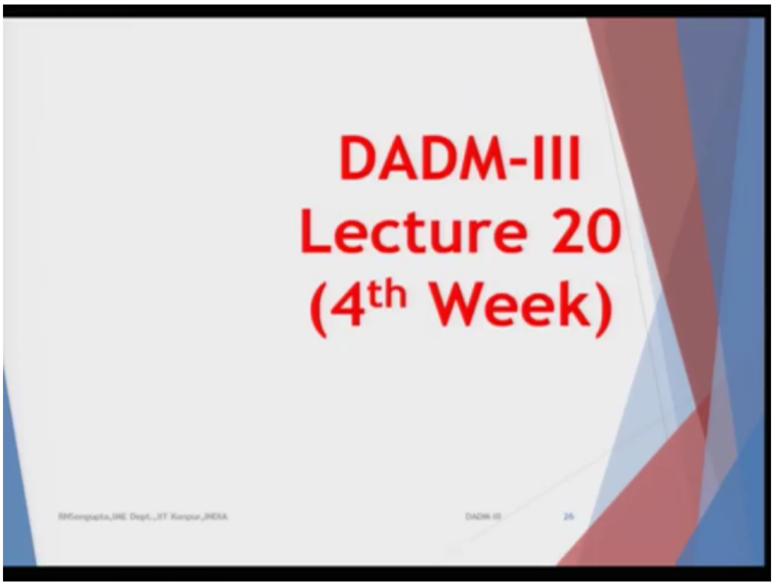
Data Analysis and Decision Making-3
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology Kanpur
Lecture 20

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you, wherever you are in this part of the globe.

(Refer Slide Time 00:26)



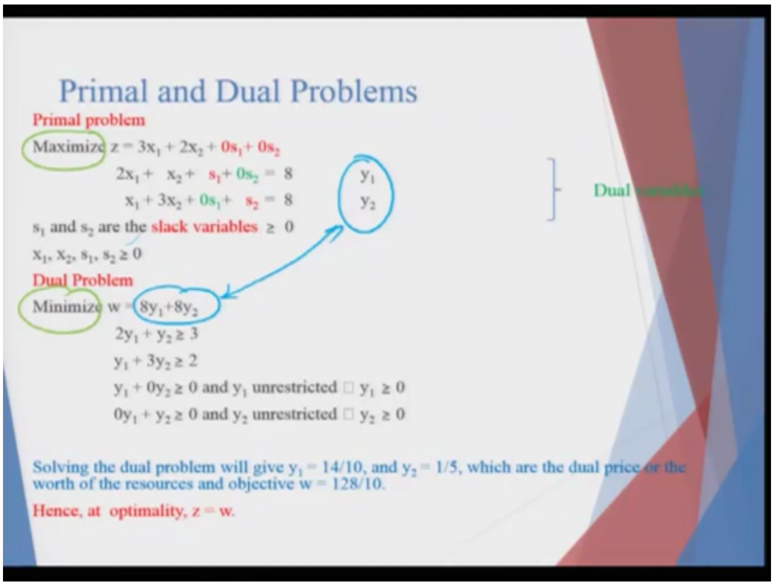
And as you know this is the DADM-3 which is Data Analysis and Decision Making-3 course under the NPTEL mock series and this total course duration is 12 weeks which we when converted into number of hours, contact hours is 30 hours in total and the total number of lectures is 16 which means that each lecture is for half an hour. And after each week which, each week being for 5 lectures we of half an hour each we have an assignment.

And you have already completed 3 assignments as you can see from the slide which is the lecture number 20 which is the 4th week ending of the 4th week, we will wrap up the 4th week and obviously we will take the 4th assignment, so obviously you have already completed 3 assignments now you will do the 4th assignment and in total you have basically 12 assignments and after the 12 assignment and the course ends, you will basically have one of the final examination. And my good name is Raghu Nandan Sengupta from the IME department at IIT, Kanpur.

So if you remember we were discussing about the primal dual problem which is more of conceptual basis that is a intuitive part where you are trying to basically solve a primal problem. So primal, dual are words so the dual can also be said a primal and its counterpart would basically be then the dual one. So if I say the primal one is the dual then the dual one basically becomes the primal, in the sense the primary problem and is opposite one which just the mirror image what we are going to consider.

We consider that maximization becomes the minimization and vice-versa greater than sign and less than sign also comes into the picture, in case if (X1), X the vectors are all greater than 0, then they become unrestricted and so and so forth and I will come to the conditions with this example which we are just discussing in the (())(02:10) of the last class which is the 19th lecture and also we will consider the number constraints or number of variables which were there in the primal one would just be the counterpart in the sense, number of variables now become the number of constraints and number of constraints now becomes the number of variables.

(Refer Slide Time: 02:34)



So if you consider this problem where I was just discussing the primal problem, it was a maximization problem so obviously the intuitive the dual part would basically minimization problem, so let me draw it, draw it means just note down how the relation, so the maximization problem becomes the minimization problem in the dual problem. Now if you consider the maximization problem or the primal problem, you basically have technically you have initially 2 variables x1 and x2 and then later on as we added or subtracted the slack or the surplus, it can be added or subtracted depending on whether it is a greater than sign or less than sign.

We basically had S1 and S2 hence the equality sign was there for the constraints. Now what we will do is that, as there are 2 constraints will consider for the dual problem the decision variables as y1, y2 corresponding to the 2 constraints which I have, point 1. Point number 2 is that, if we consider the total number of such decision variables which are there, in totality for the primal problem there are 4 decision variables because we will consider x1, we will consider x2 and we will also consider S1 and S2 which are the slacks and the surplus.

