Data Analysis and Decision Making - 3
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Lecture 02

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the world.

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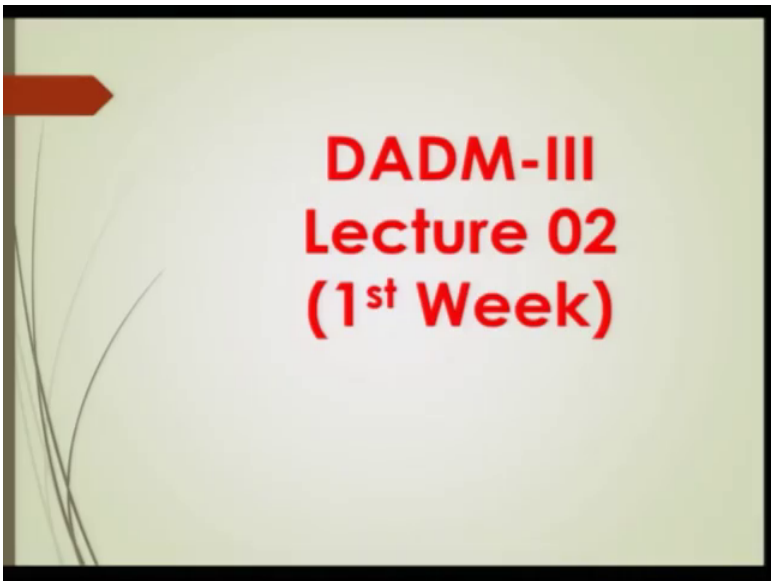

And as you know this is the second lecture in the first week for the DADM-3 which is Data Analysis and Decision Making and the main idea of this course is to basically cover the techniques and operation research all the gamut which I mentioned.

And as you know this course would be for 12 weeks, 60 lectures of total duration if you count them up, add them up is 30 hours and after each week we will be, in each week we will cover 5 lectures, after each lecture, each week of lectures we will basically have one assignment such that we will have 12 assignments and considering the 12 assignments at the end of the whole course we will have a final examination.

Now if you remember I was basically considering the assumptions of liner programming and I will giving you a feedback, so basically they were the properties and the assumption. So that slide I did not, I was not able to cover in the last few minutes of lecture 1, so I will again reiterate

that and basically further continue the discussion of linear programming. So, properties are basically as I mentioned other properties of liner programming which are inherently there and based on that we assume some assumptions and if you remember I had mentioned the assumptions of the Newton's law, I mentioned the assumptions of pendulum super hormonic motion of the pendulum, we do away with friction, we do awake with radiation of heat loss whatever it is and we still we get results in the theoretical sense which are closer to practicality and they give us a good idea.

So, linear programming is basically technique based on which we can get some ideas, is basically a tool and we are going to learn the different type of mathematical texting of the (loon) tool and if you remember I did mentioned in the first class that linear programming also has a lot of philosophical way how you model the problem, obviously that has to be taught, that has to be inculcated, that has to be brought up and sipped inside you through experience, so I will try to give some examples in different flavors they would be picked up but I will keep it minimum both would be from the point of view of two dimension one and the actually problem formulation what we use.

Now I will come to the property assumption in linear programming, so the first assumption is certainty. If you remember that I mentioned that certainty basically means the decision variables, the input parameters whatever. A certain that means they do not have any distribution, they are not probabilistic. This assumption would be basically done away with as we basically go into the area of stochastic programming, as we go into the area of robust optimization, as we go into the area of reliability optimization.

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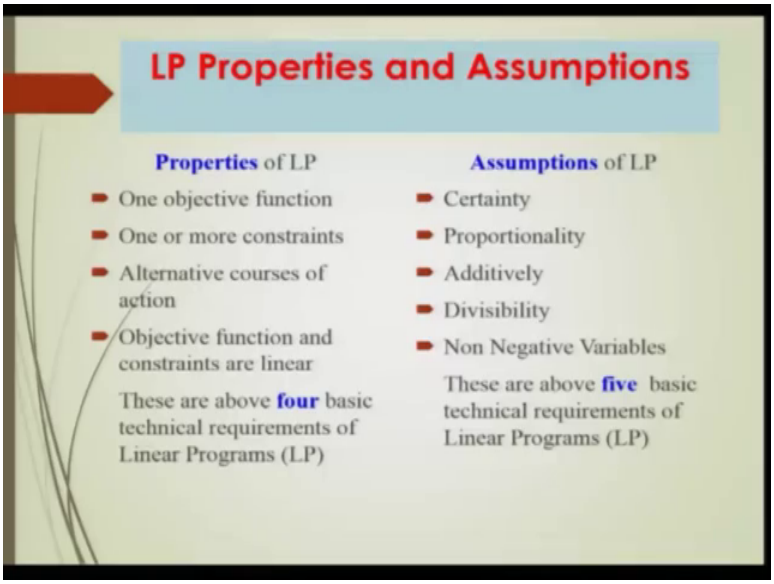

