Data Analysis and Decision Making-III
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Lecture 22

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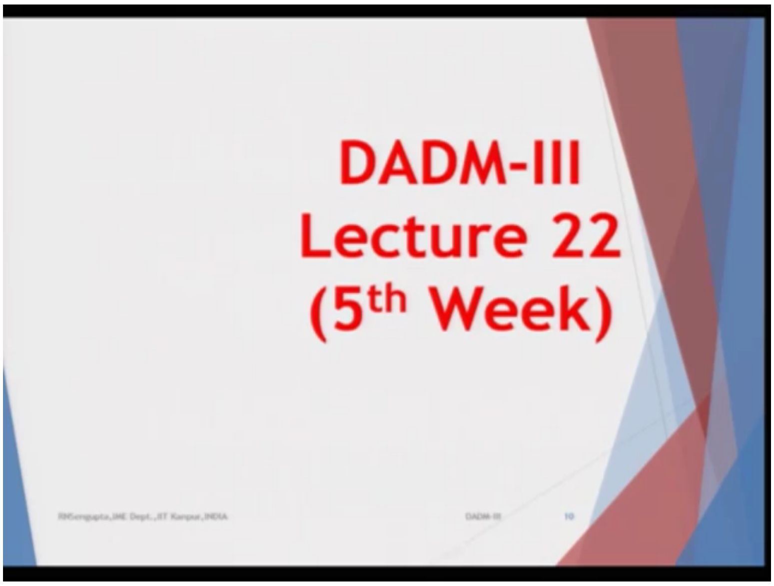

Welcome back my dear friends a very good morning good afternoon good evening to all of you, wherever you are in this part of the globe, and this is~~,~~ the DADM-III which is Data Analysis and Decision Making-III course under the NPTEL ~~mock~~ MOOC series of lectures, and as you know these total course contact hours is 30 hours which is a split and made into 60 lectures each lecture being for half an hour. And the total duration would be basically for 12 weeks and after each week ~~we~~ where each week you have 5 lectures each being for half an hour as I told you we have an assignment.

So here as you can see from the slide this ~~ere~~ is the 22 lecture that means we are in the 5^{th} week we have already taken all of we have taken 4 assignments ~~now~~ and we will complete in total 12 assignments after which you will have the final examination, and my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur so, if you remember we were~~have~~ discussing the very simple concept or simplex method that what would be the entering variable what would the exiting variable and before that.

On the 20^{th} lecture which was the last lecture or the last week 4^{th} week we ~~have~~ were also discussing their concept of primal dual problem in the sense that they are just intuit of mirror image of each other in the sense that like when you seeing your picture in the mirror your left becomes the right hand and the right hand becomes the left hand which means the maximization becomes the minimization and the minimization becomes the maximization, but obviously I will repeat it please ~~pear~~ bear with that.

The numbers of constraints become the numbers of variables, the decision variables and the numbers of distinction variables becomes the number of constraints for the primal and dual so whichever is primal whichever is to dual that is a different question, it can be maximization or minimization, and the greater than sign and less than sign those conversions have already been discussed.

So we will, we were now based on that I said that we are going to basically discuss the primal problem the maximization one and then basically go and try to solve it in the sense of minimization, one thing you should remember that we were talking about ~~on~~ unbounded~~ as~~ ness, unique solution, no solution, on the constraints rowan ~~go on~~ satisfy all these things, so the output which we have getting in the primal problem would you will get the exact same intuitive output in the dual problem also.

Because that would come out very easily because if you are trying to maximize in the sense for the primal one and if is un bound it that mean, it will keep going way from the 0 and more inside the first quadrant in the same way when you do the minimization problem because remember in that case if x is, whatever the x, s, w depending on how ~~we~~ you have been able to formulate the slack and the surplus.

So obviously the slack and the surplus would all be denoted by s but in some cases to make the differentiation we would also use the symbol small w because also remember this nomenclature of trying to basically maximize being z minimization being w would also be utilized.

