**Data Analisys and Decision Making -3**
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**Lecture 21**

Welcome back my dear friends, very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe and as know this is DADM 3 which is a data analysis and decision making 3 which is being shown on this slide. Another NPTL MOOC series and as you know this is the course another for which the total duration is in number of weeks is twelve weeks, which when converted to number hours contact hours is thirty and each lecture is for basically half an hour and each week as you know we have 5 lectures.

So total numbers of lectures will be 60 in number. Now as you also that we are already completed 4 weeks which is about twenty lectures we have already completed and after each week we have an set of assignments we have completed 4 assignments and we going to start the fifth week after another 5 lectures of half an hour each we will have the fifth assignment. And my good name Raghunandan Sengupta from IME department at the IIT Kanpur.

So if you remember we were discussing about the concept of primal dual problems that means primal dual are interchangeable words that means if I am a maximization on primal then the minimization is dual or if the minimization is primal the maximization is dual. Where we convert the number of rows or the number of constraints into number of decision variables and vice versa depending on how you want to solve the problem and the greater than sign, less than sign, unrestricted part all this things we have discussed in the last class which is on the twentieth lecture. Now as you see this is the twenty-first lecture starting of the fifth week.

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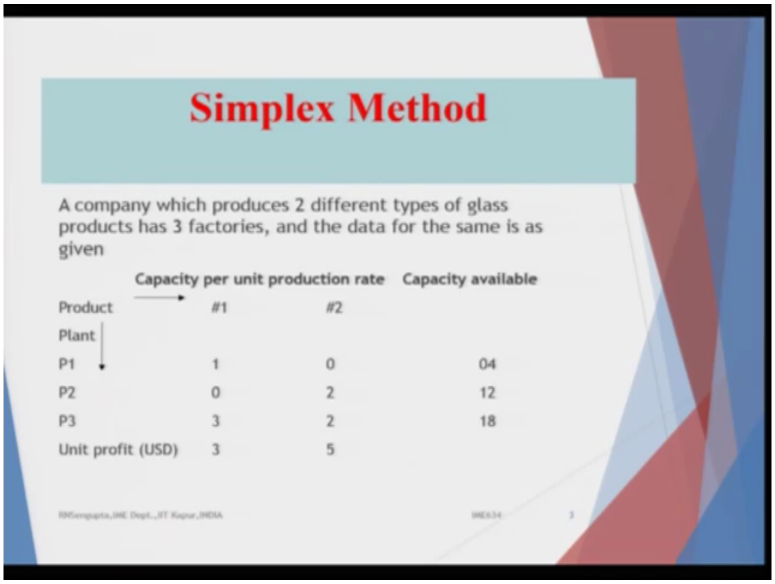


Now let us consider a problem and we will also consider its duality later on. So company consider is has 2 different products. So mark the thing as 2 different types of glass products and it has 3 factories. So if you consider there are 2 decision variables here and there are 3 constraints. So now I will be switching from the concept of how you solve the problem upon the primal one and dual one. The concept of solving would remain the same but the formulation is what is important.

So if you have 2 different types of glasses and if these are the 2 decision variables which is x 1 and x 2 and if there are 3 constraints pertaining to the factories when you convert that into the dual they would be 3 decision variables pertaining to the factories and 2 constraints got pertaining to the decision variable in the initial primal problem. Now this if it is a maximization problem you will have the counter part in the dual case which will be minimization problem.

So let us first solve it and then we will come once the overall algorithm is mentioned will come to the dual problem solution also. So company will be leaded, a company which produces the 2 different types of glass products has 3 factories and the data for same is as given below. So the plant P 1, P 2, and P 3 are given along the first column on the left hand side and the product which is one and 2which is given in the second and the third column.

These are the capacity per unit production rates and capacities available from plant one, plant two, plant 3 are respectively given as 4, 12 and 18. So they can be ten to the power 3, 10 to the power 4 does not matter, I am just giving the numbers. Which means that in plant 1, you can produce only one product which is product 1 and 0 product 2and total capacity utilization is 4. So you can produce maximum 4 units of product 1.