So relax that but for the linear programming very simplistically we will assume the property of certainty for all the (various) decision variables and the parameters. We will also consider the concept of proportionality that means the proportionally the decision variables can be broken down into minutest part as possible but now remember here this may not be possible in the practical sense. see for example I am trying to basically produce some Kgs or liters Kgs of some quantity maybe for say for example for fertilizer, maybe for some feed it to be utilized for cattle and in liters it can be the paints which are going to be produced, in liters output can be the pollutants.

Another example if I if you remember I had mentioned about the trucks being transported, so if you are travelling the total distance and you want to minimize the distance, minimize the cost the distance travelled can be to the minutest possible distance it can be in kilometers, it can be in deciliters (Deci) decameters, it can be centimeters, it can be millimeters so the distance can be found out accordingly depending on the problem. But now consider the problem is trying to utilize the minimum number of trucks, so the trucks technically can be 2 or 3 or 4 or 5, so technically they cannot be 2.3 number of trucks.

So we will consider the proportionality of concept to be there and this would definitely be utilized as a precursor when we consider the concept of this integer programming, integer programming means when you are trying to solve and find out the number of trucks for this

problem, so we will consider them to be not true in the case of the linear programming where proportionality would basically be utilized.

We will also consider the proportion mean the proportion and which there are and can be increased or decreased. Additivity we will basically consider in such a way that any addition or subtraction or addition of more products for the decision variable, more products for the parameters, more products basically for different constraints which are there, they can be added and subtracted in the same proportions considering the linearity holes. So say for example if we are trying to produce 10 liters of one paint it can be if it goes to 20 so the additivity would basically hold in the sense that the total amount of raw materials which we will try to utilize for producing that paint would be added up in the same way as you have done and proportionality would basically hold.

Divisibility which I just mentioned there was a switching between the proportionality and divisibility, divisibility would basically mean the decision variables can be divided to the minutest possible way such that finding of the results can give you the actual decision variables to the decimal points, to the fractional points, to whatever level it is required and proportionality would basically coming back to that proportionality factor, the proportionally increase and decrease of the decision variables, of the parameters would be considered accordingly. We will come to that as we solve the problems accordingly.

In the linear programming also we will consider initially the decision variable are non-negative, so obviously non-negativity would mean that we do not produce negative of some chairs, so it would be either zero or positive, so it can be 1 chair, 2 chair, 3 chair, 4 chair say for example you are a carpenter. If you are trying to produce liters so it cannot be any negative liters, so negative liters would not hold true so it can be basically 0 liters of production of paint 1 or 1 liters of production of paint 1 or 2 liters production of paint 1, depending on the problem which you are facing.

If you are trying to basically transport goods in a truck it can be say 1 truck or no truck or 2 trucks, 3 trucks, 4 trucks whatever it is. So the concept would be that we will consider the problems in the way that decision variables would be 0 or any positive number, negatives would not be there but obviously we will when we formulate the problems later on for different types of

supply chain management we can see that this concept can be brought down in the area of with negative variables also and we will solve it accordingly but give the flavor of that.

So these 5 basic (prob) technical requirements or assumptions of certainty, proportionality, additivity, divisibility, non-negative variables will be the crux of the assumption based on will solve the problems and the properties have already been discussed in the last few minutes of the first lecture and also I briefly mentioned in the beginning of the second lecture. So these would be the 4 basic technical requirements for properties and 5 different assumption based on which we will simply try to basically solve the problem for the linear programming as such.

So considering the basic assumptions of linear programming, so there was a properties on the assumptions so basically let us go into the depth of what we mean by the assumption.