Now, coming back to this the concept of unboundedness so if we have unbounded maximization solution in the sense when we have the dual problem it would be the just reverse where when we are trying to do the minimization we would not be getting any results any unique results, now, when you consider now coming back to the problem which we left yesterday so you are basically trying to discuss that which variable will ~~inter~~ enter and which variable will go out and at their stage we discuss that x2 comes into the picture x2 is one of the actual ~~distinction~~ decision variable, initially we started at 0 which is the origin.

Where x1 and x2 were 0 and the corresponding variables or the values of x3 x4 x5 which was the slack for equation number 1, equation 2 and equation 3 which is basically for 1 2 3 the corresponding constraint 1 2 3 were the maximum value.

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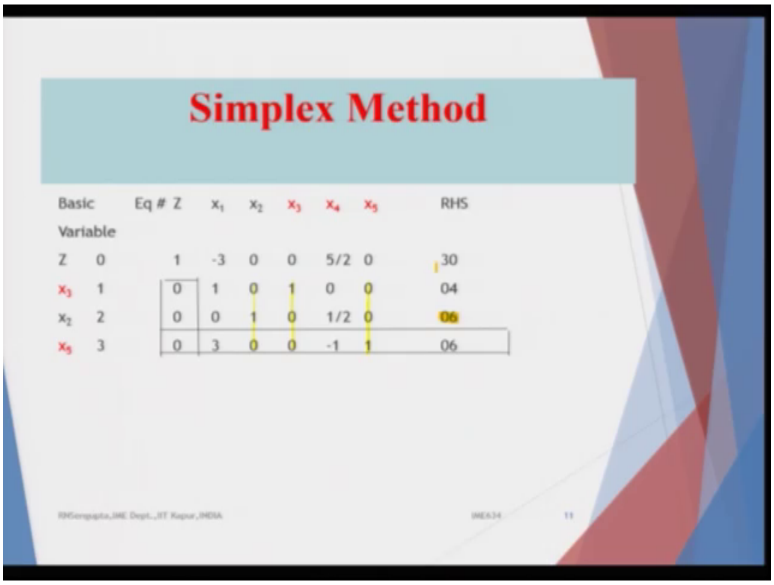

Now when we consider and we also saw. That the value of addition for 1 x1 or 1 x2 was basically given by the value of 3 and 5 respectively so we consider 5 to be the entering one for which case x2 came into the picture and the one which would leave would be that variable for which the decrease in the objective function considering the maximization one would be the least so, we found out that to be, x3 so x3 is out, x2 is comes and the end of the day solution we saw, so we will basically just constraints concentrate on this identity the entry matrix so the identity entry matrix would be basically one value would be this one, one value would be this one, and one value would be this one, based on which.

Now, you know the objective function value is 30 why it is 30 because if we multiply 6 which is the value of x2 here, this is 6 for x2, x1 is not there because the x1 value is not the basic variables obviously the x1 value is 0 here, and the slack of x3 is 4 x5 is 6 and x4 which has left is also not there so, and also remember the x2 component adding on to the overall objective function was 5 per unit of x2 so 6 into 5 is 30 which you find out have the objective function here.

Now, when you consider the next, now you will ask a question that whether you should proceed or not proceed? rRemember one thing, this point initially were 0,0 and the corresponding values of x3 x4 x5 were there, but now the once you do the iteration on the first move the values are x1 is 0 x2 is 6 x3 is 4 x4 is 0 x5 is 6. So ~~here~~ you have moved from one corner point to other depending on the movement criteria of the algorithm.

Now, we will basically again check for the maximization problem what we check~~,~~? ~~w~~We check that if there is any one the variables which is negative coefficient, coefficient becomes why, because you have brought it ~~(())(07:06)~~ from the right hand side to the left hand side so, check one x1 is minus 3 others are all positive and positive in the sense that the slacks 5 by 2 is positive which is dual and that is a basic one of the feasible solutions. Now x1 should enter, so if x1 enters obviously we will try to find out who would basically be replaced by x1.

