Data Analysis and Decision Making – III
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Lecture 25

Welcome back my dear friends a very good morning good afternoon good evening to all of you wherever you are in this part of the globe and as you know this is the DADM 3 as you can see in this slide which is data analysis and decision-making 3 course under NPTEL MOOC series and this course total contact hours is 30 hours spread over 12 weeks and if you convert into in the number of lectures considering each lecture is for half an hour this is this is total 60 number of lectures and as you can see from the slide this is 25th lecture which is the end of the fifth week and each week you after five lectures you take an assignment so you have already taken for assignment with the end of this you will take the fifth assignment and after the end of this course for 12 weeks you will take the final examination and my good name is Raghu Nandan Sengupta from IME Department at IIT Kanpur.

So if you remember in the 24th lecture at the last moment I was drawing to the diagrams trying to explain that if perturbations were there they can would initially be normal in the very simplistic sense and perturbations can be if there is only 1 constraint which is being perturbed you will basically have the univariate distribution you will solve it using the standard normal deviate and your life is done and obviously you can consider them in the simulation part because normality being a very nice distribution you can utilize the standard normal table and solve it accordingly.

Now the second question which we went into the next level was that if normal it was not it was there but if they were more than 2 (norm) normal distributions obviously within the multi (norm) normal multivariate normal case and the problems could be solved and I was trying drawing the diagram accordingly so which I will start now explaining as ~~define~~required.

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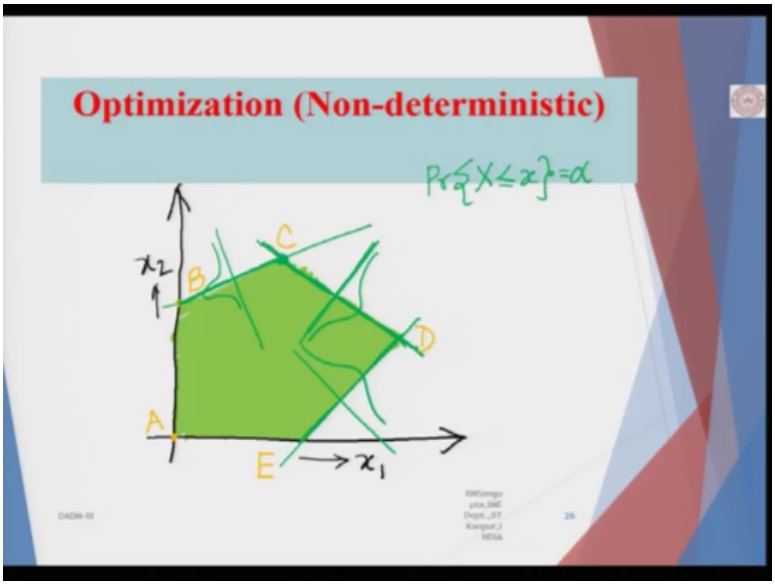

So this diagram where I stopped in the in the end of 24th lecture so now consider at Point C let me assume that the the line BC and CD are orthogonal even though in the diagram they are not being shown an orthogonal they are orthogonal so consider orthogonality ~~says~~ such that~~there~~ that you you your you are standing in front of me the plane which is orthogonal and there is 90 degrees so perturbations are there and a normal distribution is coming from your side a normal distribution is coming from exactly orthogonal plane and both of them have the same variance so they pass each other so if you consider the overall common area it will be just a simple standard bivariate distribution for which if you take the slice if you look from above they are just concentric circles you cut them concentric circles because the variance is if you consider from the plain side where the first distribution coming which is normal and from the orthogonal side although both of them are the same variance.

Now so (o) which means that in the two dimensional case it is a circle in a 3 dimensional case it is a sphere all the variance is same and they are orthogonal in the higher dimension they would be just be hyper spheres and the areas would basically dictate something which I am going to come within 2 minutes. ~~n~~Now consider in the 2 dimension case they are orthogonal but they are normal distribution but 2 different variances so consider the one which is coming from your side has a lower variance and they come one coming from the right hand side orthogonal from the right hand side which is where I am pointing my hand this is orthogonal and a higher variance.

So when they cross each other and if we look from the top they would be ellipse that means they are exactly looking look like a rugby ball which is placed in a in a sense where the

elongated surface is facing from your side to mine side because the variance is higher in and the major accesses this so if you are looking from the top there are ellipses now just reverse the the scenario where the the distribution normal distribution which is coming from your side has a higher variance than the normal distribution which is coming orthogonal to this and they cross each other it will be again the same rugby ball and ellipse but where the major axis would now be parallel to what I am standing and the parallel to your face.