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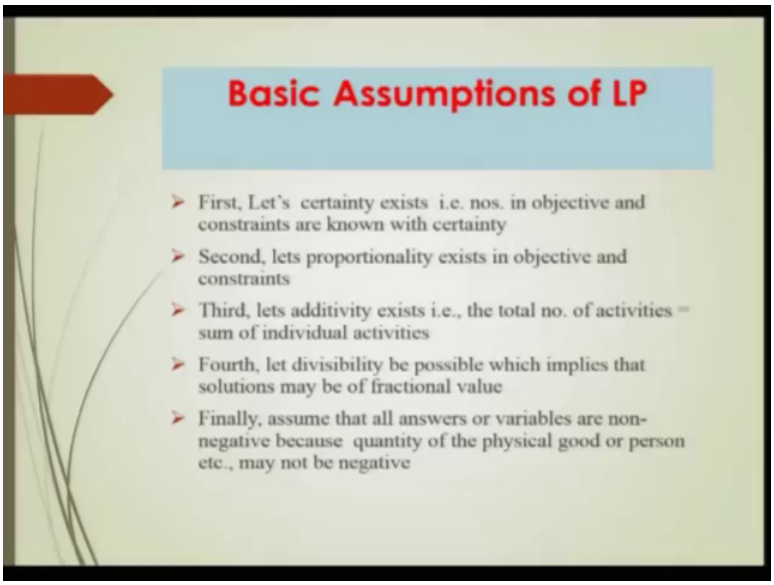

So first, let certainty exists so if certainty exists as I had mentioned that means it means that the variables which are there, decision variable, parameter which are there in the objective functions and the constraints are known with certainty that means the values may not be known but they are known that if I find out the result as 3, 3 number of trucks it would be 3, it cannot be say for example any other different value because if the this if non-deterministic concept was there obviously the numbers even if we find out can vary depending on the what is the variability of the dispersion,

If we are saying that we are going to produce say for example 3.5 thousand liters of paint 1, it would obviously mean that the values which are produced are sacrosanct such that any deviation would not come in to the picture because the decision variables are non-probabilistic. So second would basically will be proportionality concept which will be there in the object function and the constraints which I have already mentioned, so the proportion increase and decrease would be considered for both the objective function and the constraint depending on the variables which I found out.

So obviously if you remember there are different alternatives, so if there are different alternatives there would be different levels of decision variables, if there are different levels of decision variables it would mean the proportionality concept will be there both for the constraints as well as for the objective function in order to basically to keep in mind that (one) that assumptions would be important in order to solve the problem.

To give a very simple example I will consider obviously they can be done in the problem but we will initially not consider that say for example I am selling 100 items, consider them items as some numbers and if I say for example send 200 of then obviously somebody would basically demand a reduction in prices, some rebate would be given to me. So initially we would not consider the rebate or the reduction in the prices because we will consider the assumption such that it would easy for us to basically discuss the problem as it is required.

The third additivity concept would be that the total number of activities which are there, activities means what are we are trying to do in order to achieve our objective function would basically be the sum of individual activities required together. So it basically we are trying to produce chairs, manufacture chairs in a position to manufacture tables, we are producing some paint, we giving some services to the customer who are coming to the bank, we are giving some services to the people who are there in the tailor machine counter or in the counter where they are booking the tickets or getting the refund in airline industry or in say for example in the railway counter.

We will consider that if individually they do the work, the total sum of them would basically be equal to the sum of all the individuals together and this will come up in the discussions so if you are trying to understand what these assumption mean they would basically come out in the discussion when we solve the problems or give very simple examples.

Fourth, which here said is that let this divisibility be possible or should be there which would imply that the solutions as I said can be in fractional values, even though I did mention time and again and it had warn you that if you are trying to find out the problem solution for the number of trucks obviously divisibility is not possible but we will still stick to that based on the fact we will try to utilize the concept on linear programming in order to solve the concept of integer programming concept.

But in the general sense the divisibility is there and finally all the answers of the variables will be non-negative because the quantity, the physical goods produce, or the number of persons who are working or the number of machines which are being utilized whatever the decision variables are or the number of hours you are trying to utilized they cannot be less than 0. So we will consider them all to be non-negative and proceed accordingly.

Now to basically to what we have discussed is basically the overall general feel of what we mean by linear programming. So now I will basically go into the concept of general historical perspective of linear programming or it is nothing to do any theoretical concept but I will just mention few names and remember these are the forefathers for linear programming concept or optimization concept and later on we will also check different people who have been there in the area of nonlinear programming, in the area robust programming, in the area of reliability programming so on and so forth and we will discuss them as we proceed.