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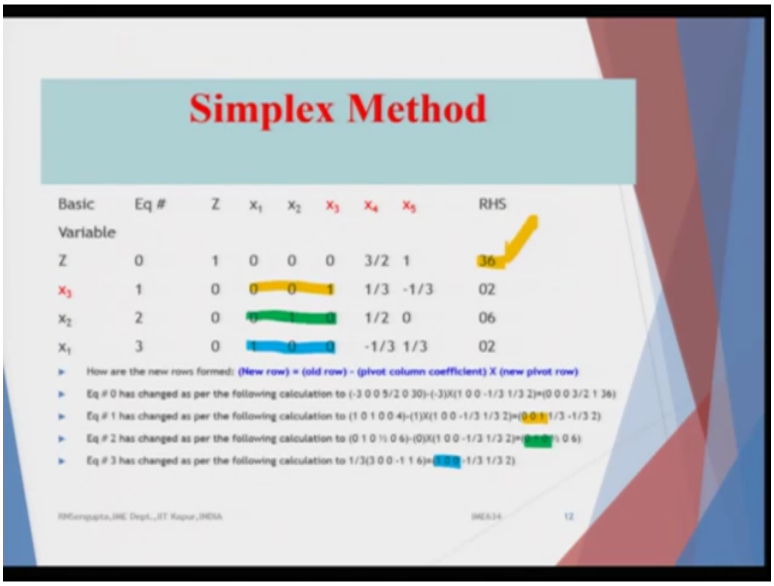

So now, when x1 enters and as you can see from this graph as x1 enters obviously x5 leaves so if x5 is leaves again ~~we~~ you will basically consider the division of the right hand value, column value for the b vector which is being changed divided by the corresponding coefficient for the pivot column elements and based on that we can basically find out the~~,~~ Pivot not the element the Pivot ~~P~~ column and the actual value based on which will do the swapping and try to find out the calculation would be the Pivot element which would be the common ~~s~~cell based on the Pivot row and the Pivot column, Pivot column means based on which we going to swap or change.

Now again we basically follow the same policy that means you are now trying to the policy is that do not be too much bothered about what is the equation, main thing is that the one which comes in side which is now x2 x1 technically that particular overall row corresponding to the basic solutions which is now basically you have x3 x2 and x1 here, these values in the corresponding ~~s~~cells would be one what do have mean by the word corresponding ~~s~~cells, corresponding ~~s~~cells means that if I considering x3 and I am considering the ~~entry~~ identity matrix only for x3 the identity value would 1 and for x2 and x1 it will be 0.

Why, because once you want find out x3 the right hand value which is there and the transform B vector would give you exact value of x3 which you have trying to proceed. ~~s~~Similarly, when I consider x2 I consider all the ~~s~~cell value corresponding to the identity value, identity value means the top most columns are also corresponding to x3 x2 and x1 so only the common ~~s~~cell which will be one would be for x2 rest x3 and x1 would be 0 hence, when you find out x2 it would basically be given by the transform to B ~~s~~cell which is there on the right hand side.

similarly, when I go to x1 the corresponding values which would be there for x1 ~~s~~cell the common ~~s~~cell if you look from above x1 and from the right x1 it should be 1 rest of the value should be 0 because when I want to find out x1 the corresponding B ~~s~~cell on the right hand side of the transform B would give me the value of x1, so let us check, so if I consider x3 leave ~~at this~~ aside Z so these are parameters are 0 so let us see so, the basic variables here also x3 x2 x1 so let us not be bother about x4 x5 so x3 ~~s~~cell values for corresponding to x1 is 0 which is fine x2 is 0 which is fine x3 is 1 and these values are not there so obviously their affect would not be there.