So technically, now you will basically have 2 decision variables in the dual and 4 constraints. So let us check how they are, so we all know the S1 and S2 are the slacks or they are greater than 0, if they were surplus obviously they would be less than 0 but we can convert them with the minus sign to be greater than 0. So in totality again I am repeating we will have (S) x1, x2, s1, s2 as greater than 0, now y1 and y2 are the dual variables.

So if you consider the dual variables, they would come here. Now how the conversion is done? So if the dual variables are y1, y2 the right hand side of the constraint which is 8, 8 would basically be the parameters corresponding to the dual variables y1, y2 so if they were 3 constraints, they would be consider the right hand side of the equations were 8, 8 and 5 then the dual variables would be y1, y2, y3 and the corresponding objective function in the dual problem would be 8 y1 plus 8 y2 plus 5 y3, so 5 y3 I just consider that is why I am saying. But in this problem the objective function would now become 8 y1 plus 8 y2 as it is written there, in the minimization problem which is the dual problem.

Now let us go back to the constraints, now as there are 4 different decision variables they would be 4 constraints so let us first count whether the constraints numbers are right, so you have the first constraint in the dual problem, the second constraint, the third constraint and the forth constraint. Now how are the constraints formulated? So let us concentrate on the parameters or the variables corresponding to A in the primal problem, so what are the variables? 2 and 1 for x1, 1 and 3 for x2, 1 and 0 for s1, 0 and 1 for the s2, so let us go one by one.

So the first problem which is and obviously (let us) if I go back to the maximization problem which is the objective function of the primal problem, the parameters are 3 for x1, 2 for x2, 0 for s1 and 0 for s2. So let us take all the x’s, so x1, so x1 would basically be the problem which is 2 into y1 plus 1 into y2 and there is 3 on the right hand side, but which variable it will be? I will be basically a greater than type depending on the conditions which we have put.

So the first constraint becomes 2 y1 plus 1 y2 is greater than 3, the second one would be 1 y1 plus 3 y2 is greater than 2, the third constraint would basically be 1 into y1 plus 0 into y2 because the parameter value multiplying s1 for the second constraint is 0, so it will be 1 y1 plus 0 into y2 is greater than equal to 0 because this is 0 here and the last constraint which is the fourth one for the dual problem would be 0 into y1 plus 1 into y2 is greater than 0.

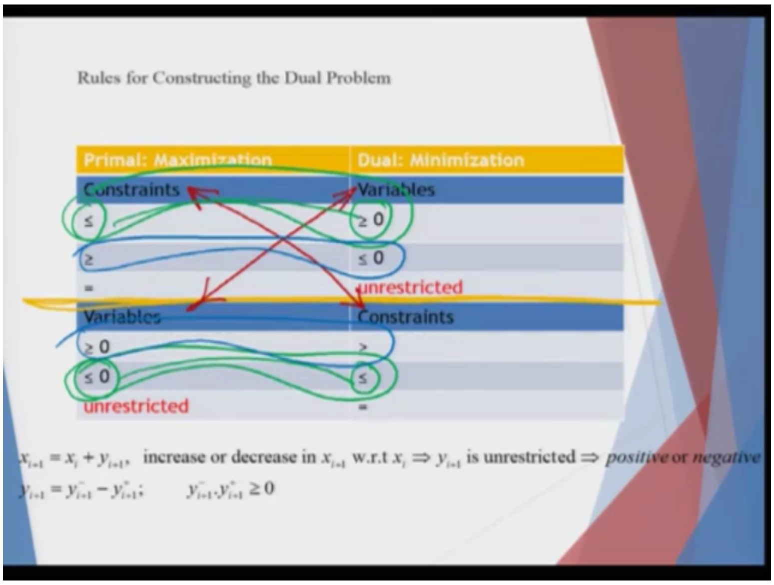
So let us see them one by one, the first one is 2 y1 plus y2 is greater than 3, second one is y1 plus 3 y2 is greater than 2, third one is basically y1 plus 0 into y2 is greater than 0 which is right and the last one is the fourth constraint in the dual problem would be 0 into y1 plus 1 into y2 is greater than 0 and y1, y2 are unrestricted and the values are given accordingly.

So solving the dual problem would give us a value of y1 as 14 by 10 and y2 as 1 by5 which are the dual prices or the worth of the resources in the objective function would be same as it is 128 by 10, if you remember the problem when you have solved the value it was coming out to be 12.8. So the maximization problem would also lead to the same problem in the minimization problem when you considering the primal and dual.

But the variables which we will have basically y1 as 14 by 10 and y2 as 1 by 5 would basically be the dual or the worth of the resources which would be basically corresponding to the fact of s1 and s2, which will come later on when we solve a primal dual problem accordingly. Hence, at optimality remember that as we approach the mirror, the z which is the objective function which is the maximization problem and w which is the objective function in the minimization problem would both merge and the value should be exactly be the same as it is 128 by 10 for both the problems.

So, let us put the rules for constructing the primal dual problem, so primal dual are the words a maximization problem can be the primal one, the dual can be minimization problem or the primal can be the minimization problem, the dual can be maximization problem.

