Data Analysis and Decision Making-III

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Lecture 23

Welcome back my dear friends, very good morning, good afternoon, good evening wherever you are in this part of the globe. And as you know this is the DADM - III course which is data analysis and decision making - III course under and NPTEL ~~noc~~ MOOC series. And as you know this the DADM - III course like DADM - I and DADM - II its total duration is spread over 12 weeks. And as you can see we are in the 5th week, 23rd lecture which is with the third lecture in the 5th week. And this total contact number of hours for this DADM - III, like DADM - I and DADM - II is for 30 hours which have been converted into number of lectures is 60 because each lecture is for half an hour. And each week considering this the 5th week we have already finished 4th week. That means four weeks. So each week we have five lectures of half an hour each. And after each week we have an assignment. So you have already completed 4 assignments. And we are with the 23rd, 24th and 25th lecture. We will go and take the (the) 5th assignment. And after the whole course, you will basically take the final examination.

So if you remember we are discussing about the simple simplex method. And I gave you how the solution basically changes depending on the entering and exiting variables, depending on the whether you are trying to do, the maximization problem or the minimization problem. And based on that decision, like for the maximization problem you bring everything on to the right, I would be repeating many things time and I again, please bear with me. So you take the maximum in the negative sense. Because it is you are brought into the right hand side, that will be the entering one. Depending on the fact that it will increase the objective function maximization to the maximum degree or maximum quantum. And that would basically become the pivot column based on which you will rotate, rotate or basically you do the conversion. And the pivot row would with the one for that variable which will leave the system such that in the maximization problem or decrease in the objective function would be the least.

Hence we take the ratio of the right hand side which is the B value, I will always consider that as the B value. B vector divided by its corresponding Coefficient that is the unit based on with B divided by its corresponding cell will give me per unit reduction in the objective value if it is the maximization problem. And that would be the exiting one (exist) exiting one. And if there are more than 2 contenders for the entering one, you will take both of them. But also consider that the ratio for which its minimum for that exiting ones, if one we can and find out with minimum we will take that. If both of them are the same, then obviously you have to basically use one of them. But individually it means there are two different corner points depending on which you will get the maximization concept.

If you continue doing this observing the fact that (the the) the base of the matrix corresponding to the basic variables which are there on the left most column, those corresponding cell values would be one, rest would be zero. Such that when you put that basic solution to the one value which we have on the right hand side. That will give you those exact solutions such that putting them in the objective function whether there is a slack or the surplus or the actual variables will give you the objective function maximum. So we saw that it changed in our last example. It changed from 0 to 30 from 30 to 36 and after 36 obviously any negative values for the entering variables who are not there. So you have to basically stop there.

And the minimization problem now obviously when I talk about the minimization just the reverse. That means if is the minimization one you are trying to you basically find out that the fall that the decrease in the objective function would happen the highest increasing whatever it is. And exiting one would also become considered likewise. And again contenders for ~~for~~ 2 more 2 or more points which you are (exit) entering and exiting would be basically be clarified that as at the two corner points you can have~~. S~~same solution for two different values of the corner points.

And for the unboundedness we discussed that (as you) as you keep changing the entering and exiting ones, they (would) would be not going out of those x1, x2, x3, x4 whatever these variables are depending on how many slacks and surplus which you have. And how many actual real variables which you have. But the z value will be unbounded. It will keep increasing for the optimisation problem, if it is maximization and in the case if it is minimization it will keep decreasing.

So we were discussing the last day on the 22nd lecture. So what is the directions we consider? So directions obviously what I have been discussing time and again are the same thing which I have been written in words.

