0.

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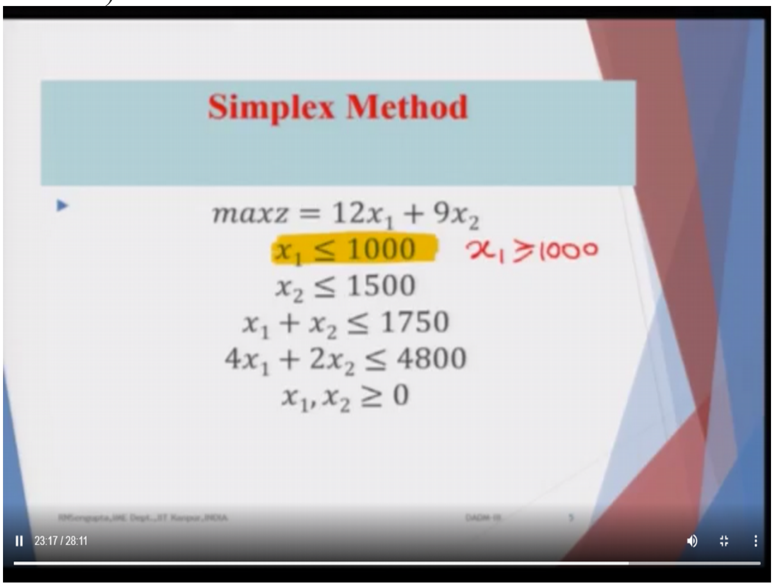

Now remember when you are formulating the equality sign and basically trying to put them as a greater than and less than sign, so in that case if you go back to the first equation, so the first equation would be….So the first equation which has equality sign would have a less than sign, correspondingly you will also have x 1 is greater than equal to 1000. So if you combine them obviously you will get in that. Also, it will come down to the fact that the slack or surplus which was here, so in this case less than equal to would have a slack and surplus.

So in this case both as slack and surplus would be added. So the overall effect of the slack and surplus in the optimum solution would be 0. That means there is a both an excess as well as a dearth. So in the actual optimal solution, if you are if you are get the actual optimal solution where all the utilization on the materials can be utilized to the maximum possible extent considering the solutions are only pertaining to x 1 and x 2 here as in this problem, in that case the extra number of slack or surplus which will have here, so one would be x 3 dash 1 and another would be x 3 dash 2. Dash 1 and dash 2 I am basically implying the slack surplus corresponding to the greater than equal to sign for the same equation.

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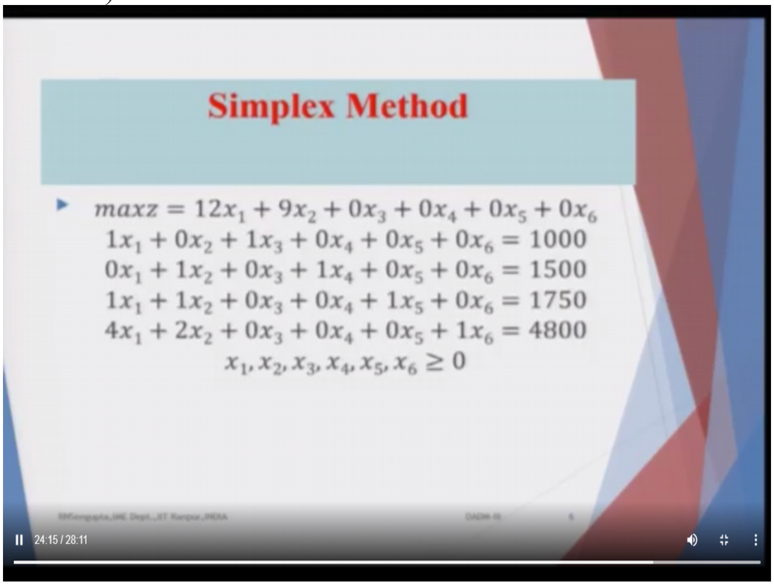

Similarly when you have in the original stage, this equality sign is there and none of the slack or surplus are there.

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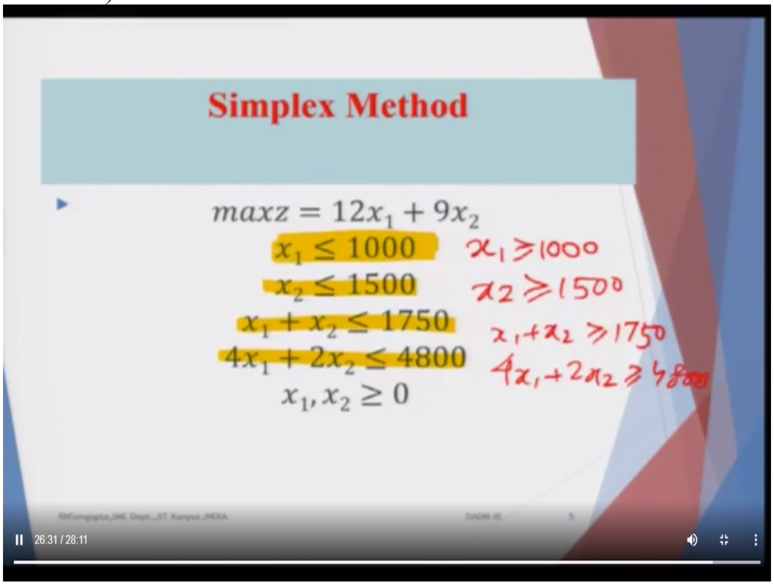