Similarly, for plant 2 depending upon setup which is there you can produce 0 number of product 1 and 2 number of product which will be produced. So per unit capacity utilization what I am saying and capacity available is 12 consider that product 2 requires 2 machines and we have 12 total machines in plant 2 so you can produce maximum of 6 products in plant 2. Finally, when you come to plant 3 it has got 18 total units of capacity utilization. Consider that each unit of product 1 requires 3 units of capacity utilization and each unit of product 2 requires units of capacity utilization.

And if I consider this production obvious there would be profit, so unit profit I am considering so obviously it means that I am calculating using the concept of selling price and cost price. So the unit profit which I get from product 1 irrespective of the plant is 3 units and the unit profit which I get from product 2 irrespective of plant 3 is 5 units. So if I want to basically formulate the primal problem considering it's maximization problem it will be x 1 and x 2 is basically for product 1 and product 2. So the unit production if it is 3 and 5 it would be 3 x 1 plus 5 x 2 which you want to maximize corresponding to this 3 constraints product to plant 1, 2 and 3.

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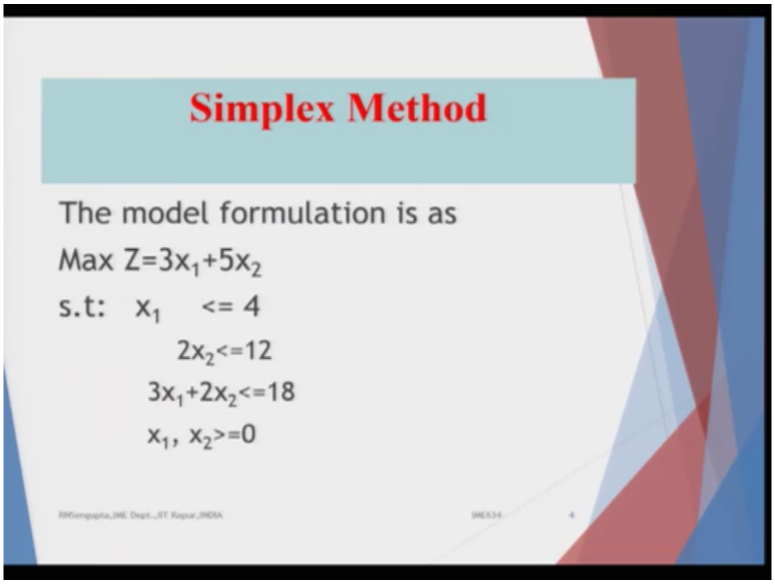


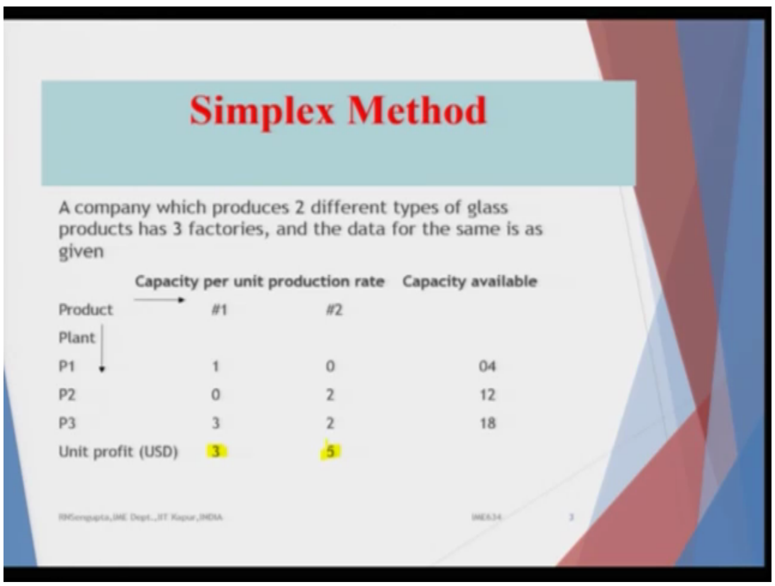


Now the modern formulation if we do it which is very simple it basically comes the maximization of 3 x 1 plus 5 x 2 depending because 3 x 1 plus 5 x 2 because if you see so this 3 unit is coming from 1 unit of product 1 produced this 5 unit is coming from 1 unit of product 2 produced.