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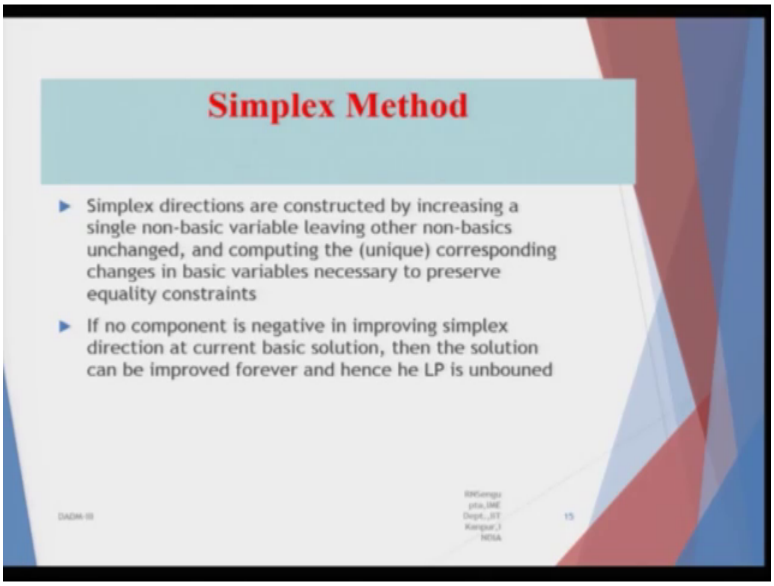

So the simplex directions are constructed by increasing the single non basic solution. Now remember the point important fact is that it has to be a basic feasible solution, feasibility is very important. Because if you find out the constraints and the point satisfying is one of the point which is actually possible but it is not inside the feasible region. Then it does not make any sense in trying to solve it.

We have considered very simple examples earlier. The single non basic variable leaving would be leaving such that other non basic solutions, basic variables remained unchanged. So which is very important because when you are changing from one corner points to the other, we are only considering that there is a change of one variable at a time. Because the movements would be happening such a way that you would consider the movements taking place. Such that, if in the table if you saw, there was only one variable which is entering. So 1 enters, obviously it will push out the other. So that will be only the single one.

So the movement of the corner points are such that there is only one change which is happening in the variables, point 1. Point number 2, you should also remember that when we are considering the entering and the existing ones, we are considering the simple concept as I mentioned is dy dx will take the maximum or the minimum direction depending on whether it is a maximization or minimization problem. But this dy ~~and~~ dx, I did not specify it should be the del y del x, that is the partial differentiation based on fact that when we change one of the variables with (respect) respect to the others kept fixed. We are trying to find out the rate of change of the objective function which is the maximization or the minimization problem based on only one change.

That means one (basic) one variable comes in another variable goes out. We are not going to consider more than one in the simplex method. So that is very important. So here the point it mentions, the simplex directions are constructed by increasing a single non basic variable which was not there in the basic obviously. It will one will be coming inside and one will be going out. Such that other non basic would be kept unchanged because they keep changing obviously you cannot find out in which direction the rate of change is the maximum or the minimum, for the problem of optimisation.

And we will compute the unique corresponding change in the basic variables. As we did like x1 comes x3 goes out or x3 comes x4 goes out, whatever it is. Irrespective of the fact that whether x1, x2, x3 are the slack or the or the surplus or the actual variables whatever it is. That would depend on how the problem construction and how the (process) procedure of solving the problem occurs.

So the corresponding change in the basic (variable) basic variables would be necessary to ~~piss off~~ preserve the equality constraint. Because remember when we have constructed the tableau, the tableau has been constructed in such a way that addition and subtraction, plus or a minus, whatever if we have put. For the slack and surplus have been done in such a way that the equality sign holds for the right hand side which we have the B value.

So if somebody is entering, somebody (is) is going out that equality is always present. If not what is the problem is, if you remember in the last class I mentioned that the values of at any state, when you stop you take a snapshot. The variables which are there on the left hand side, these are the corresponding coordinates for that corner point. So when you equate them to the right hand side which is B, corresponding to the fact that you have obtained in the identity matrix corresponding to those x values only equality sign would mean that those are the exact values of x's.

Such that when you put them and find out in the objective function they would give you the exact value of the objective function at that point. So as you keep moving the equality sign is, it holds always. And the new sets of x's would be given by the new sets of B's which you have. Once you find out this x's, you put them in the objective function. You have already got the objective function which is maximization or minimization as it is. And then you decide whether you want to proceed or stop.