So when we put the value actually x3 would be 2 that means the slack value for x3 is 2. ~~w~~When I go into x2 let us check the x1 ~~sell~~cell is 0 which is fine x2 ~~sell~~cell is 1 which is fine x3 ~~sell~~cell values I am just correspondingly checking is 0, x4 x5 are not there so it is basically half and 0 so the value which on the right hand side is 6, similarly, when I consider x1 again x1, x1 ~~sell~~cell value is 1 x2 and x3 ~~sell~~cell values common values are 0 so their value is 2.

Now as you have transform it technically the value has also come down to 36, ~~Hotstar~~ How we do transformation is given in the bullet point which I am just hovering my electronic pen, now in the objective function what it was~~works~~ I am going slow but please bear with me so what was the objective function?~~,~~ ~~o~~Objective function was basically x1 some factor multiplied by x1, some factor multiplied by x2 plus 0 into x3 plus 0 into x4 plus 0 into x5, so x3 x4 x5 are ~~rolled~~ ruled out so even if x3 is 2 it does not come into the objective function which is fine.

Now when I go to with the value of x1, and x2 remember the values which I had what basically 3 and 5 that means 1 unit of x1 is going to add up you 3 units profit 1 unit of x2 would basically add up 5 units of profit so, let us check 6 units of x2 so 6 into 5 is 30 2 units of x1 which is 3 into 2 is 6 so 30 plus 6 is 36 so let us check the objective function is 36 which is right.

That means now we have moved from the second stage we have move to the third stage the corner points where the corner points now the coordinate is x1 2 x2 6 x3 2 x4 0 x5 is 0 ~~on~~ and the corresponding objective function value is 36. So let us check whatever we did in the row column multiplication it can be basically we are doing only the row the multiplication in us to ~~be~~ basically convert that into the identity matrix because you remember I am repeating it your actual policy was A axis is equal to B so you are trying to covert A by pre multiplying or post multiplying by A invers and that becomes I so, I into x is equal to A invers B or B A invers and that is the basically the final result what we are~~n~~ aiming at.

So here the new row would be formed by the concept that the new row is equal to the old row minus the Pivot column co efficient so, obviously ~~we~~ you have Pivot column so ~~we~~ you will take the corresponding element based on which row you are doing multiplied the new Pivot row and the, so column- wise you would pick up each element which is there in the Pivot column and you will basically multiply the corresponding Pivot row which is there, which is basically fixed and when ~~we wants we~~once you do that, very interestingly we will get the values accordingly which is what I will reroute ~~wrote~~ the values are absolutely given just pay attention here.

~~We~~ You will good understand that what we are doing is that converting the rows corresponding to the fact that they~~is~~ slowly get converted into identity matrix. So in the equation 1 that will change in such a value that the old column was basically minus 3, minus 3 was basically for x1 00 was basically for the corresponding basic variables which were there, 5 by 2 0 in 30, 30 was basically the last value in that 0^{th} row which was basically the objective function, so~~,~~ objective function would now be changing from 30 to 36.

So let us check that now minus so the Pivot column element is minus 3, minus 3 is basically corresponding to the value of x1, because that was the only ~~contain that~~contender which is going to enter the system based on which x1 would come into the system. ~~m~~Minus 3 into the initial new Pivot row which you already have is 1 0 0 minus 1 third, 1 third in ~~to~~2 so actual value which you get is 0 0 0 which is basically the ~~now~~ norm as should be 0 0 0. 3 by 2 is basically for x4 1 is x5 and see interestingly 36 comes out which is what we have already rolled about the objective function.

When I go to equation 1 which is for plant 1 or constraint 1 that will change according to the concept where the old column was our O row was 1 0 1 0 0 for 4 minus 1 this minus of 1 is basically the Pivot columns corresponding element which is there, which is 1 and obviously the Pivot row which is fixed is given as 1 0 0 minus 1 third, 1 third and 2 and the final vector which you get, this is what is interesting and pay attention to the particular values, pay attention to 2 because 2 is coming here which is basically the value of x3 at that stage.