So now I am mentioning I may not be able mention, I may skip or in an advertently or carelessly I may skip of some of the names so it does not mean other people are not famous so this is just as I was reading or whatever information I have I am just trying to give a feed that who are the famous researchers who are basically considered as a the forefather or the father of this area of operation research.

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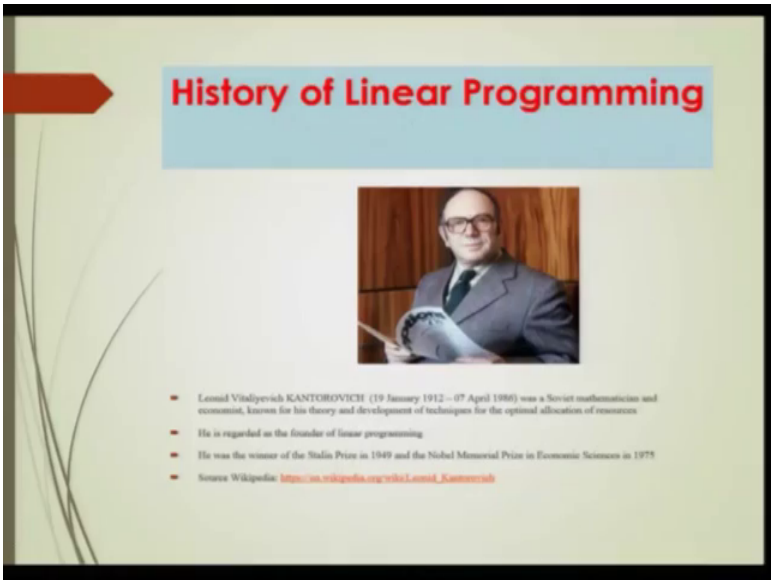

So the first person is is Kontorovich, who was born in 19, in 12, on 19 January and he died on 7th April, 1986. He was a Soviet mathematician and a economist also he has worked in the theory of development of optimum allocation of resources to find out the best possible combination. He is basically consider as the father of the linear programming and he was also awarded the noble memorial price in economics. Obviously this economic prize does not come from the Nobel foundation it comes from the bank and he was awarded the Nobel prize in economics in 1975 and he was also given the Stalin prize in 1979.

So the link which is given here the 4th bullet point, the sources of Wikipedia you can basically read it and try to understand who was this person and it definitely will inculcate some interest that why operation research is very important and being utilizing very big way in different of the area of resource allocation or supply chain management, marketing, in finance, in variable science and so and so forth. It is definitely utilized in a very big way in engineering fields of electrical engineering, in mechanical engineering and all this areas.

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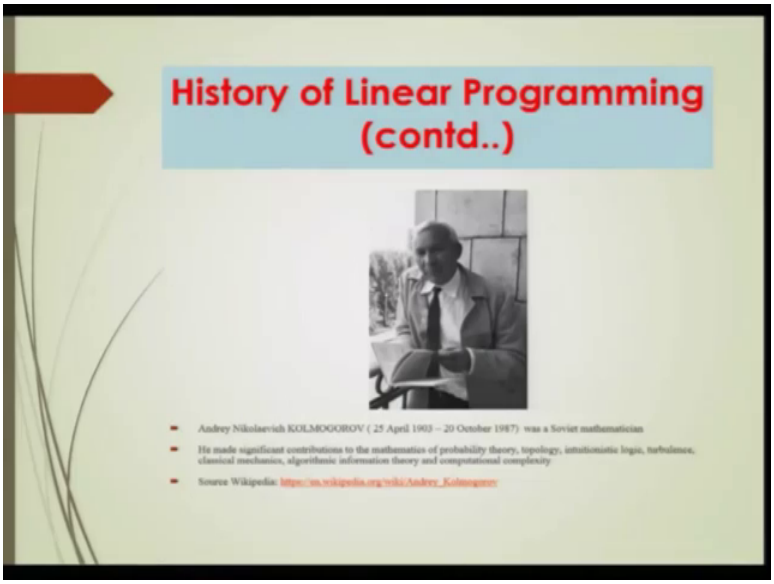

The second person about whom I am going to mention is by the name of Kolmogorov, so the people who are from the areas of statistics definitely must have heard the name of Kolmogorov in equality and his different types of famous proves which have been proposed by Kolmogorov. He was also a soviet mathematician, he was born on 25th April, 1903 and died in 20th October 1987. He made significant contribution in the area of mathematics of probability theory and topology because if I consider statistics basically and from the very simple point of view I will basically divided the area of probability and other would basically be of statistics.