So it is 3 x 1 plus 5 x 2 and one of the constraints if you remember I mentioned that plant 1 can only produce product 1, plant 2 can only produce product 2 while plant 3 can produce both of them. So if you bring them into the situation it will be x 1 is less than equal to 4, 2 x 1 because we are utilizing 2 units from plant 2 to produce the product 1 unit of them so if you are producing x 2 unit of them it will be consumption will be 2 x 2 and why it is 12? Because on the right hand side the constraints of utilization in plant 2 is 12.

So it is 2 x 2 less is than equal to 12 and for plant 3 total utilization is 18 corresponding to 3 units being consumed by product 1 and 2 units being consumed by product 2. So it is 3 x 1 plus 2 x 2 is less than equal to 18. Now remember here all them of all them are less so obviously if they are less you have to understand that what you have going to add slack or surplus depending upon the problem. So obviously it would mean that if you are able to produce (7:42) product in plant 1 all utilization is hundred percent.

That means you can produce maximum 4 units of product 1 similarly if you produce all units in plant 2 all capacity utilization is maximum then you only produce 6 units of product in product 2 in plant 2 and similarly the combinations can be thought of or if you produce basically some combinations of product 1 and product 2 in plant 3 and obviously as per the assumptions these products have produced in positive numbers, so x 1 is greater than 0 and x 2 is greater than 0.

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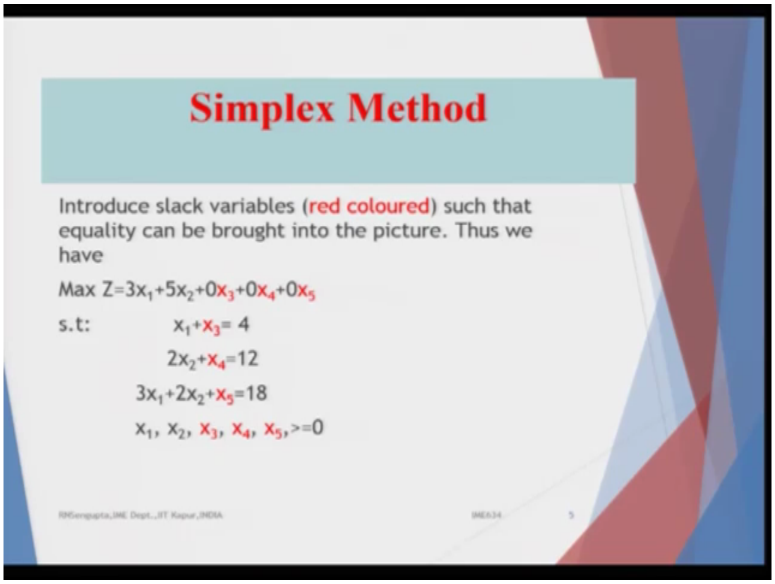


Now we will introduce the slack variables, slack because they are lagging behind so you have to add slacks. So to introduce the slack variables which would be red in colour for less than equal to sign and obviously you would remember one thing if you are converting the less than sign to the greater than sign corresponding to the dual problem so in place of slack the surplus will come with a negative sign.

So introduce the slack variables which are shown red in colour, such that equality can be brought into the picture thus we will have the problem the maximization so the slacks would be what? There are 3 equations they are with 3 slacks which we can denote by s 1, s 2, s 3 but we have denoted it as x 1, x 2, x 3, x 1 x 2 is basically for variables, so obviously they would be x 3, x 4, x 5. So the first constraint would be x 1 plus x 3 which is equal to 4 where x 3 is the slack corresponding to the first constraint.