So the corner points where we have obvious (())(15:57) has a value whatever the values of x1 x2 x4 x5 is immaterial (())(16:01) we will come to that but the value of x3 is now 2 as it should be, now and these vales which are before number 2 I am going to that later within 2 minutes.

So when I go in to plant 2 which is equation 2 when I do the transformation the old row minus the Pivot column coefficient which is basically 0, 0 was there already so, obviously that overall transformation would not be affecting a multiplied the new Pivot row would give us the value of 0 1 0 half 0 6 so this 6 is important because 6 gives us the value of x1, and finally when we go into the plant 3 which is equation 3 multiplying that in that case obviously you will be dividing by the corresponding value of the Pivot row and the Pivot column element.

So, when that is done the value comes out to be, because why we are dividing by 1 third remember once we divide by 1 third this number 3 which is the Pivot sellcell would basically be converted to 1 that means we are forcing that to be one of the element in the identity matrix as 1, this is where it becomes so, it is basically 1 third and you divide the overall new Pivot the whole column that becomes actually only pay attention with the last sellcell which is 2 which is basically the value of x1.

Now another very interesting thing is I am repeating it please look into it, so when I mention x3 x2 x1 and here x3 x2 x1 so, this whole thing is basically looks like an identity matrix obviously it should be the principle diagonal but, do not worry about that I am just swaping it in such a way the B side right hand side also changes, this has already come out here, so let us check I am not interesting in equation 1 so, I am only interesting in equation -2 3 4 so, the values are 0 0 1, 0 1 0, and 1 0 0 is exactly what I get. So let me use a different color it would be easy.

So this is 0 0 1 which is 0 0 1 here, this is 0 1 0 this is 0 1 0 here, this is 1 0 0 this is 1 0 0 here, so we have basically obtained the actual identity which is marked in color in the row, column transformation.

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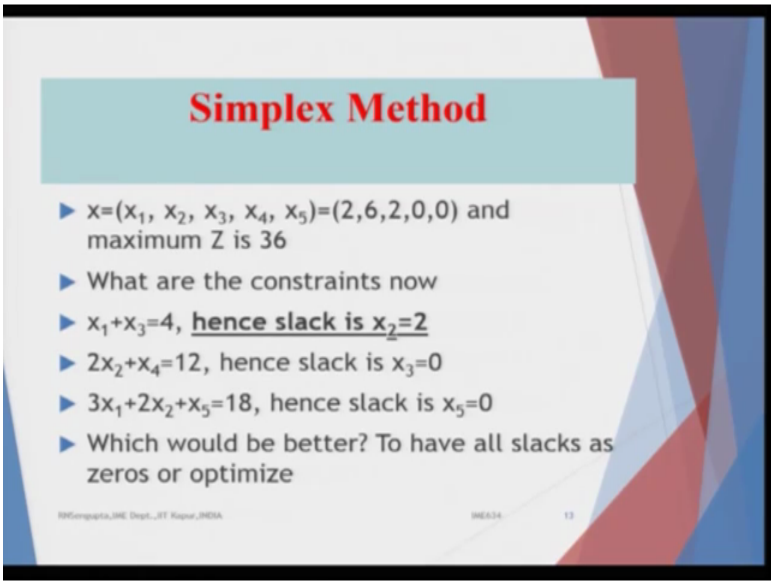