(Refer Slide Time 08:49)



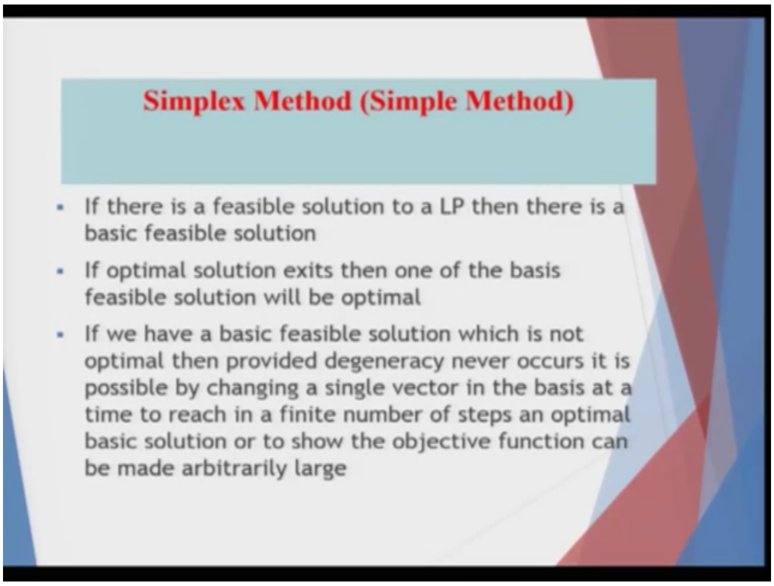
So in the primal problem it is maximization problem, the constraints equals to the variables and the variables becomes the constraints as we saw, that means x and y have to basically be differentiated and the general nomenclature is the primal variables are given by x, x the vectors x, x1, x2, x3, x4 so and so forth. While in the dual problem, the variables are given by y, y1, y2, y3, y4. Now remember one thing, I did mention about m and n, so in the primal problem if the size of the matrix is m cross n that the size of the matrix in the dual problem just reverses it becomes n cross m because the number of duals, numbers of constraints and number of variables are just interchanged numbers if you remember.

So if the constraints are less than type, they become in the dual problem they become greater than equal to 0, if the constraints of greater than type they become basically less than equal to 0 and for equality signs for the constraints they become just unrestricted depending on how you have been able to formulate the problems. While in the primal problem the variables as I mentioned just few minutes back becomes the constraint for greater than type so if you see which is very interesting, so this part which you have drawn, this or written this whole thing which you have written, they are just the mirror image with each other. I will just write it down one by one, use the color brown.

So the variables becomes the constraints here, the variables, variables are for the primal dual if you see here, if you see the constraint of less than type, constraint of less than type, they actually becomes greater than 0, they actually becomes less than 0. So these are the same, now if they are of the greater than type, it becomes less than 0, greater than type becomes, so greater than type here becomes less than 0 in both the cases and if it is equality sign it becomes the unrestricted problem.

Now what we are considering in this problem is basically just a simple way of (solution) finding a solution for a maximization on the minimization problem, in the least effective way considering that a sparse matrix because if you consider A is a matrix so if the number of rows or the number of columns are very large in any one of the cases we will be tempted to convert that into a dual problem such that solving that problem becomes very easy from the computational point of. Now I will just go to very basic theory and discuss the primal basic feasible solution and then come back to the primal dual problems again.

(Refer Slide Time 12:05)



So if there is a feasible solution to an LP then there is a basic feasible solution based on which you can start, so if you are sure that there are set of feasible solutions so one of them have to be the basic feasible solution based on which we can start of a iteration (process). It does not mention any thing about the in feasible solution I will come to that within few minutes.

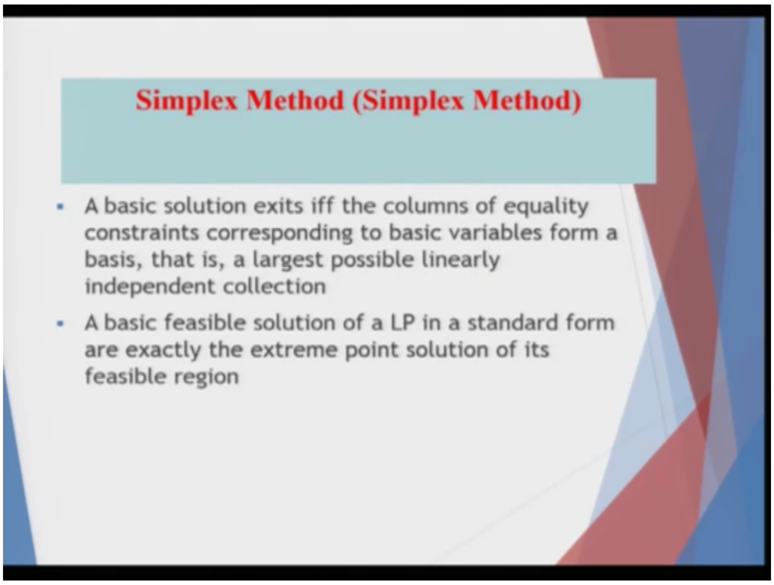
If the (optimum) optimal solution where you are trying to get the maxima or the minima exist, then one of the basic feasible solution has to be an optimal one, whether the maximization problem or the minimization problem, so what you are trying to do is that, in very simple words you will restrict your search algorithm to only the corner points which are basic feasible solution and one of them have to be optimum, if there is a unique solution or two or more of them can be optimal depending on the number of solution which you have can be more than one.

If we have a basic feasible solution which is not optimal then provided degeneracy does not occur, degeneracy that means you keep changing the variables of the entering and the exiting one they basically go into a circular loop and the objective function keeps increasing or decreasing in the infinite fashion, arbitrarily large or arbitrarily small depending on objective functioning you want to maximize or minimize.