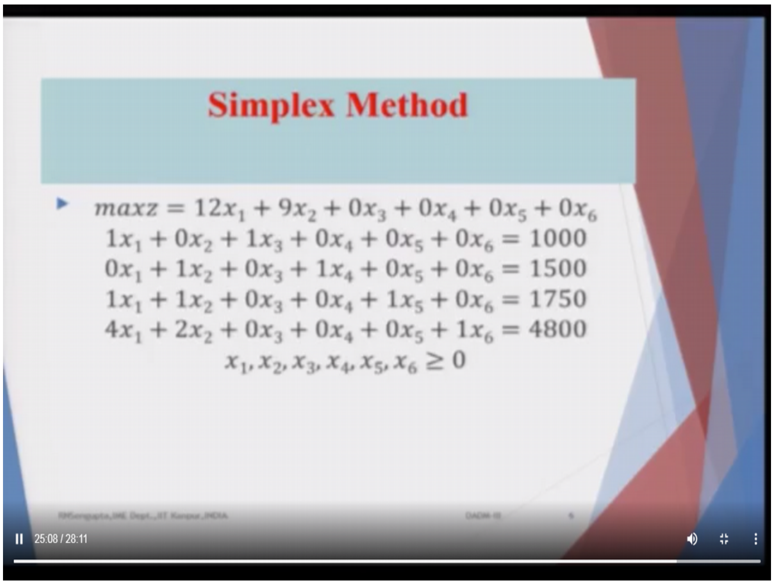

So when you of formulate, you will have x 2 is less than equal to, x 2 plus some slack slash surplus is equal to 1500 or less than sign which you are putting and in the other case you will have x 2 is greater than equal to 1500 another a slacker surplus in the opposite direction.

So obviously they would be counterbalanced. I am using a word counterbalance in order to make the explanation simple such that in the optimum solution the actual utilization of x 2 would be done in such a way that there would not be any slack and surplus pertaining to the solution. In the similar sense when I have x 1 plus x 2 is equal to 1750 in the same equation, third one, so again this will be 1. And the other one will be, so another, equality means they are being exactly utilized, remember that. So obviously whether the solutions are applicable not applicable, that will come in the second stage. So if equality sign is there, the exact amount of x 1 and exact amount of x 2 considering their continuous variable, they will be combined in that proportion depending on the parameters which is there for these constraints such that they will exactly match the right hand side.

So in this case, the slack and surplus will have for both this saffron colored equation and the red colored equation would be such that they would technically cancel out. That means everything is utilized. Similarly, when we go to the last equation, here 4 x 1 plus 2 x 2 is exactly equal to 4800. When you bring it here you have a less than sign and you have a greater than sign. And you can 4800 put the slack and surplus, solve it and then you get technically should get the optimum results. Well, none is left, not excess not basic required to the reach the optimum solution.

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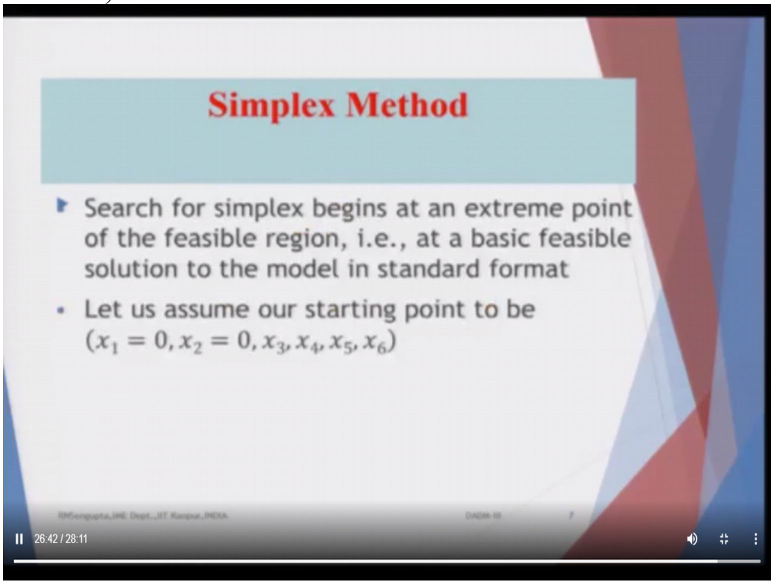

So in this simplex method, so you will search for the simplex you will begin an extreme point which is the basic solution and the basic feasible solution, you will basically have the standard format. So let us assume you start at x 1 is equal to 0, x 2 is equal to 0, x 3, x 4, x 5, x 6 are some points. So, whatever the points are, so which means now remember, technically you are starting at the point where nothing is being utilized, everything is slack and surplus and actually you want to end where x 1 would be some positive value, x 2 will be some positive value, x 3, x 4, x 5 technically should be 0 such that equality, the utilization has been optimum because there is a less than equal to so that equality sign has been met and you have reached the objective function in the maximum possible way.

With this, I will end the 26th lecture and continue discussion about the simplex method and revised simplex method in the later class. Thank you very much and have a nice day.