Now extend that to the case where the surfaces where the normal distributions are interacting they have the same variance but they are not orthogonal so obviously it is not the same way how you will basically perceive the ellipse with the major and minor axis the major minor axis in the initial 2 cases were 90 degrees to each other now they would not be 90 degrees to each other in the case when you have if the variances difference.

Now if you extend that to the case of higher dimension it is basically an ellipsoid and ~~and and~~ higher dimension ellipsoid would basically give you an idea that trying to find out the common area now what is that comma really I will come to that is the common area where the overall probability would be maintained for all the combinations so if I am when I am saying probability of multivariate distribution doing less than x, x is also vector is equal to some alpha value it would mean the common area where all the ellipse where all the circles or the hypersphere basically the total common area of the hyperspheres of the sphere is such that it will basically be true for all of them inclusive now consider that so with this we have considered that normality if it is done how will basically we tackle it so in general if it is normal 1 dimension no problem 2 dimension bivariate orthogonal no problem higher dimension no problem variances are same different no problem if the variances are same or different but if they are not orthogonal you could be basically transform them into orthogonal surfaces and use the concept of normality and proceed.

Now what happens in the case if they are themselves not ~~()(6:59)~~ non norm ~~non normal~~ so obviously trying to find out a (rev) a very wired shape is coming from your side depending on if it is an exponential distribution or whatever it is and another orthogonal plane another distribution is coming then the common area is no more a sphere or a circle or just like a (ban) balloon it is not that so finding on the common area where the overall probability will be satisfied depending on the constraints would be very difficult to calculate you can visualize but it will very difficult to calculate so how would it look is exactly the diagram based on which when I stopped was what I was basically trying to point out now I will with

this discussion whatever I I had I will come to the diagram and explain it and then go into the sensitive analysis. So these 2 blank slides which I had mentioned I thought I will draw but I will skip and then come back if it required.

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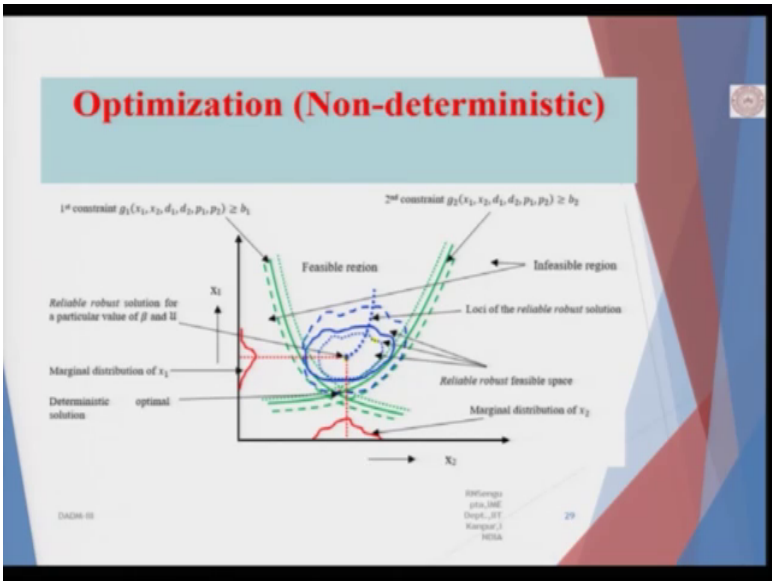

So this is the diagram which I was talking about it looks a little bit complicated but it is not whatever I have discussed if you have listened to it carefully will understand so and this again I am doing it in in a 2 dimension one it can be extended for a higher dimension you will understand so along the y-axis we have X 1 variable along the x axis near the X 2 variable and consider I would not be drawing too much because as such the colors are very very close to each other it will be difficult for you to differentiate just I will hover ~~might~~ by the electronic pen so so consider this and this the green yellow color I will just hover it.

So consider the yellow colored pen which is hovering this green bold line is the are the two constraint and in the initial key example which I considered they were straight lines because they are linear but not linear that does not matter so this is the first constraint which is given by G 1, X 1 X 2 are the decision variables D 1 D 2 are the deterministic one P 1, P 2 other parameters which are probabilistic for the time being consider the probabilistic part is not there the second constraint is G 2 X 1 X 2 D 1 D 2 P 1 P 2 are not there still I am writing and is greater than b 2 so in case if they are greater than b 1 and greater than b 2 and their deterministic the common point which is here which is a red dot if you can see as I am hovering the electronic pen is the deterministic optimum optimal solution. So if you solve the problem using any of the method which you have considered you will get the answer give the

result find out the objective function whether maximization minimization and be happy and your work is done.