If no component is negative in improving the simplex variable concept. Which is to where you because you are bringing it to the left hand side, at the current basic solution, then the solution can be if ~~if if~~ the non negative ones are not there. And if you think that the sign is not changing and one would always enter and one will always go out. And ~~with~~ if this cycle continues based on the fact that the objective function will keep increasing which means that you ~~have to~~are slowly tending to move more inside the first quadrant. Away from point 0 such that you would not have any unique solution~~. I~~it will be an unbounded one. Just reverse the concept, if you go into the third quadrant or you basically I am trying to solve the problem considering the duel problem. You will also see that as you keep increasing or decreasing as the case may be, the objective function would become more and more negative. Hence the minimization problem would basically mean that you ~~would~~ basically go to minus infinity.

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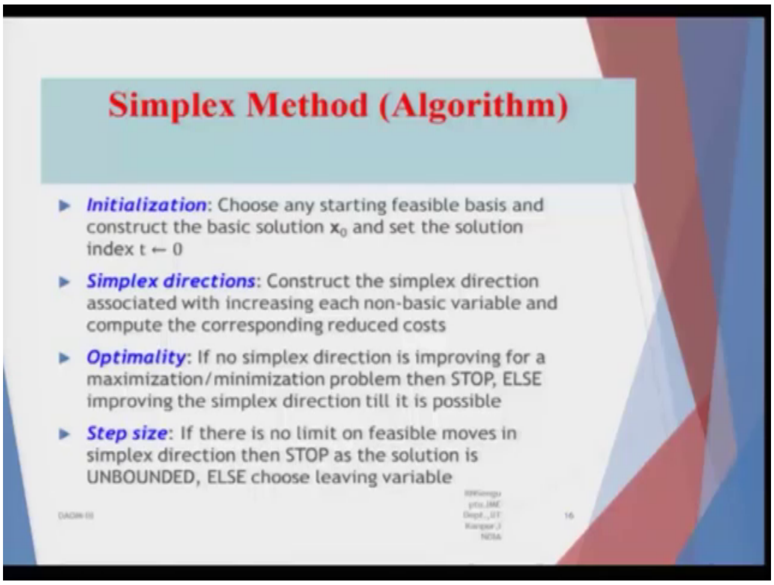

Now the simplex method for ~~for~~ the (initialize) the all over algorithm works very simply. What I have been talking about is in words. I will just mention them in a stepwise (formula) formulation. So what have I discussed is just a formalization of the procedure how you proceed. So you will initialize, so what you want to initialize is that you want to find a point from where you want to start. So what is that point. That is the basic feasible solution which I told that in many of the cases is the point zero based on which you can proceed, provided it is feasible.

So in initialization choose any static feasible basis and construct the basic solution x naught which is that iteration point that t is equal to zero. Considering that you have started your overall process at t is equal to zero. And Set the index at t is equal to zero and then move. So t is just counter, there is nothing to do with time. So you will basically change. So as you are moving around the (count) around the corner points, t will change 1 to 2, 2 to 3 and so and and so forth ~~answer for~~. So the simplex direction works like this, construct the simplex direction in which direction you will move associated with here the important point is, associated with increasing each non basic variables and compute the corresponding reduced costs.

So reduced cost in the sense that you will try to find out that as you are moving, remember one thing this is important. As you are moving you are trying to move in the maximum possible direction. Now the maximum possible direction would always be where the del y, del x or del f, del x, f is the objective function, x is basically any one of the x's based on which you would proceed~~. I~~is basically maximum or minimum depending on how you are looking at the problem from the maximization problem and the minimization problem. So as that occurs when you go to the next point, the contenders are dy dx would definitely be there any one of the x's. But the rate of change of the function would now basically be increasing as a added as a decreasing rate. So basically you are trying to slowly reach the objective function which is maximization. So you will keep reincreasing in this. But the rate of change (of) of (this) this equation would happen ~~open~~ at a slower rate.