So let me read it, if you have a basic feasible solution which is not optimal, then provided degeneracy never occurs it is possible by changing a single vectors in the basis at a time to reach the finite (number) in time to reach the optimum solution in a finite number of steps which means the number of this corner points can be very large, but if you consider any basic feasible solution not the infeasible solution, basic feasible solution, you can keep moving in one direction depending on the maximum on the minimization problem such that you are bound to reach the optimum basic feasible solution depending on the problem structure which is there.

Now main question is that in which direction which will move? Which we have already discussed. So it reads that in the basis, if the single vector in the basis at a time to reach a finite number of steps and optimum basic solution is obtained or we can show the objective function can be made arbitrary large depending on if it is an unbounded solution.

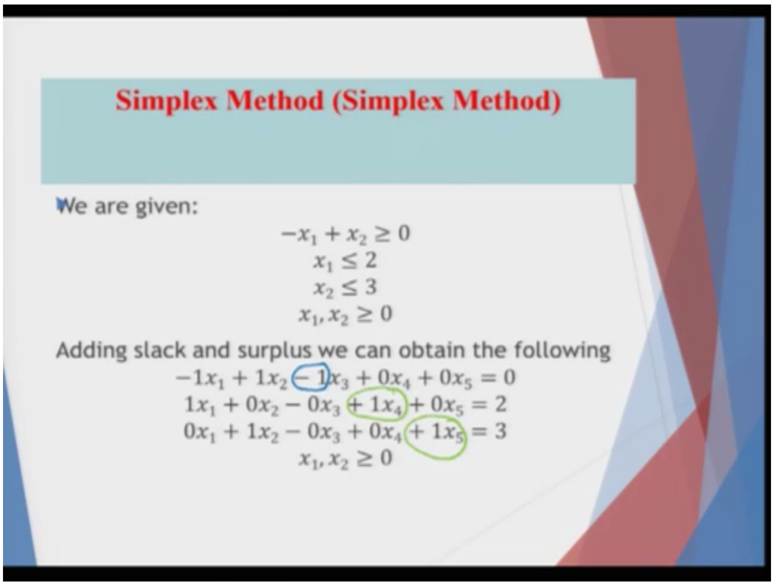
(Refer Slide Time 14:28)



The next point is that a basic solution exists if and only if the columns of equality constraints corresponding to a basic variable forms a basis that is a largest possible linearly independent collection of vectors are there such that it gives us the basic feasible solution. So, if you have a set of basic feasible solution, it would mean the basic feasible solution are such that the convex combination of this basic feasible solution would lead to (you) true the optimum solution based on which you can reach the maxima or the minima.

A basic feasible solution which is the last point, a basic feasible solution of an LP in a standard format are exactly the extreme point solution of the feasible region as I have mentioned that trying to basically move along the corner points from the basic feasible solution would definitely lead to the optimum solution which is also a part and parcel or an element of the basic feasible solution.

(Refer Slide Time 15:31)



Now let us consider the simple problem, problems in a very simple manner, so considered we have the objective function and we have the constraint also, the constraints are like this minus x1 plus x2 is greater than 0, that means we have to basically add or subtract a slack and surplus depending on the sign of the equality sign, x1 is less than 2, x2 is less than 3, and obviously x1 and x2 are greater than 0 as per the nomenclature or the assumption for linear programing.

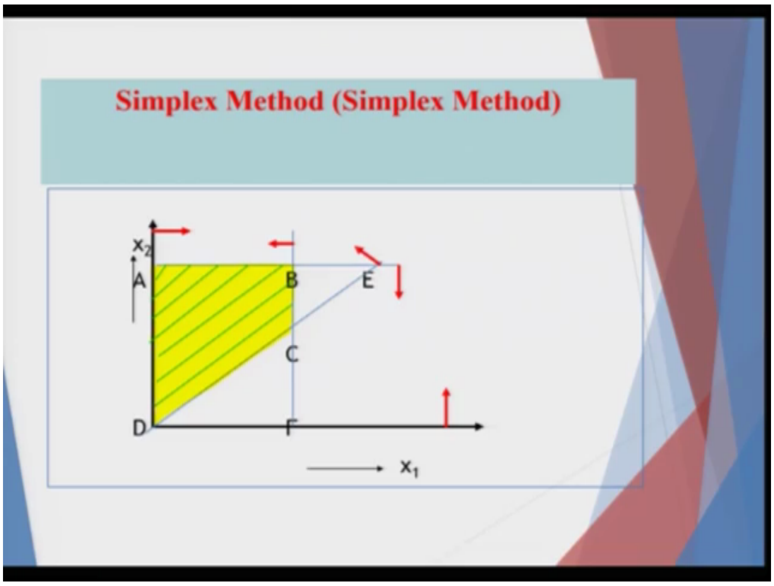
Adding slack and surplus we can convert the following constraints into this way, the first equation becomes that as we are so, if it is greater than 0, that means we have to bring down that constraint on the left hand side such that is exactly equal to 0 that means we will be adding or subtracting the slacks depending on the sign which I mentioning in time and again, so the constraint basically becomes minus 1 x1, which was already there plus 1 x2 minus x3 because we are trying to decrease their left hand side of the equation and we are also adding 0 quantum of x4 and x5 depending on the slack and surplus which are there for the second and the third constraint, which we will see that later very simply.

So on the right hand side it is 0, the second constraint obviously you will basically have 1 into x1, plus 0 into x2 because x2 is not there obviously it will multiplied by 0, plus 0 into x3 or minus 0 into x3 depending (on) ok now the question is that, whether you plus or minus that does not matter it is plus 0 into x3 plus 1 into x4 depending on the variable which we have, which is basically slack or surplus for the third constraint.