Now what happens if by itself initially the constraints are non-deterministic due to the fact that p1 p2 has come which means if you if you if you go back to one of the problem that it was that the plant 1 plant 2 plant 3 had some different constraints on the maximum capacity similarly for Machine 1 machine 2 number hours was 8 now it is no more 8 it is changing I consider that due to the policies of the of the company you can (eith) either use 7 hours or you can use 9 hours some distribution is there some probability for the number of us being utilized for machine 1 is there so in that case there would be a perturbation of the Green Line both inside in the feasible region and outside in in the infeasible region.

So obviously in feasible region would not be true for the deterministic case but for the produces case some part would be infeasible which is initially would be considered as feasible which means the overall if I if I consider normal distribution now the which I guess q considering the normal distribution for the constraints if they coalesce or they come together then the overall common area would be a circle for a 2 dimension one as it is the case or a sphere in the three dimension one if you have X 3 also vertically up and the common area would ~~add~~ or with the center of that sphere or the circle would basically be the the non-deterministic of the probabilistic solution and the overall area it will cover will give you the probability or the reliability of the result so depending on the area discovered if it is 90 percent 92 percent 95 percent it will give you how good or bad your results are how reliable your results are how robust to your results are.

Now consider as I mentioned the distributions Percy are non normal and how I am mentioning that and how I am trying to basically portray that is in these two red marginal distribution which I have drawn so the dotted lines which were there it basically the dotted green one and the dotted this more small dotted one and large ordered ones are basically the perturbations which are happening for the constrain and it is in the 2 dimension one it can be done in higher dimension also so this green so this red color distribution is the marginal distribution of X 2 similar considering that you have FX 2 similarly the red one which is each you have and they are orthogonal ~~optimal to~~ remember for the time being we are considering them orthogonal ~~particular~~ to be true for X 1 and X 2 variables and this red one which is along X 1 is the marginal distribution of X 1 so if they were normal as I mentioned I am

again repeating it common areas a circle sphere and so on and so forth hyper sphere depending on dimension.

Now if they are not normal overall contact area the common area would be a very weird shape so as the probability increases and decreases the liability the general area which you see where I am hovering my my this electronic pen the yellow one initially is the the probability based on the fact that the center point which is here which is the center of gravity or the common point of the C G or say for example the the mean value of the common between both these distributions.

So one is coming from 90 degrees another is going here they meet and the center of gravity of the center of this joint distribution of F X 1 X 2 which I am not—— able to derive but I am only able to give you a simplistic visual view for that so this point which is the blue one is the reliable robust solution with some level of probability depending the value of reliability which I put on the perturbation of constraint 1 and constraint 2 so constant 1 constraint 2 can be ~~over~~ of ~~eight~~ different reliabilities but I basically find ~~another~~ the common value based on which I can find out the the common point.

So this is this this solution which I have the blue one is the center of gravity and as I increase and decrease my level of reliability or as the perturbations increases and decreases the movement of the the first constraint and the second constraint deterministic part would be far wider inside the feasible region and outside the feasible region but obviously we will only keep a limitation and discussion of the feasible region hence the area will increase or decrease depending on the perturbation and it may be possible that as the perturbation increases decreases or as the constraint changes the loci of that point or the center of gravity basically keep moving more inside the feasible engine obviously ~~so~~ such as that it will give us and the probabilistic point based on which you can say that with certain level of reliability our answer will be true that reliability is basically coming from the value of beta so this as I mentioned is the loci of the reliable robust solution it will go in (s) more inside depending on the level of reliability which we will assign for that.

Now if it is 3 dimension one it will be easier for you to understand if the figure which is there and drawn so it will be a very weird shape which will keep expanding contracting depending on the level of beta which I am assigning for the answer as such.