The next constraint would be 2 x 2 plus x 4 is equal to 12 where x 4 is a slack corresponding to the second constraint and the last equation which is the third one would be 3 x 1 plus 2 x 2 plus x 5 is equal to 18 where x 5 is basically the slack corresponding to the third constraint. So here also as we know we will formulate the problem where x 1, x 2 is obviously greater than 0 but we will also bring the slacks to be greater than 0. So this means x 3 x 4 x 5 are greater than 0.

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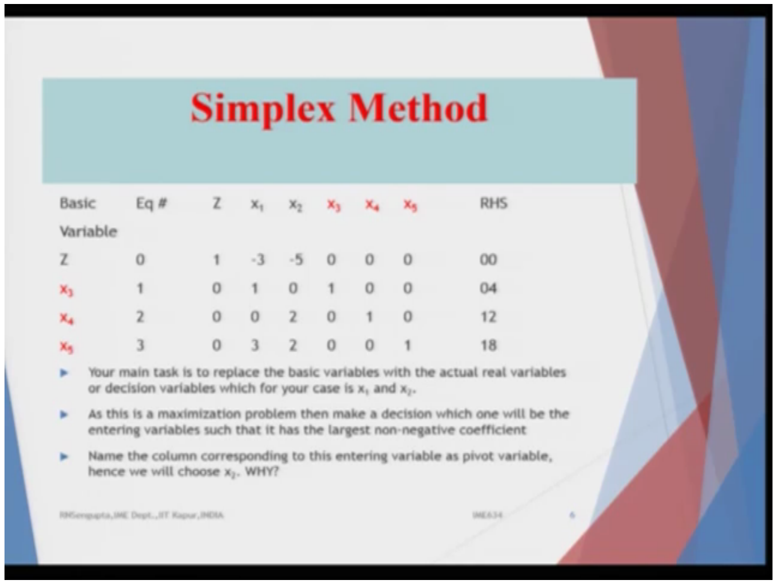


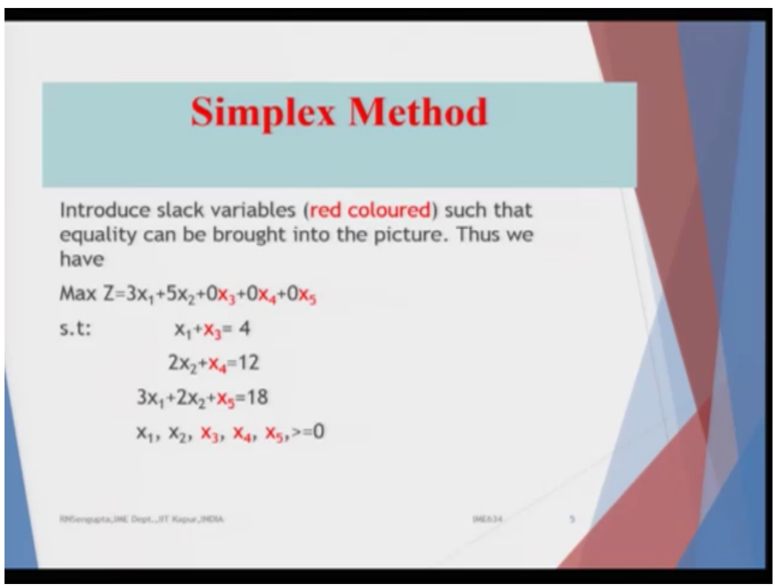


Now we will basically write the tablo, tablo is basically from where we are into start. So we can have different basic feasible solutions. So we will consider the origin considering the origin is a feasible solution. We will consider the origin as the starting point, so how do you do that? So we will write the basic variables along the left most column so the first variable which you write is Z which you want to maximize.

So now if x 1 and x 2 which is 0 point it is a basic feasible solution so if you put 0, so let us come back here. If we put 0 for the first equation so x 3 becomes 4 if we took 0 for x 2 in the second equation x 4 becomes 12 and technically as x 1 and x 2 are 0 in third equation x 5 will be 18. So hence when we start the actual vector corresponding to the basic feasible initial solution is 0, 0, 4, 12, 18. Now pause ask yourself is this a feasible solution?