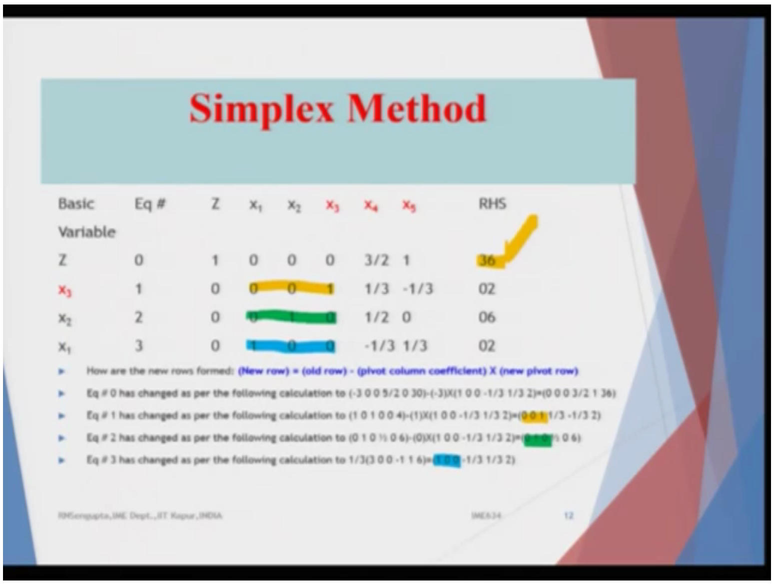

Now, we have at the stage what is the answer, the vector x has basically x1 as non 0, which is one of the solutions which we want x2 is non 0, which is the solution as we want, x3 is a slack which is there, x4 and x5 are not there in the solution. So let us check what are they, x1 value is 2 x2 value is 6 x3 value is 2 x4 value is 0 x5 value is 0. Now we have already double check that is 3 into 2 plus 5 into 6 is 36 and that is the objective function, so are first step is satisfy. So the vector x column or row whichever you denote the maximum value is

36. Now let us check that, what are the constraints at that stage, I should not be doing that but, I will tell you why I am doing that and then I will proceed.

Let me first discuss why I am doing that, the stage when you stop, stage where you stop, now this was an objective function where you want to maximize so, in that rule was basically whatever constraints you ~~wrote~~ brought from the right hand side to the left hand side if they are negative you will proceed but, let us see are they in a negative values, for the 0^{th} equation so they are not, which means that we have reached the optimum solution because any one of the ~~contain~~ contender ~~us~~ if they come into the objective function, and replace another the actual objective function will now decrease from 36 lower.

So obviously, we have reach the ~~pick~~ peak and we would not proceed that means the dy dx is basically on the down one~~,~~ trend now based on the fact ~~affect~~ that we have consider the concept using simple ~~in~~ differentiation concept. Which means that we have reached the optimum value now let us go into the constraint, so there were 3 constraints where the first one was x1 plus x3, x1 was the actual ~~distinction~~ with decision variable, x3 was a slack, so~~,~~ here x1 plus x3 what is x1 is 2, what is the right hand side is 4 so 4 minus 2 actually x3 is 2.

So is to 2 yes, it is 2. Because you have seen the vector which we have just found out the value of x3 is 2 that means there is slack that is un-utilized concept of 2 units till left, that means from plant 1 ~~we~~ you have utilized the overall production for product 1 ~~we~~ you have ~~put,~~produce say for example ~~to~~ 2 kg, ~~to~~ 2 tons, ~~to~~ 2 billion tons, whatever it is. And the overall extra left amount would be 2 which is coming from the slack, so that means plant 1 is not operating up to the capacity as it should because there are still some materials un utilized.

Now let us come into the second constraints what are the second constraints were 2 x2 is less than equal to 12, so we have added the slack which is 2 x2 plus x4 is equal to 12, so let us check what is the value of x2 in the final solution it is 6 so 2 into 6 is 12, so 12 is there on the left hand side 12 with the right hand side so, the actual value of x4 is 0. So does it match, our answer, yes, if we go into the vector x we will see the value of x4 is 0, which is the second last ~~sell~~cell is 0, so that means over a utilization from plant 2 has been 100 percent such so that, slack for second constraints is 0 so, we have utilized everything.