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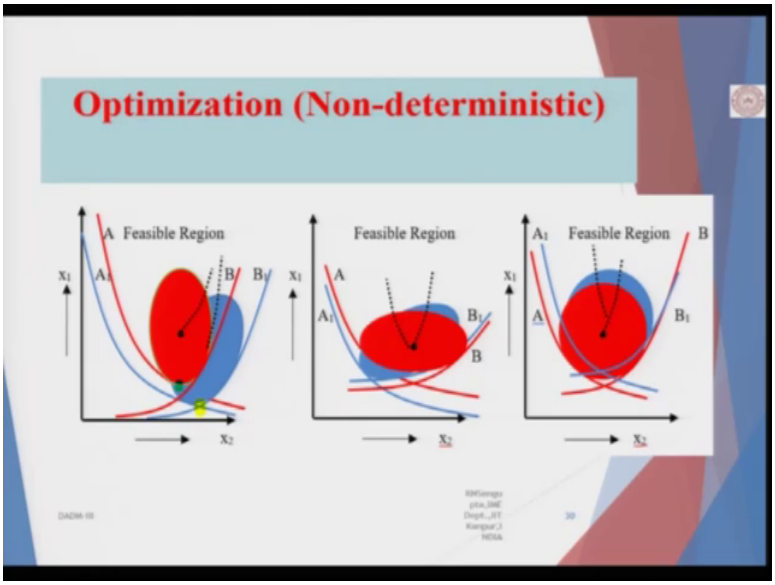

Now this diagram which you have already discussed I told verbally but I am coming to that so consider that if it is a normal case so this this again I am going back to the normal case or the symmetric distribution so you have X 2 along in the x direction and you have X 1 along the y direction and your feasible region which is which changes depending on the perturbation which is there so the blue line which is b 1 and the blue line which is a 1 their common point the deterministic one was where I will just mark it where this this deterministic point is but due to perturbation it moves inside and the the orthogonality of the constraints are not there if we check here they are not orthogonal so obviously the ellipse would be such depending on the variance being higher and low in whichever direction you are considering whether in the X 1 direction of the X 2 direction the loci would basically move inside in order to basically portray that how the reliable solution moves and what is the overall common area.

Now if you consider the red lines of A and B the deterministic point would be the common area which is this so somewhere here so I basically use different color somewhere here so because the color~~s~~ i~~st~~ blue it is difficult for you to understand and again the loci would move in such a way that the feasible region obviously would be meet and the overall area of the red one or the ~~other~~ blue one the (da) whole shaded region would give you the ~~the~~ reliability of that point so depending on 90 percent 92 percent 95 percent what you have. ~~n~~Now if the variances of any in this case the variance of X 1 is larger in diagram 1 which is the leftmost panel and in diagram 2 you basically have the variance of X 2 is higher and obviously here the the concept of normality or the orthogonality is being considered to make our life simple.

So as A and B for the red part when there is one one solution depending on the reliability and the A1 B1 part depending on the perturbations of the reliability changes you have 2 different deterministic point and they move the loci moves inside the feasible region to give us the idea how the probabilistic value for the solution changes and which areas you will basically have in and around that probabilistic point such that you ~~will~~ are able to say that with certain level of certain reliability 92 percent 95 percent whatever it is basically be determined the value of beta that you are sure your answer is correct.

Now third prior panel which technically should have com~~ma~~ on the left part is when both the the variances of X1 and X2 are same and they are orthogonal so if they are same and then trying to find out the center of gravity for the point where again the loci would move inside the feasible region and the overall area would be basically given by the level of reliability which you have now remember one thing reliability of the beta so if you remember the beta value is probability of X in the univariate case your probability of X less than equal to a small X is greater than equal to beta or alpha whatever you written.

So if you consider the value of beta it is basically the overall area which you have inside the circle or the ellipse now it can also be portrayed that as a circle of a certain radius where the beta value can be utilized to draw the radius and it will give you because you remember the beta value would always be between 0 and 1 so that will give you the unit circle maximum or a circle of lesser size such that it will give you a perception that how the circle will be drawn on the how the ellipse will be drawn depending on on you are trying to take same variance or higher variance for either X 1 or X 2.

Now what we need to is that we found we need to find out the optimum values of the decision variables under the set of constants whatever you have whatever we have been discussing and here the case is the probability of g j is and it can be h k is also but I am just taking one set of constraints only in order to portray that probability of g j's x d and p where x is the decision variable d is the deterministic one p is the probabilistic one is greater than equal to 1 less than equal to bj's for J is equal to 1 to capital J.