Yes, because all of them are non-negative hence you can start your iteration process based on the point 0, 0, 4, 12 and 18. So let us check what we have said actually matches. Now we will write in the second column which is here at the equation number so 0 would be a equation number corresponding to objective function whether maximization or minimization it does not matter. The first constraint would be numbered as equation number 1 so let us see so let us first see the equation 1.

So if you remember I mentioned that what you do is that you bring the all the variables on to the left hand side to the equality sign. So now it was initially 3 x 1 plus 5 x 2 was the objective function which was trying to maximize now when you bring the slacks in to the position your will basically have 3 x 1 plus 5 x 2 plus 0 into x 3 why 0 because there is no component of the slack which going to come in to the objective function. The 4th term is plus 0 into x 4 because the component for the second slack which is coming from the second constraint is not applicable for the objective function plus last term would be 0 into x 5 where x 5 is slack corresponding to the third constraint which would not have any bearing on the objective function.

So when we bring all this terms to the left hand side you will have minus 3 x 1, minus 5 x 2, plus 0 into x 3, plus 0 into x 4, plus 0 into x 5. Here the red colour denotes the slacks so now if x 1 is 0, x 2 is 0 and automatically x 3, x 4,x 5 even they have the non 0 they are 4, 12 and 18. Because there parameter value by which they have been multiplied individual are 0 hence the objective function when you start of is 0 which is shown here.

Now I want to basically optimize now let us go to the constraints 1, 2, 3. The first constraint if you noted down it was only related to x 1 so now if you consider x 2, x 3, x 4, x 5 is equal to 4 obviously the variable corresponding to x 1 is 1 which is here. So I will mark the equations accordingly so it will be one while the corresponding parameter values corresponding to x 2 will be 0 because x is not there in the first equation x 3 will be one because x 3 is coming out as a slack for the first constraint x 4 is not there because is the slack corresponding in the second constraint and x 5 is 0 because it does the slack corresponding to the last constraint.

So hence the parameter values which will be multiply z is 0 obviously as per the norm because we will consider that as the nomenclature 1 x 1 plus 0 x 2 plus 1 x 3 plus 0 x 4 plus 0 x 5 and the right hand side is 4 because if you this equation actually what it means is that as x 1 is 0. So obviously it will become 0. X 2 is technically multiplied by factor 0 so it would not be coming to the picture. Because x 2 even if an as such it is also 0 x 1 and x 5 are being multiplied by 0 so they would not come to the picture.

So only thing which you have is basically one into x 3 is equal to 4. Now let us come to the second equation and I will use a different colour second equation. Now has in equation where x 1 is 0 so obviously it means 0 into x 1 x 2you know that it will be multiplied by 2 so 2 into x 2, x 3 is not there in the second constraint so it is multiplied by parameter 0, x 4 is there in this constraint which is the second one so x 4 is multiplied with the value 1 and x 5 is the constraint pertaining to the , the last constraint so obviously it would not be there so is 0. Now if we see the equation it will be 0 into x 1 plus 2 into x 2 plus 0 into x 3 plus 1 into x 4 plus 0 into x 5. X 2 is 0 so obviously this vanishes x 3 and x 4 are not there so the value of x 4 if you put it on the right hand side it becomes 12.

So now we are double checking the value of x 3, x 4 are 4 and twelve respectively. Let us come to the last equation so this the value which we have x 1 was there x 2 is there and obvious I will put the colour of 0 ya this is true, this is true. Now I come to the last equation let me use the colour of blue so the third equation had 3 x 1 plus 2 x 2 plus x 5 is equal to 18., so x 3 and x 4 are not there so they being multiplied by value 0. So let us check 3 into x 1 plus 2 into x 2 plus 0 into x 3 plus 0 into x 4 plus 1 into x 5.

Now what is there on the right hand side? 18, and why it is 18 because if you put x 1 as 0, x 2 as 0 so everything x 1, x 2, x 3, x 4 vanishes so the slack value x 5 is 18 so let us mark these values and also I will use a colour scheme of this. Now let us pause and look at this, this matrix look at this matrix what is it? Is an identity matrix.