Now here I will just step aside and tell a few facts (about) about the initial assumption which you took. If you remember in linear programming we assumed that there is a linearity in the addition and subtraction of the (object) of the variables. That means 1 unit increase 1 unit decrease in the variables x's. Whatever the x's. When I ~~have to~~ mention the word x's (or) or B or or the y's, when it is a dual problem, remember I am talking about the vectors. Because x is a set of vectors, similarly y, similarly B or A is a matrix. So any addition or subtraction of any one of the x would always keep increasing or decreasing your objective function with the same quantum.

That means in general we are not considering a decreasing return to scale or an increasing return to scale. As they may be the cases in one of the methods which is DEA, data envelope analysis which we have considered long time back in DADM - II. So as we construct the (simple) the simplex directions associated with the increasing each non basic variable. Then we will compute the corresponding reduced cost and produce the direction where the increase or decrease happening the most or the minimum. Now you want to check the optimality. So if no simplex directions in the direction wear where you we are moving is improving. That means it is not giving us a higher value of y or z as the objective function. That means you remember, we took the value of 30 then it increased to 36. Now after 36 if you saw technically there was no entering variable. Consider that we do not (consider) (we we) we ignore that. We forcefully bring one of them into the as the basic solution. And forcefully take 1 out in the basic solution which is optimal at the point of 36.

See for example forcefully bring one and forcefully take out one. The result would be point 1, the objective function will decrease from 36, point 1. Point number 2 that it would be in the case that the slacks and the surplus which could have been zero in in the actual theoretical sense. Would now no more be zero. Because there would be some extra amount for either (production) plant 1 or plant 2 or plant 3 or machine 1 or machine 2, however the problem has been framed with the constraints, there would be some extra amount (16.18) extra amount has been not utilized. Or it leaves still to be unutilized. So those would be coming to the picture because in that case the variables based on which you want to optimiseoptimised ewould not reach their re actual maximum value in order to give you the maximization or the minimization problem.

So if no (simple) simplex direction is improving for a maximization and the minimization problem, then you will STOP, you would not proceed further. Else improving (simple) in the simplex direction till it is possible you will continue. So you will check if it does improve continue improving find out the x the point, point means the vector. Find out z. Now after you find out z go to the tableau, double check is there an entering is there an exiting. Answer is yes, do that. Find out the value of z. But obviously unboundedness and all these things properties would be checked. But if we will continue doing that till you reach the unique solution, stop it.

Step size, if there is no limit on the feasible moves in the simplex direction which is important because in which direction we will keep moving that is not being given by the tableau, final tableau which you get. So each time you have some negative values, in this in the top most zeroth equation and it says that somebody should enter comedy somebody should go out. So if this keeps happening you will find out the variables, basic variable and the optimal case. There is no optimality here, so the rotation or the change of this variables will be happening time and again. Which will point to the fact that your objective function will keep increasing or keep decreasing depending on the maximization or the minimization problem.

So it says that if there is no limit on feasible moves in this simplex direction, then stop as the solution is unbounded both in the negative sense and the positive sense. Else choose the living variable and the entering one and basically repeat the process accordingly.

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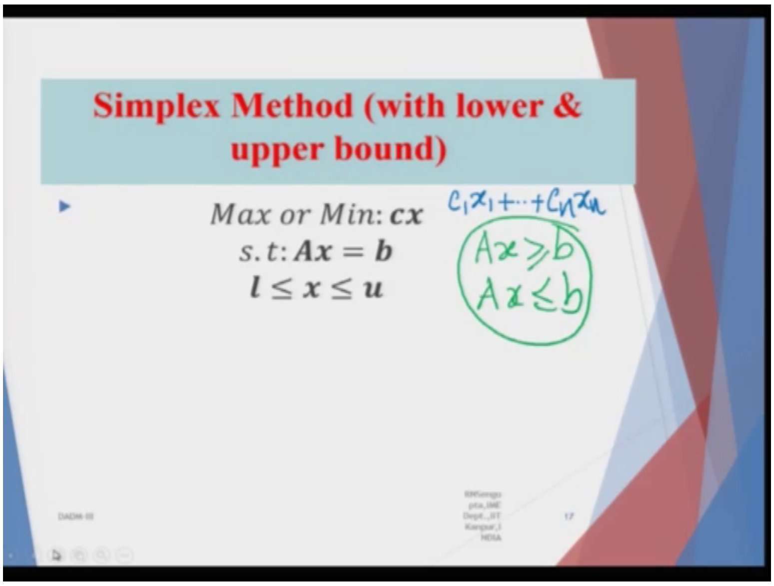