And on the second constraint and the last variable for the second constraint would be x5 corresponding to the slack or surplus which will get in the third constraints. So equality sign on the right hand side is 2, now remember one thing, if it is greater than sign obviously it is minus, if it is less than sign, obviously this is plus.

So when I go to the third constraint, the equation would be, there is no x1, it will be 0 into x1 plus 1 into x2, x2 is already there plus x3 is not there, x3 was basically corresponding to the first constraint, x4 is corresponding to the second constraint, so it will be 0 into x3 plus 0 into x4 plus 1 into x5 because this is a less than type, so it will be added equal to 3 on the right hand, in the right hand side.

(Refer Slide Time 18:15)

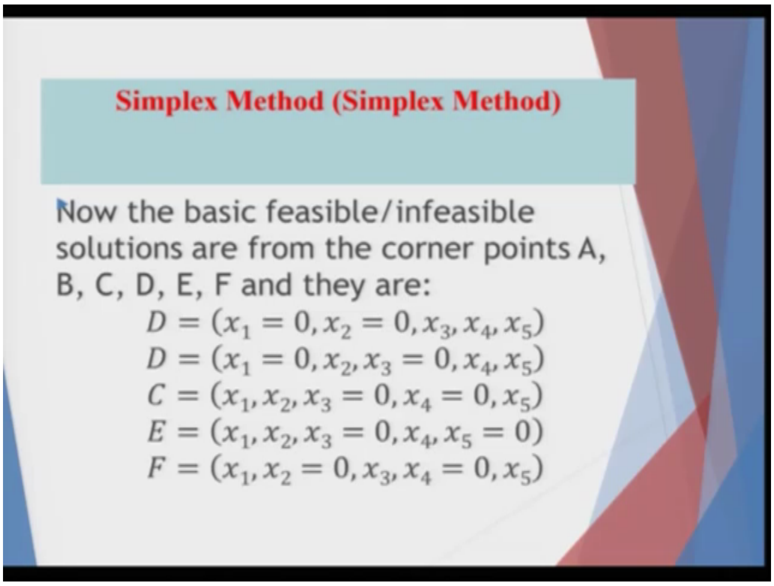


So now the equation if you solve the constraints, the overall feasible region is now I will go slow one by one, the overall feasible region if we draw x1 along the x axis, x2 along the y axis corresponding to the fact that we are considering a 2-dimensional problem in initial case obviously it will change to a multi-dimensional problem depending on how we have been able to add s1, s2, s3, s4 or x1, x2, x3, x4 that is the slack and the surplus.

So the constraint regions are, the feasible region is A, B, C and D if you solve them you will get it, but very interestingly thing is that the point E and F are basic but they are infeasible solutions because the values of the slack and the surplus at the point E and F are negative, I will come to that later on, so if you consider the overall boundary right hand side of y-axis which is basically the first and the third and the fourth quadrant above portion of the x-axis which is basically the first and the second quadrant and the corresponding region of the other (2), other 3 constraints, one is on to the (right) left, one is on to the top, one is to the bottom.

So if you consider the overall region, the region feasible region is this, so what I have hashed I am trying to basically mark it accordingly, so I am basically marking it so there is no confusion. So this is the feasible region and the corner points A, B, D, C or A, B, C, D if I go clock wise are the basic feasible region or feasible points sorry and the point E and F are basic but they are infeasible point, I will come to that within 2 minutes.

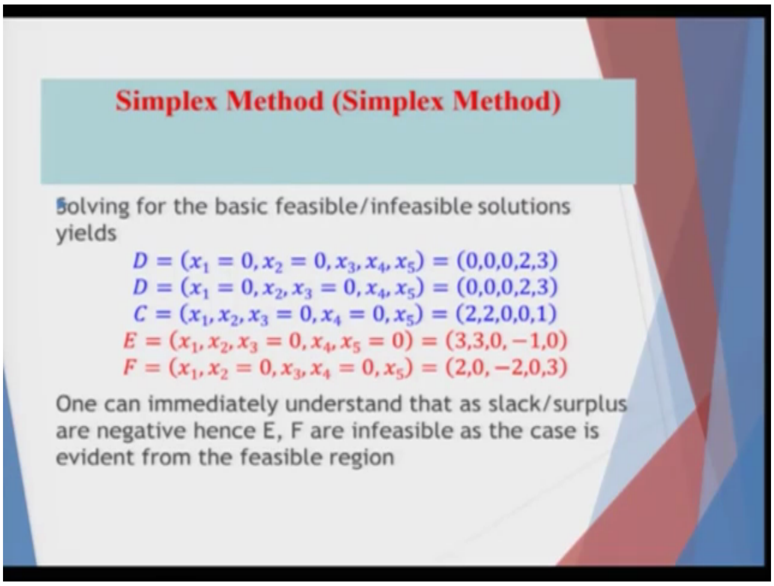
(Refer Slide Time 20:45)



So now the basic feasible or infeasible regions or the solutions are the corner points which are mark, which is A, B, C, D, E, and F so when i mark them the corresponding values which I have I am marking it very carefully so, the basic points are if I consider D, C, E, and F the corresponding values are for D it is x1 and x2 are given for D it is 0, and x3, x4, and x5 would have basically the characteristic depending on how we have been able to solve the problem.