So what we want to do is that convert because x 3, x 4, x 5 are the basic feasible solution from a starting so technically when we Stop the duration the values pertaining to slacks x 3, x 4, x 5 should be 0 actually should be 0 because it would mean that we have been able to utilize the overall utilization of machine of plant 1, plant 2, plant 3 to the maximum possible extent as that we are able to produce the product 1 and product 2 to maximum possible extent and gaining the maximum profit.

So actually what we want to do is that once we finish so in place of x 3, x 4, x 5, x 1 and x 2 would definitely be there and anyone of the other slack so you will have either x 1, x 2, x 3, x 1, x 2,x 4 or x 1, x 2, x 5, x 1 , x 2 would be non 0 and this 3 or 4 or 5 whichever the slack variable would be there, would be 0 corresponding to the fact that overall utilization is 100 percent in all the 3 equations. So if it is, if it is x 3 is 0 by calculation it means that x 1 and x 5 are obviously not there x 3 is also being forced to be 0 depending on the utilization.

Now let us read the problem your main task is to replace the basic variables with the actual real variables or decision variables which are x 1 and x 2 for your case it is x 1 and x 2 as I mentioned and there would be additional slack variable also. As this is a maximization problem then we should make a decision which one of the variable should leave and which one of the variable should enter. So who are the contenders to enter? Actually the contenders to enter is x 1, x 2, x 3, x 4, x 5 actually and the contenders to leave are x 3, x 4, x 5.

So if x 2 is entering it may be possible that anyone of x 3, x 4, x 5 can be leaving. If x 1 is entering it may be possible that x 3, x 4, x 5 is leaving, so we will check the validity of that statement. So as this is a maximization problem then make a decision which one will be the entering variable as that it is the largest non negative coefficient. Again I am repeating I have reported it so if you consider the largest this largest non negative coefficient in the sense one additional unit if x 1 being produced or one additional unit of x 2 being produced will increase your objective function by units of either 3 or 5 that means z will now become z initial is 0 plus 3 or 0 plus 5 depending on 1 unit of x 1 or x 2coming to the picture.

Now you name the column corresponding to this entering variable as the pivot variable so they would be a entering variable and they would be a existing variable and we know the common element to that column and row would be the pivot element based on which you will do the calculations. Now we will choose x 2 the reason is very simple because addition of x 2 would be increase your objective function by 5 units. while addition of x 1 will increase your objective function by 3 units. Because it is a maximization problem so the x 2 should be the entering one.

Now let us see who will leave because x 2 if it comes it will push out one. So let us see who is that one. Now what we do is that if 5 due to plus 5 addition for x 2 if this is going to add so let us check the rules so if the variable enters one has to leave which is true we now have to decide which variable will leave. Now we will calculate the ratios, what are the ratios? Ratios would basically be per unit addition or subtraction how the objective function is changing whether is a maximization or minimization problem. So let us read it.

Calculate the ratios, what are the ratios? One ratio is right hand side we have the factors so it is 4 by 0 because in case if it x 2 is coming and x 3 is going for pertaining to equation 1 the overall loss which will you face loss and profit depending upon how you looking in the problem the maximization problem or minimization problem would be 4 by 0 for x 3. Loss or profit which will you pertain for per unit of leaving of x 4 will be 12 by 6 which is 6 and 1 unit of leaving either not for x 4 but for x 5 would basically have a effect of 18 by 2.

So let us read calculate the ratios of 4 by 0 not defined hence they would be left out 12 by 2 will be 6 for x 4 and 18 by 2 would be 9 for x 5. So choose the which has the ratio is the smallest because why it is smallest we will consider for the maximization problem number one who is entering would give you the maximum benefit and number 2 who is leaving would basically not being affecting your objective function to that possible set.

Because if one variable is leaving obviously the technically objective function would coming down but that reduction in the objective function if it is a maximization problem should not happen to that possible extent so you will take the minimum value and the one who is entering would basically give the maximum credit to the objective function hence it increase would be maximum that is why x 2 and leaving one would basically be the one for which is the lowest one. So column marked is basically the pivot column corresponding to basically the value of x 2and you will decide the corresponding pivot element accordingly.