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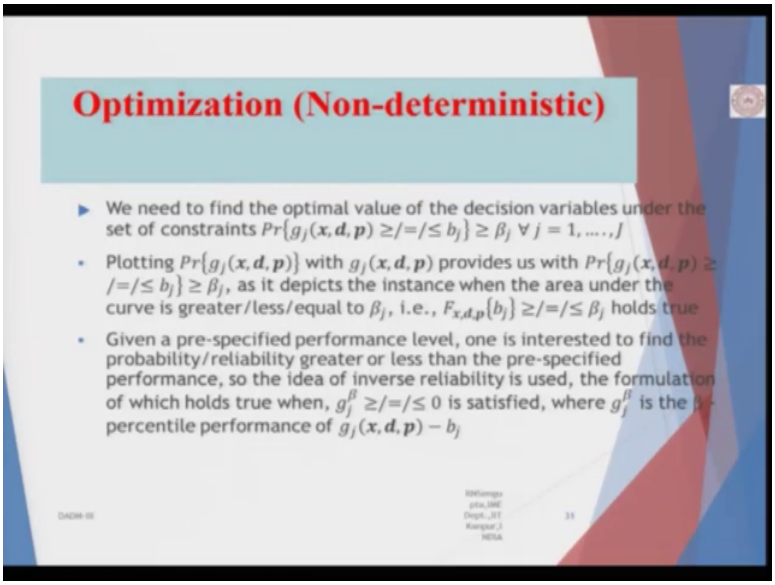

So (prop) plotting the probabilities so you are plotting the probabilities along with the constraints and would provide us some information of how the values of betas are there so I am I plot the values values of the function gj's and then I try to find out that as the probability changes how the overall surface changes so it will be depicts the instance when the area under the curve a less than greater than less than equal to depending on greater than sign equal to sign less than sign for the value of of A.

What we are considering is that if you have a joint distribution what you want to find out is the the multivariate I am not talking about the normal case the multivariate distribution of the of the cumulative distribution is such that the overall probability is less than equal to greater than equal to or less or equal to the value of beta based on which you are trying to process and I am trying you to find out so because why it is a multivariate distribution and you are taking capital F of X is that you are trying to find out the joint distribution not the marginals so they would basically be coming at the joint distribution where all the probabilities have to be met hence you would basically count from either the left hand side if it is less than or if it is greater than you will basically consider from the right hand side.

So what you will do is that given a predefined performance level so the performance level will basically be coming from the problem formulation which you have beta you will be interested to find the probability or the reliability such that it is greater then less than or equal to a pre-specified performance value which would be given by the fact that you will basically do the orthogonal transformation and find out the gj's to the power beta which is basically greater than equal to or less than equal to or equal to 0 so what you are doing is that you are

trying to transform beta on to the left hand side and trying to find out the probabilities which will satisfy that based on that you will basically find on answer so the perturbations are now being considered where if you consider AX is equal to B greater than B less than B or equal to B you are transforming that B the vector onto the right hand side putting a 0 on the right hands ~~and on on the right hand~~ side and then recalculating to find out what is the probability based on which you can solve.

So the idea of inverse reliability would be utilize the formulation of which will hold true such that g suffix j for each of these j is to the power beta is is greater than equal to less than 0 as satisfied where they would the g beta would give you the percentile performance of the constraints such that the unique transformation which you are going to do from the probability sense to the percentile conversation would be true and this one-to-one transformation will give you the values of g b's g betas based on which you can do the calculation.

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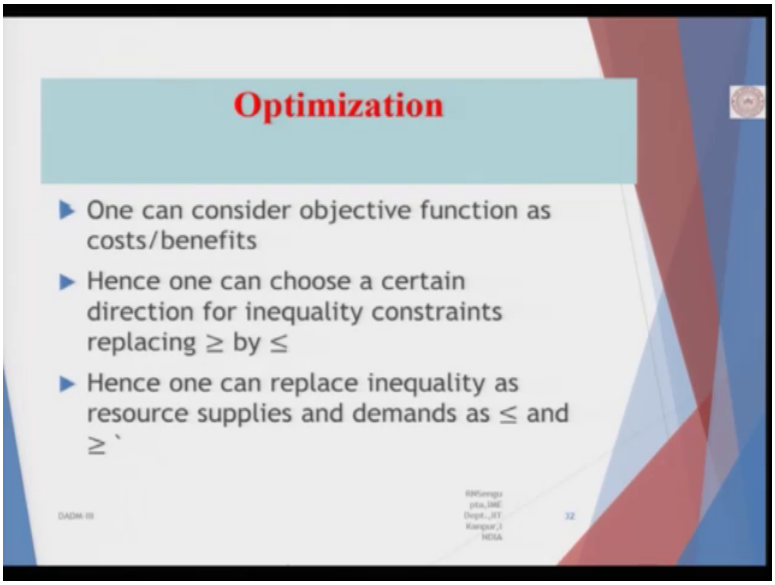

So once you are considering that you will come you will consider the objective function as when we are doing the perturbation so when you are doing so there there the perturbations which I mentioned was from the sensitive analysis but they can be different ideas of perturbations so one can be you can consider the objective function as a point of cost-benefit analysis hence one can choose a certain direction for inequality so if it is cost or a benefit so if it is cost you will basically consider them as negative if it is benefit you will consider them positive.