Now we will consider that ~~if~~ there are with lower bounds and upper bounds. So if you remember in the initial case of the maximization minimization problem, we had the example where x was for all greater than zero. So which means that if you consider the minimization ~~or~~ and maximization bounds, so here the problem is like this, you want to minimise or maximize cx. So why, why I am saying Cx? These are all vectors or matrices that is why they are bold.

So if I consider cx, cx is basically… I would use… So this is c1x1 plus dot dot plus cnxn. And here we are considering, because as the~~i~~re is equality sign we are considering the slacks and surpluses are all subsumed in this equation. So ~~then~~ it may be possible out to this n, n is 20 consider. So the first 10 are the actual variables, while the next 10 like 11, 12, 13, 14, 15 out of the 20. So this fives are all slacks and 16, 17, 18, 19, 20 are the surpluses. So all these are subsumed here. Now why this equal to~~ese equality~~ sign, because the moment I ~~has~~ said that all are subsumed in this equation. It means that even if which early it is, let me use....So even if they were this, so these are bold remember A and x are bold. Or this, so all of them converted using the slacks and surpluses to the equality sign.

So we have now Ax1 is (greater than) is equal to b and Ax is equal to b depending on slacks and surpluses, how they have been added. So obviously it would mean that the matrix of size A is m cross n, where the rank would be the minima of m and n. And we are taking the column operation accordingly. So rank is the lower one of the minimum of them.

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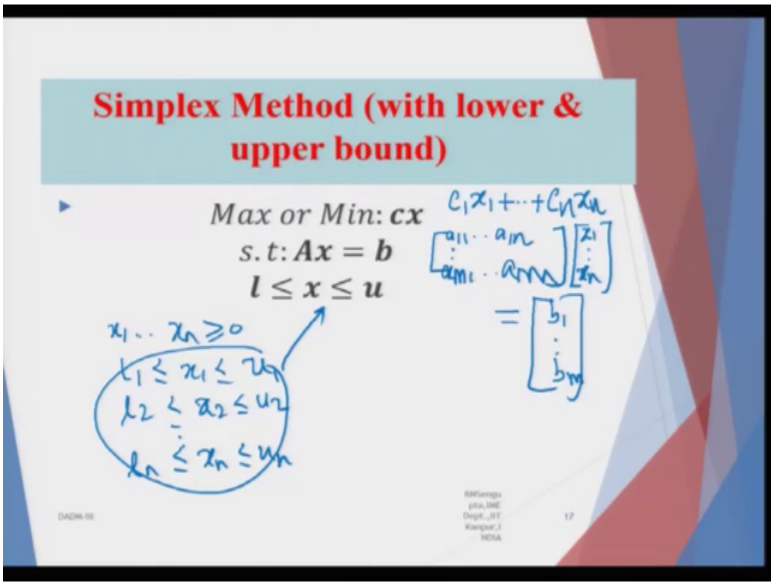

And if you remember, and if you remember you had x' is greater than 0. So now you are basically, x's are between l and u, lower bound and upper bound. So obviously x1, if I write it here. So this would be a11 till a1n, am1 till am into x which is, so there are n number of variables obviously. So want to need to multiply, that is equal to, initially what was that? m cross n for a, (n) n cross 1 for x. So m cross n into n cross 1 would give you m cross 1 for b. So the matrix multiplication is valid.