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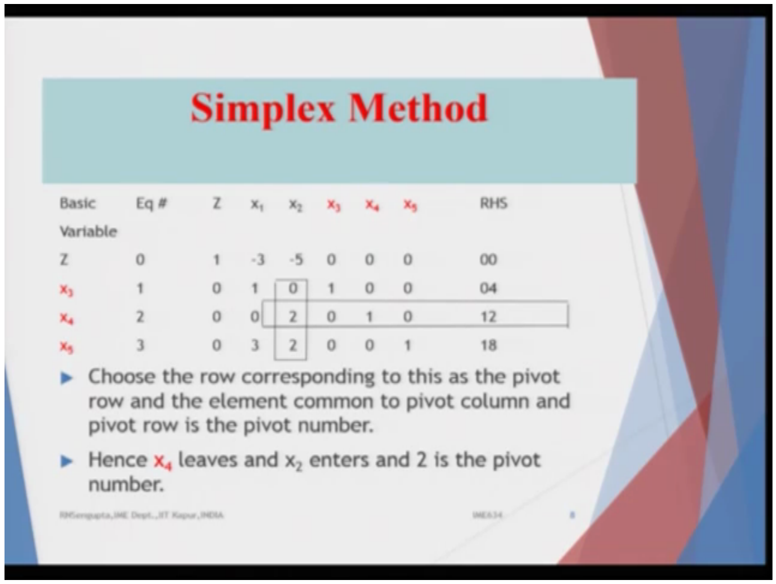


Now we will consider the value because 4 by 0 is undefined  so we will basically take the value if 12 by 2 which is 6 that means 1 unit of x 3, x 4 leaving would decrease technically decrease your objective function by 6 units. 1 unit decrease of x 5 would basically decrease your objective function by nine units. So we will take the first one choose the row corresponding to this as the pivot row and the element common to the pivot column and pivot row which is true the pivot element based on which we will start the calculation.

Which means based on that we will start the calculation to find out that how we are going to change and utilize the Gauss Jordan method to convert that initial basic feasible solution in the final solution. Hence x 4 will leave which is given as red and x 2 will enter and 2 is the pivot element based on which we will do the calculation.

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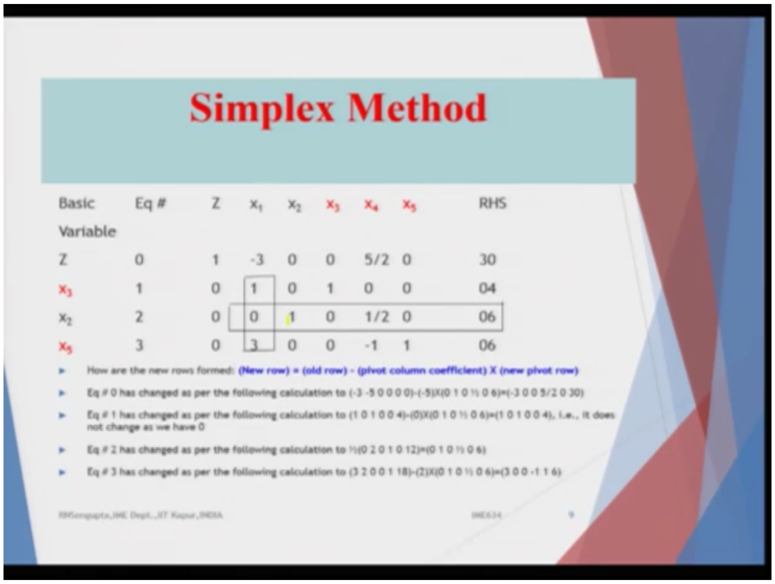


So once this is done and if x 2 comes what we will actually do is that we will basically aim the elements multiplication and addition in such a way that we will slowly convert pivot element corresponding to x 2 because x 2 has entered and x 2 is also there in the row so the column and the row the actual value would be 1 because in the end of the day the whole row corresponding to x 2 should have only one element which is one and the right hand value which you will have at the right most column would you the actual value of x 2 based on which you will do the calculations.