So he is considered a very stalwart in the area of probability theory. His worked in the area of topology, turbulences, classical mechanics and trying to build up algorithms and information technology and computation complexity. So any information about him if you are interested just click the name in Google or in Yahoo wherever it you are comfortable with and just do try to get some information about this gentleman by the name of Andrey Nikolaevich Kolmogorov from Russia. He was also a pioneer in area whose ideas and propanones and different concepts were utilized in operation research are utilized at based on which the operation research field has been build up.

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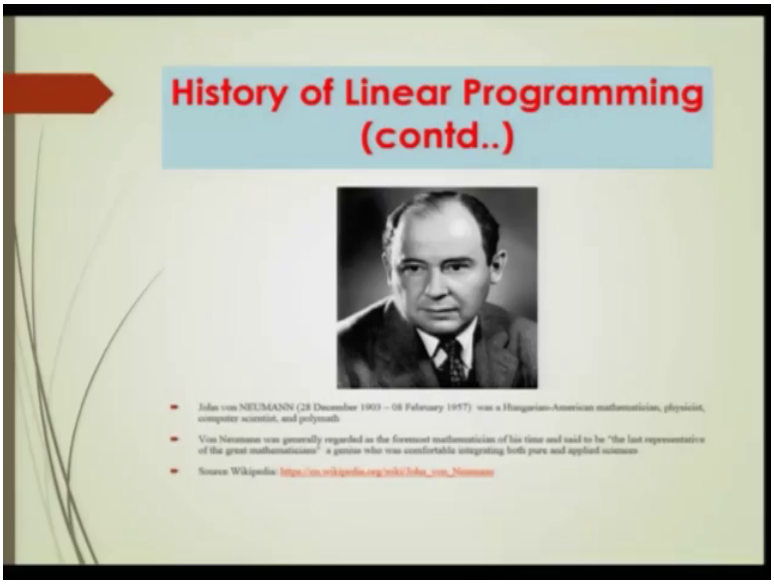

The third person about whom I am going to talk is basically is John Von Neumann and many of the people who are there in the game theoretic concept who study game theory know that Von Neumann was considered the one of the fathers of game theory and his classical work in game theory obviously later on Nash came but game theory and the concept of game theory and how it can be utilized mathematically with mathematical derivation has been proposed by Von Neumann and other person.

So john Von Neumann was an Hungarian and American mathematician, physicist and he was a computer scientist and he was considered as a polymath, so basically during world war he migrated to U.S.A and basically settled there. Von Neumann was generally regarded as the foremost mathematician of his time and he is said to be the last representative of the great mathematicians and is considered as a genius. So he was born in 28th December, 1903 in Budapest and died is 8th February, 1957 in USA. So sources about Von Neumann can definitely searched in the Wikipedia and definitely you can go through.

They may not be exhaustive but that definitely they would give you some idea and who this persons were, why they are famous? What, based on what work their concepts, their ideas have been utilized in operation research? Obviously their effort is there in different areas but I am just taking their names because I found them to be very interesting work had been done in the area of operation research.

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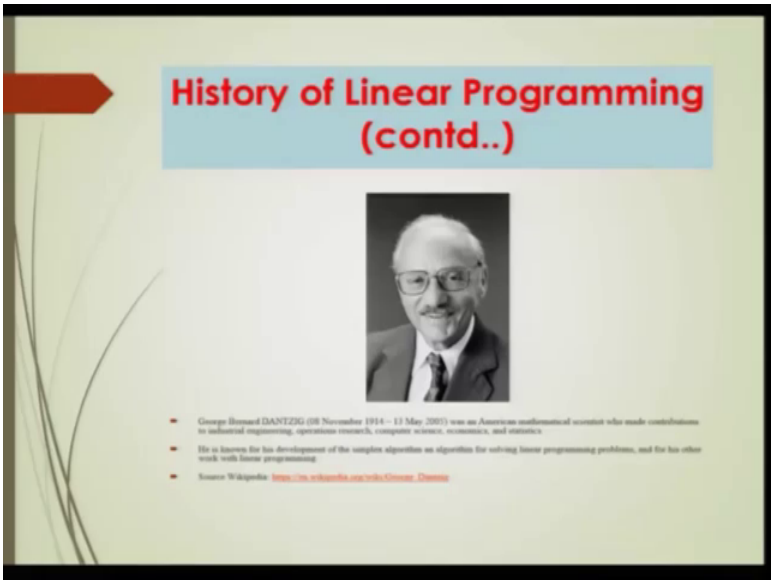