So if you are considering the corner points which is origin where you put x1, x2 as 0, and solve the problem to find out what is the feasible point, whether is(())(21:27) the feasible point or infeasible point and but it will be basic infeasible, feasible region depending on how you have been able to formulate the problem. the point C would basically have x3 and x4 as 0 and x1, x2, and x5 would be depending on how the problem has been formulated. The point E would basically have x3 and x5 as 0 while x1, x2 and x4 would basically take the points, how we have been able to solve and finally the point F would have basically x2 and x4 as 0 while x1, x3 and x5 would take the points depending on how we have been able to solve the problem accordingly.

(Refer Slide Time 22:11)



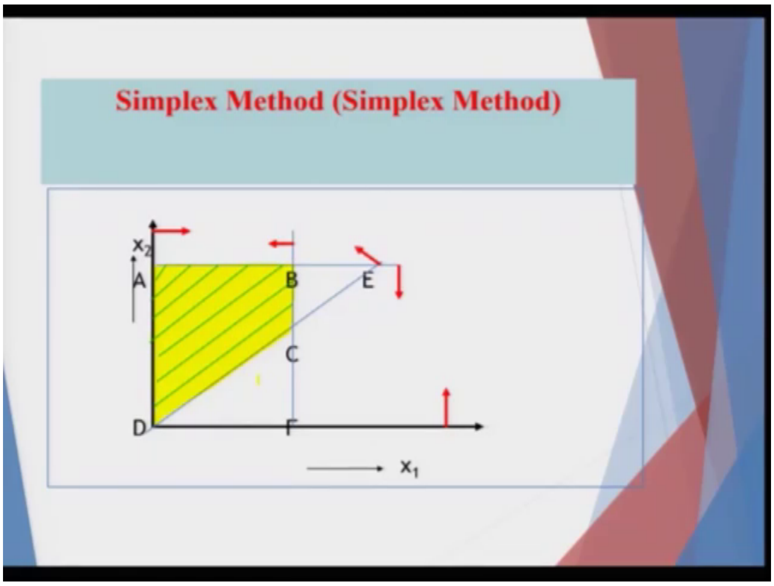
Now when I solve them, solving does not mean its a linear programing you just put this equation solve the simultaneous equation and get the points. So solving for the basic feasible and infeasible region I divide into 2 color scheme, blue one are the feasible region, feasible point sorry and red one are the infeasible region I will mention them both of them as basic feasible, basic infeasible. The basic feasible points are like this D would basically have x1, x2, x3 as 0, 0, 0 and the slack or the surplus corresponding to the second and the third constraint which is x4 and x5 would basically have the values of 2 and 3.

Now pause here 1 minute, 2 and 3 are positive hence they are basic feasible points from where we can (task) start the search. Let us check at C, for the value of C x1, x2 is 2 and 2 respectively while the value of x3, x4 and x5 is 0, 0, 1 so if I consider the slack and the surplus as amongst them is x3 is 0, x4 is 0 and x5 is 1, which means this is also a basic feasible solution based on we check and start the search. From where I start the search that will is a different question.

Now when I consider the point E and F and I solve them, the values which are coming out for x1, x2, x3, x4, x5 for E, R respectively like this 3, 3, 0 minus 1 and 0. So here I pause the values of x1 and x2 are fine, they are 3 and 3 which is perfect, but if I look at the values of x3, x4 and x5, x3 and x5 are 0, which is also fine but the value of x4 is minus 1 hence it is not a basic feasible solution its a basic infeasible solution or in basic infeasible point.

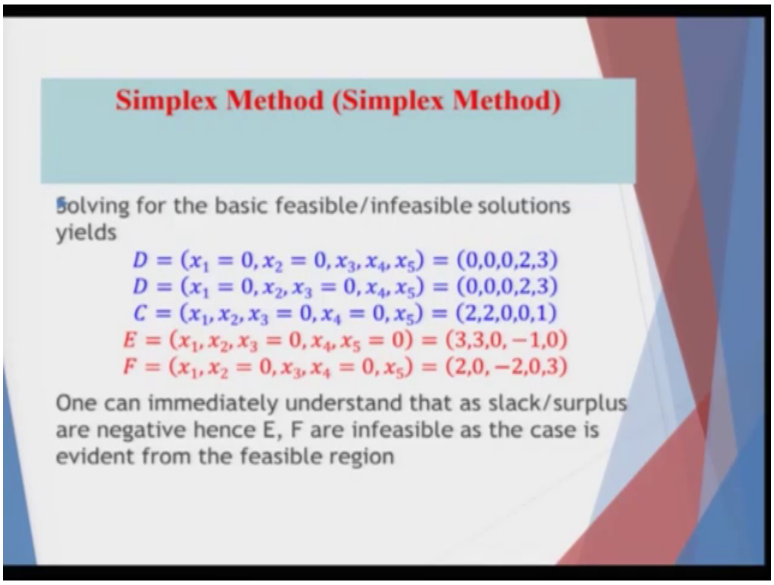
Now when I look at the point F, the points are for x1, x2, x3, x4 and x5 are respectively as 2 0 minus 2 0 and 3, so if I consider the value of x1 and x2 they are 2 and 0 which is perfect, but if I consider the value of x3 it is minus 2 while x1 and x5 are 0 and 3 which is fine but as x3 is minus 2 hence it is a basic infeasible solution. Ok again I am repeating, the values which I have got is just simply solve the equation one at a time and then get them, that is all.