Now let us come to the third constraint depending on the plant 3, so what was that constraint is was 3 x1 plus 2 x2 is less than equal to 18, so we have added the slack which was 5, so now let us basically put the values of x1 and x2 in that equation so, what is it, is 3 into 2 which is because x1 is 2 3 into 2 is, 6 and x2 is 6, so 2 into 6 is 12, 12 plus 6 is 18, so left hand side is 18, right hand side is 18, which means that x5 is 0, that means the slack corresponding to the third constraint is all 0, so we have been able to utilize the third constraint corresponding to the plant 3, totally.

Now if actually if we were able to so, this is the situation we have stop but are satisfied, technically I am mentally not satisfied because due to the reason that I would like to utilize that 2 kg or 2 billion tons or 2 tons whatever is there for slack corresponding to the plant 1. Not being utilized is something which is irritating me, but the fact is that, if I try to utilized that, I will basically break, I ~~would~~ will basically exceed that constraint because the only product which I can make, in plant 1 is x1, so if I basically utilized that to make x1, obviously that would will ~~the~~ violate the constraint, ~~will attending that~~violating the constraint obviously it would have, 2 effects one positive one negative.

~~p~~Positive in the sense that it will definitely increase the objective function, provided x2 is at the same level but, if we come to the third constraint you will see that, x5 is 0, which means that we have utilize everything. So, increasing x1 it would have been obviously x2 we have been, we will have to decrease such that, the total amount which will keep produce at plant 3, is kept 18. Now consider that, x5 is still 0, so if x5 is still 0, x2 has decrease so, decreasing x2, does it increase are objective function totally from 36 value. If that is true then I am happy, if that is not true I am definitely not happy.

That is point 1, point number 2 also remember it may be possible that if I concentrate on slack number the first one which is x3, then having and then effect would also increasing it would also been that they may be some x5 slack which is happening so, x5 becomes positive in that case x2 would decrees so if x2 is decreasing x would also come into the picture so, x4 is also become positive, so in the sense that increasing x1 and if we have to decrease x2, if we see the objective function, is increasing from 36 it makes us happy, such that we will be able to go ahead and basically plan it accordingly.

But, pause here, we know that per unit increasing x1 would give us a profit of 2 units while, per unit increase of x2 would give us a profit of 5 units so, what is better, obviously we will be happy, that if x2 increases by 1 unit, while x1 is increases by 1 unit, the additional profit would be basically 5 minus 2 which is 3. So, it considering all these things you can definitely prove, that any further improvement would not be possible, that mean it is possible that at the end of the day, some unutilized material is left any one the constraint, depending on how you have been able to formulate the problem. Now let us go into so, I will come to the dual problem of this, later on.

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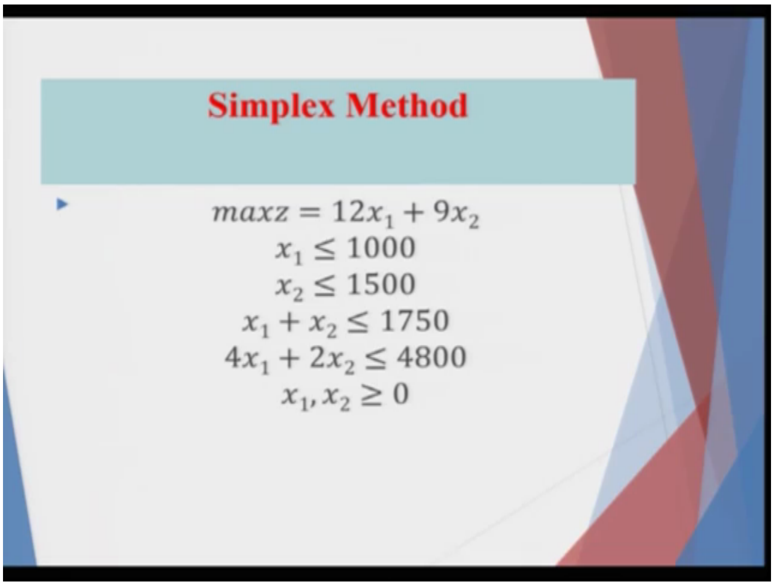

so let us consider another problem where, you have maximization of 12 x1 plus 9 x2, I am considering all of the ~~mean~~ them in~~that~~ the 2 dimension one very simplistic sense it can be expanded accordingly so, there is no issues about that, so the constraints are x1 is less than equal to 10000, x2 is less than equal to not 10000 sorry, it is 1000, x2 is less than equal to 1500, x1 plus x2 is less than equal to 1750 or 1750, and ~~for~~ 4 x1 plus 2x2 is less than equal to 4800.