Now if I look at the bounds, so the bounds are like this. Technically we already had the all greater than or equal to zero. So what we now actually have is l1 is for the first one, l2 is for the second one. Similarly last one. So all these things would basically is given by the equation here. So what can be the possible formulation. Like say for example, you are in a factory and it says that (in in) in the factory your total number of production of x, even if the constraints are such that you cannot produce more than 20 of the first paint. And the first paint has to be produced in in in quantum of minimum of five. That means five litres, 5 tons, 500, 1000 litres whatever the case be. You have to to produce for the paint 1 in the the in the paint manufacturing process.

You cannot produce more than say for example 10 tonnes or 10000 litres whatever it is for paint 1 also. So in that case, x1 which is the variable for the paint 1 would be bounded between 5 and 10. So it will be x1 is greater than equal to 5, is less than equal to 10. Similarly you will have 4 paint, 2 paint, 3 so and so on. Depending on that you can have the constraints also.

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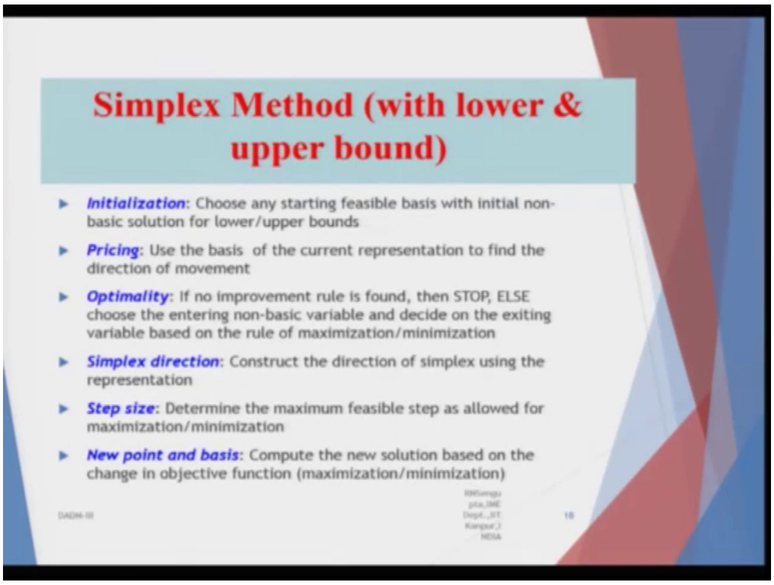

So now ~~do~~ the simplex method with the lower and upper bound, so in the initialization choose any starting feasible basic solutions with initial non basic solution for the upper and lower bounds. So you will basically consider the upper and lower bounds in such a way, so that they would basically give you the feasible solution. That means what you are doing is that you will have the feasible set which is fine. But you (chose) choose the the upper and lower bound. Or you consider the all upper and lower bound in the sense that they would be one of the sets in the as the starting basic feasible solution. Because obviously if it is a (content) if it is a basic feasible solution it may be possible it is the optimum solution.

Now we will consider the basic of the current representation to find out the direction of the (moment) movement. So the pricing would (basic) this pricing is that, as you bring on to the left the left hand side the positive value for the negative value would give you overall importance. So the optimality, so if no improvement ~~on~~ and the rule is found, then you will stop the iteration process. Else you will choose the entering non basic variables. So again the same concept they would be entering non basic, it will throw out one or push out one of the basic (exiting) existing one. And replace the entering one replace the exiting one and that will become the new basic solution.

Then you find out the values of B would be recalculated using the pivot column, pivot row the pivot cell based on which you will do the transformation. As you are doing that your zeroth equation which is corresponding to the z value for the minimization. Or if it is w for the (maximum) z value for the maximization and if it is w for the minimization, you will recalculate it and do the calculations again. So if no improvement rule is found then stop, else choose the entering basic feasible basic, non basic and decide on exiting variable based on the rule of maximization and minimization we will continue that.