(Refer Slide Time: 24:51)



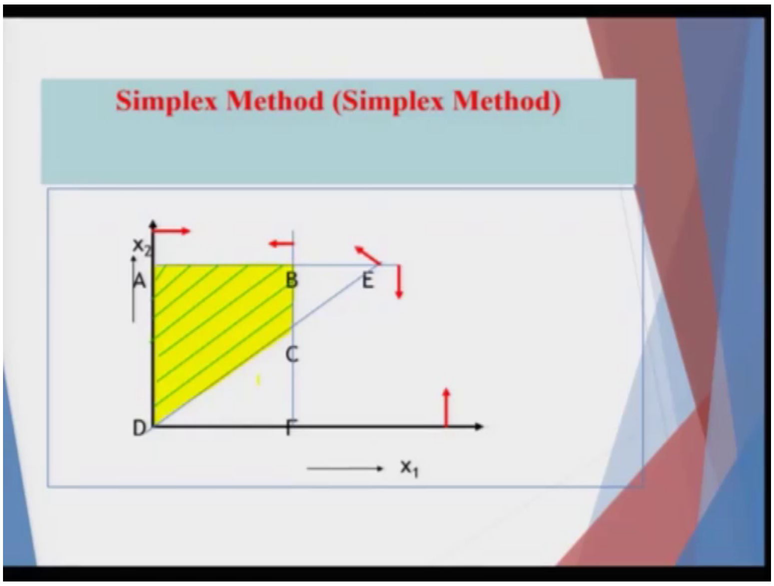
So the values which I have for A, B, C, D are AB is not given.

(Refer Slide Time: 24:57)



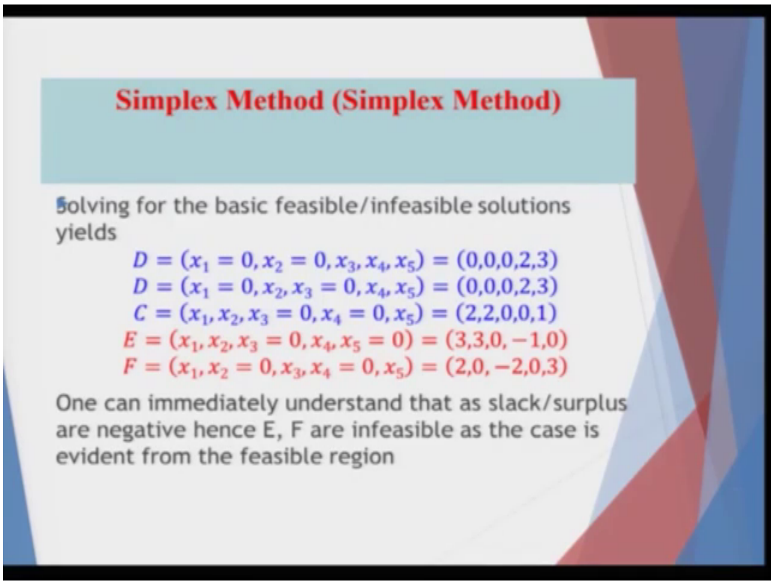
CD points are given which have colored blue hence they are basic feasible solution, if I consider E and F.

(Refer Slide Time: 25:06)



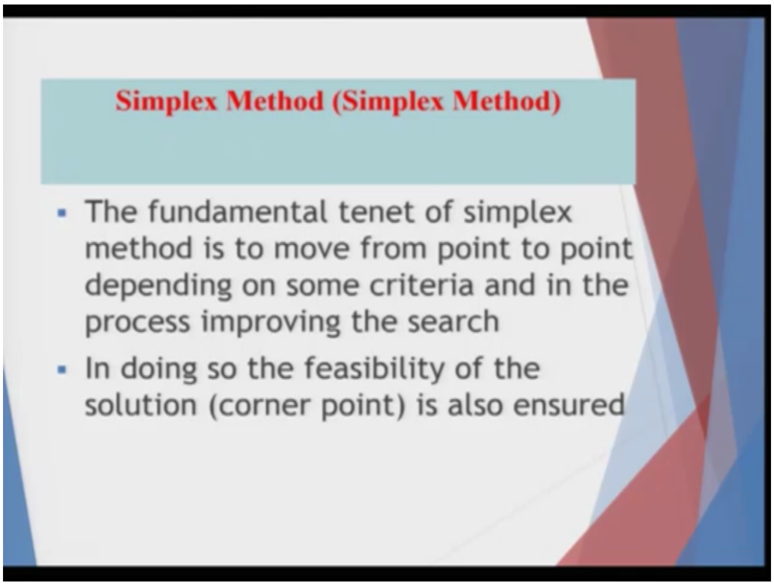
E and F are here and here if as they are basic infeasible solution from the diagram also is very clear that the yellow region which is the feasible region does not have the point E and F. So hence starting our algorithm from point E and F is not at all advisable.