Now, if I basically talk on the same point as I have been talking for the first problem it would be, so now, I have to basically add~~t~~ the slack so the ~~a~~ slack would be one would be added ~~to~~ for the first one would be added to the second one, one would be added to the third one, one would be added for the forth one. So consider the slacks I am not mentioning them as x1, let us consider that I basically use the symbol as s1, so it would be s1, for first s2 for second, s3 for third, s4 for fourth. So, the equation actually would be x1 plus s1 is equal to 1000, x2 plus s2 is equal to 1500, x1 plus x2 plus s3 is equal to 1750, and the last one would be 4 x1, plus 2x2 plus s4 is equal to 4800.

So, as I add the slacks in order to bring the equality in all the 4 constraints the objective function also will change which would be 12 x1 plus 9 x2, plus 0 s1 plus 0 s2 plus 0 s3 plus 0 s4 is basically the objective function. And obviously you will have the constraints as x1 x2 and ~~now so~~also, the slacks s1 s2 s3 s4 as greater than 0.

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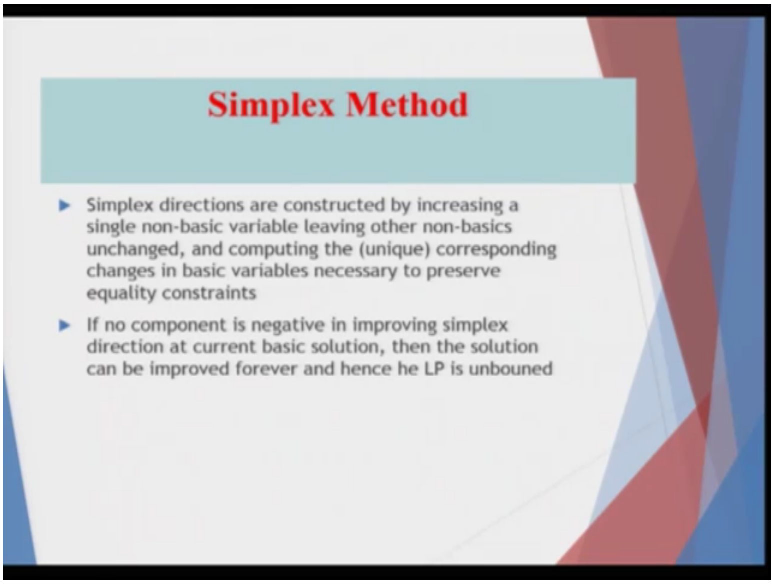

So we will basically take the simplex reduction as constructed by increase in the non-basic solutions as I have already discussed and the entering and the existing one would basically depend on, the maximization on the minimization problem which I have stated, but, preserving the fact that your movement would all be along the boundary points corresponding to the basic feasible solution.

If no component is negative in order to increase your simplex method, considering the maximization problem will stop, and obviously it may be in the unbounded case that the entering variable, come again and again, such that the feasible solutions would basically keep changing or they would be basically left at certain different combinations of all the basic solutions are and the slacks or the surpluses but, the objective function will keep increasing if it is on the upward trend, similarly in the negative sense when you are trying to do is solve, the dual problem it will keep decreasing to negative infinite, or in minus infinity depending on the problem how its has been done.

So with this I will close this 22 lecture which is the second lecture in the fifth week and I continue to discuss more about the dual and primal dual problem accordingly in the next class, have a nice day and thank you very much.