You will construct the direction of simplex using the representation as you have done it. So you will basically consider entering exiting, entering exiting. But also consider that at what part you will stop that will be dictated by entering and exiting one. Whether they are actually feasible, because there is no non negative one. Then obviously it will mean the case of maximization there is no one who would replace any one of the basic variables in order to improve the objective function.

You will determine the (maximum) maximum feasible step as allowed for maximization and minimization in the same way. Find out the rate of change of the function. Again remember it is del y del x, del f del x. So x is basically the vector you will consider del y del x1, del y del x2, del y del x3, del y till del xn. And (change) check the one for which the rate of of the increasing the function the maximum. That is why you are taking the value of the minus negative being possible. Because if you keep all of the other variables fix, then you will try to find out that as one unit of x1 or x2 or x3 changes. What is the additional profit which is going to come the objective function by what unit it will change. So ~~T~~that is given (by) by the value of 5 or 3 whatever was there in the maximization problem.

So you will determine the maximum feasible step as allowed for the maximization and minimization and continue considering the exiting ones based on the fact that which one will you leave. So that will leave which ~~will~~ we basically bring down you an objective function. The least if it is an optimisation maximization case. You will compute the new solution based on the change in the objective function for ~~the~~ maximization and minimization. And you will keep computing it in the the same way as you did for the tableau, you will keep computing it till it is possible to consider that there is one entering and one exiting variables. But obviously look into the fact that entering and exiting are not happening in such a way that your actual objective function is unbounded.

And obviously in that case it unbounded means that ~~that~~ it will keep increasing till infinity both in the positive sense or the negative sense.

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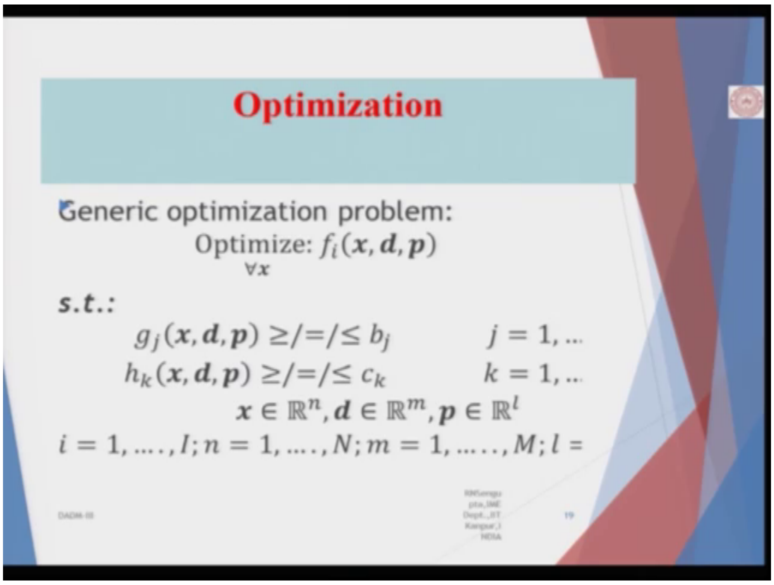

Now this generic optimisation problem I already stated, I will state this in very simple terms. So here the reason why I am stating is that in all of the problems I have been stating that the, this parameters based on which you are trying to optimised are fixed, but they need not be. If they are not fixed ~~means~~ which is what, your constraints may change. Like say for example if you consider machine 1, your actual utilisation is 8 hours. What is it becomes 9 hours. So if it is deterministic. We solve the problem and accordingly if it is non deterministic we will solve the problem in a likewise manner.

So Again I will state we have an optimisation problem with repetition of one only. If I is 1 in the first set you have a deterministic one second set you have a problematic one. And we will consider the solution in such a way that it will be it will be possible for us to find out the concept of the shadow prices (and the) and the slack and surplus in such a way that we can comment intelligently what is going on in simplex method. With this I close this 23rd lecture and continue more discussion along the process as you proceed for the simplex method. Have a nice day and thank you very much.