(Refer Slide Time: 25:25)



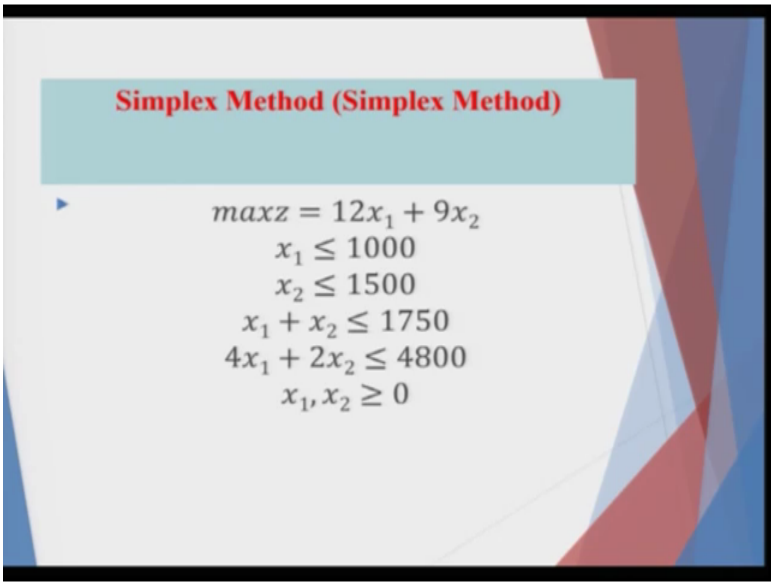
So one can immediately understand that as slack and surplus are negative hence E and F are infeasible as the case is evident from the feasible region diagram which as shown.

(Refer Slide Time 25:35)



The fundamental tenet of simplex method is to move from point to point which I have already mentioned, depending on some criteria and the process improving the search as we move from the corner points and trying to basically maximize and minimize the objective function, we will consider z as the maximization objective function and w as the minimization objective function. In doing so, the (feasible) feasibility of the solution or the corner points is also ensured because if you are moving from feasible region to feasible region which means that they are the corner points of the feasible set. We are not going into the infeasible region or infeasible points that is very important to understand.

(Refer Slide Time 26:16)



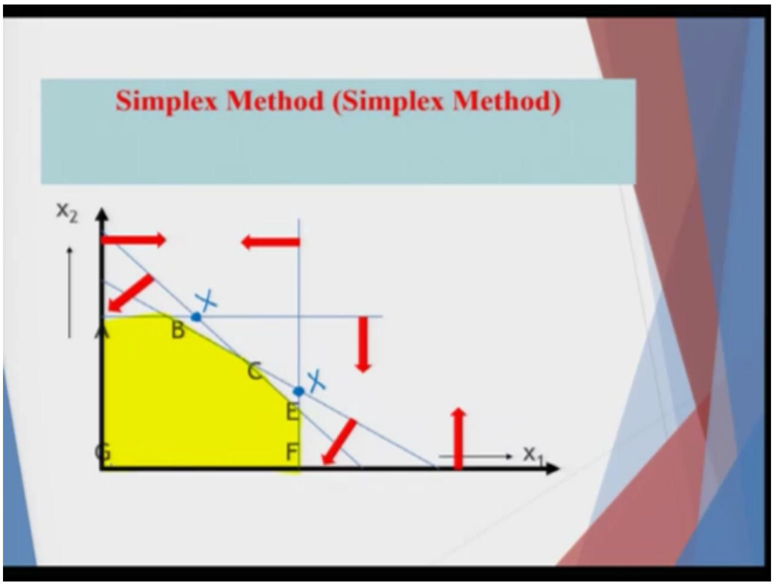
So consider this problem we have the (max), the simplex problem where you have 12 x1 plus 9 x2 and you want to maximize it, the constraints are correspondingly given as x1 is less than 1000, x2 is less than 1500, x1 and x1 plus x2 is less than equal to 1750 (1750) and 4 x1 plus 2 x2 is less than equal to 4800 and x1 and x2 are greater than 0, so now here pause 1 minute.

So before I go to in the next line you are now quite well acquainted that if I add some positive value to the first constraint that is x1 plus s1 it exactly becomes 2000, point 1. If I add s2, x2 plus s2 and it becomes exactly equal to (15,000) 1500. If I add a third variable s3, so x1 plus x2 plus x3 would exactly become to 1750 and if I add a fourth slack or the surplus depending on how the problem is being consider, but this is a less than sign so obviously you have to basically add the value such that it comes up to the value of 4800, so the actual equation now would become 4 x1 plus 2 x2 plus s4 is less than equal to 4800.

But as you are adding s1, s2, s3, s4 for constraint 1, constraint 2, constraint 3, constraint 4 the objective function would also change accordingly such that the objective function would now be 12 x1 plus 9 x2 plus 0 s1 plus 0 s2 plus 0 s3 plus 0 s4 as the maximization problem where the constraint would now become now I will show it within 2 minutes but listen to me carefully it will be x1 plus 0 x2 plus 0 s1 plus 0 s, not 0, s1 plus 1 s1 plus 0 s2 plus 0 s3 plus 0 s4 is equal to 1000.

Similarly, the next constraint which is a second one would become 0 x1 plus 1 s2 plus 0 s1 plus 1 s2 plus 0 s3 plus 0 s4 is equal to 1500. The third would one become 1 x1 plus 1 x2 plus 0 s1 plus 0 s2 plus 1 s3 plus 0 s4 is equal to 1750 and correspondingly the last one would become 4 x1 plus 2 x2 plus 0 s1 plus 0 s2 plus 0 s3 plus 1 s4 is equal to 4800 and the last set of variables which is very intuitive would be all of these x1's, x2, s1, s2, s3, s4 are all greater than 0.

(Refer Slide Time 29:22)



So let us show it, so the region is like this, your actual I will when I consider the solution the actual region is like this, your feasible region if you plot it, so this is the feasible region, your corner points which are basic feasible points are A, B, C, E, F, G now if I consider this point let me mark it with the not a red color let me mark it blue one, this point is a basic infeasible point, this point is also basic infeasible point.

So our search procedure would never start from these 2 basic infeasible point, they would be anyone of the point which is amongst A, B, C, E, F and G and we will consider that in details in the next class, have a nice day and thank you very much.