So let us see, so how are the new rows formed, the new rows form would be the old row minus the pivot column coefficient into the new pivot row. So if you are considering that concept so the pivot element is 2 so that 2 would definitely be utilized in that calculation so that the common element which I mentioned few minutes back the common element which is there between x 2 as a column and x 2 as a row should be one.

So let me consider so equation one has changed as per the following calculation based on this fact. Initially it was minus 3, minus 5 corresponding to x 1 and x 2 and all x 3, x 4, x 5 was 0 so it is there x 3, x 4, x 5 and last one was basically the objective function which was initially 0. So this element pivot column coefficient was minus 5 there because that was basically the entering one and the new pivot row which you would have technically initially if you consider let us consider this.

When you convert it when you have basically divided and convert it would basically be 0 corresponding to x 1, 1 corresponding to x 2 which I just mentioned 0 corresponding to x 3, half corresponding to x 4 and 6 corresponding to x 5. So when you do this vector multiplication so the actual value which will come out in equation one would be minus 3 for x 1, 0 for x 2, 0 for x 3, 5 by 2 for x 4, 0 for x 5 and the right hand side value has now become 30 which means the objective function has increased from unit 0 to 30.

So if you stop here let us stop it will give you an indication, so if we just pause here let us have a look I will read the equation 1, 2, 3. I will read the equation 1, 2, 3 first and then I will come to what I was going to explain. So equation one has changed as per the following calculation because the element based on which you will multiply 0 because that entering, exiting variables, pivot element is being converted so it will be 1, 0, 1, 0, 0, 4 and as per the new would the old column which you had, old row which you had and the element which is multiplies 0 hence the actual value will remain because you are multiplying by 0 remains the same it becomes 1, 0, one, 0, 0, 4 that it does not change any value because you are multiplying it by 0.

Similarly equation 2 would be as has been changed as per the calculation because you are now dividing by 2 only so divide by 2 would basically make it as 0 1 so this one is very important to note. What I am trying to basically I am trying to convert that actual bring that actual value as a, as a identity row or the column in the actual calculation when it ends and the third equation changes according to the same concept by new row is equal to the old row minus the pivot column element in the new pivot row and values comes out to be 3, 0, 0 minus 1, 1 and 6.

Now what I was going to repeat let me come back now if the calculations suddenly stops there you do not want to replace it so you will try to check that this value of 30 in the objective function how did it come. Now let us go back to this calculation now let us check that what is the actual identity matrix which you have. So Identity matrix now which you have looks very interesting. So it is 1 unit in factory 1, plant 1, 1 unit of if x 3 is only being produced x 1 and x 2 are not there because x 2 was not being produced in plant 1.

So obviously the total slack of x 3 always remains 4 now if I go to plant 2 now plant 2 does not have any x 1 which is fine plant 2 does not have any value corresponding to this, this of x 3 is also been not there and the value which is basically corresponding to x 4 even the parameter is half but the actual value of x 4 is 0 so obviously half into 0 would basically give you a 0 value and x 5 is basically 0 in plant 2 because it is not a slack for the second constraint.

So actual value which will get in x 2 would be one into x 2 is equal to 6. So that will give you the value which is corresponding to, to 6 units of x 2 is being produced. Now pause here what was per unit profit which is coming by 1 unit of x 2 it was 5 so 5 into 6 is your objective function value which is 30 here. Finally x 5 if we consider your getting so x 1 is not there so x 1 value is 0 ,so 3 into 0 is 0 x 2 is in plant 2 obviously you will see that the plant 2 does not have any value which is being produced for the value of 2.

And x 3 value is 0 and x 4 value is minus 1 but the actual value which will have for the slack corresponding to plant 3 is x 5 which is 6. So with this I will explain this in the next steps and come with the dual problem also and then see that how they merge and they give you the answer I will continue in the 22nd lecture the discussion about the primal dual problem in the simplex method and it can be increased to any higher dimensions also. Have a nice day and thank you very much.