So maximization problem will be converted into a minimization problem or minimization problem to the maximization problem obviously you will be asking that does it seem that we are trying to do a primal dual problem the answer is that in order to find out the solution I did tell you that number of constraints becomes the number of variables and vice versa in order for the solution of the problem but here we are only considering from the ~~quantity of~~ point of view of sensitive analysis and trying to do the problem.

Hence one can choose a certain direction for inequality constraints ~~repressing~~ replacing the greater than on the less than type and then basically replacing the plus for the minus, minus for ~~+~~ the plus we can also replace the inequality as a resource supply and demand in the less than type of a greater than type so if it is a factory where it has to supply 20 million tons of say for example fertilizers.

So obviously the constraints would be depending on how you have problem it will be a less than less than type in the sense that you will definitely utilize those amount of products so it is like this I have a factory which needs definitely 20 million tons so our constant formulated in such a way that the constraints would be definitely greater than equal to 20 million because any product or any amount coming less than 20 million is not allowed any product coming exactly equal to 20 million and more than 20 million would be allowed based on which we can formulate the problem.

So it it can be like in your inventory storage and the number of amount of goods you can store is maximum say for example 20 boxes so any level 20 19 18 is possible but any level 21 22 is not possible so obviously you will formulate the problem accordingly such that constraint would not be violated considering the practical problem which you are facing or the constraint which you are facing.

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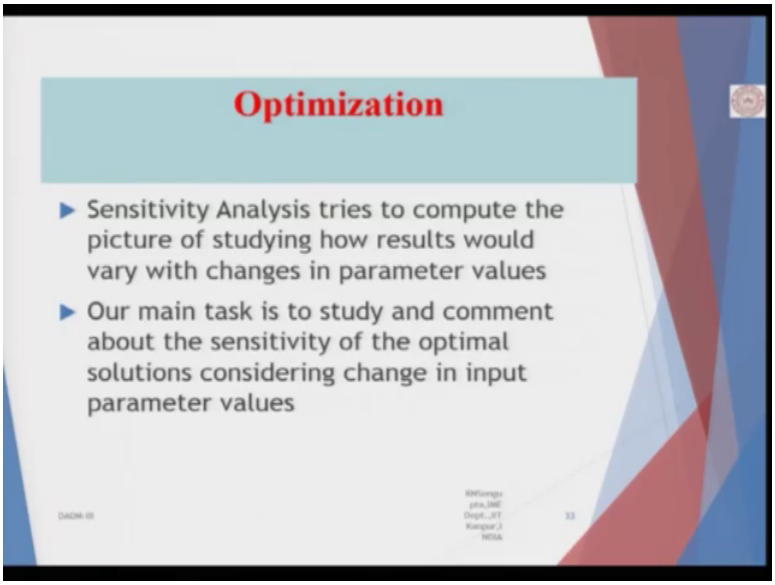

Sensitive analysis we will try to compute so what we considered is that well I am going to come to that sensitive analysis we will try to compute the picture of studying how results would vary by changing the parameter values the parameter values is that d and p what we consider if you brought up how they will change our main task would be to study and comment about the sensitivity of the optimum solutions considering change in the input parameter values.

So they as the input parameter values are changing p or d how the constraints are shifting inside if you can if you remember the last diagram which I drew and the phagon on the 24th lecture based on which I start at the 25th lecture the constraints were linear as the the constraint changes so they can either move inside that is why your feasible region will shrink or they can move outside depending on you are trying to increase the constraint levels like in the machine problem I said you can use either out of the 8 hours you can either decrease it to 7 hours or you can increase to 9 hours so 7 hours would mean that your overall constraint is decreasing and if it is decreasing remember also or it is increasing remember also that would have an effect on leave aside the answer that will have an effect on the slack and the surplus variables based on which we can say that yes it is possible that if we basically reduce our one of the constraints or an increase one of our constraint or excess material or more materials which we will need can be made to 0 such that even if the ()(29:24) is not used (ye) yet you are able to use our resources to the maximum possible extent.

With this I end the 25th lecture and continue more discussion of the sensitivity analysis in the sixth week starting in the 26th lecture have a nice day and thank you very much.