Now another person who's name would be occurring time and again is by the name of George Bernard Dantzig so Dantzig has two classical books if you these books have not been mentioned in the reference list, so he has two classical books in the area of linear programming and nonlinear programming. So they are available in India in the paperback edition by Indian publishers but the classical books were there published in the by the foreign publishers. So he was in 8th November, 1914 and he died in 13th May 2005. So he is Dantzig's work in linear programming, in especially in simplex method is considered the basically the framework based on which all his progress has been made.

So he was a American mathematician scientist who made contribution in industrial engineering and application in OR in industrial engineering and operation research, computer science, economics and statistics. He is known for his development of the simplest algorithm as I told, an algorithm for solving the linear programming concept and how you proceed to find out the solutions, so the concept based on which you proceed, conceptually it is very simple so how you how you proceed we will consider that later on when we solve initially the simplest graphical method and then go into the later for trying to solve the problem. And he is also known for his other work with linear programming concept. So you can find out about him as mentioned in the in the Wikipedia link of George Bernard Dantzig.

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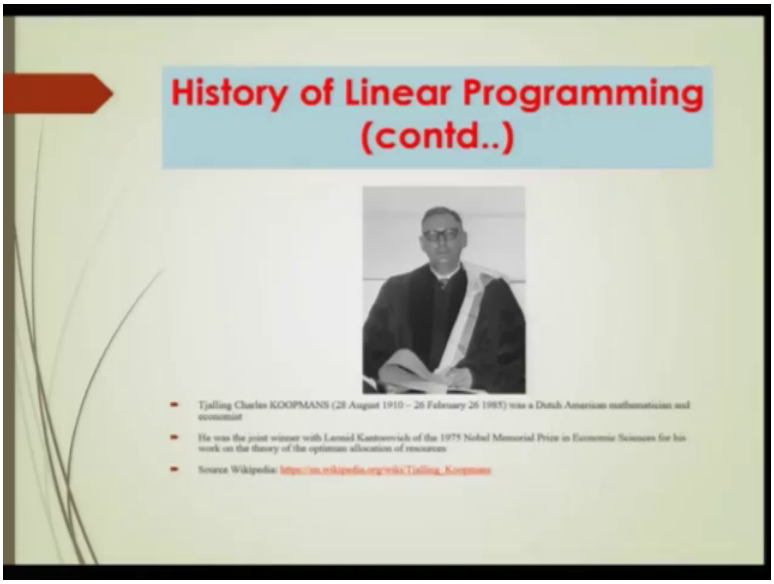

From the economics point of view many work had been done so all these obviously all of them are mathematician but Charles Koopmans he was who was a Dutch American scientist, so he worked in the area of concept of resource allocation from the economics point of view. So resource allocation may be say for example you have different type of goods and you want to basically give different type of benefits to the society or may be say for example you have different nutrients, your money amount of money which you want to basically utilize for purchasing this nutrients is limited and you and to give the maximum nutrients or maximum calorie value to the kids who are in the hospital, so how will you basically manage that?

Or you have different goods to supply from one part of the country to the other part of the country and you want to (maxi) transport the maximum amount of goods, try to utilize the least number of trucks, try to basically travel the least amount of distance because travelling the least amount of distance travelling utilizing the (maximum of the) minimum number of truck would basically reduce your cost but at the same time you want to increase the benefit of what you are trying to transport.

So these areas are very interesting in economics and Koopmans is considered the pioneering figure who had worked on these area and used different mathematical models. He was born in 20th August 1910 and he died in 26th February 1985 who was a Dutch American mathematician and economist. He was the joint winner of the Kantorovich price with along with Kantorovich

for the 1975 Nobel Memorial Prize in Economics as I said that this price is given by the bank of Sweden.

For his work in the area of optimum allocation of resources and how they can be basically optimized in order to maximize or minimize your objective function whatever it is. So information about Koopmans can be found out in the following links, so this links may not be visible to you or and if you do not worry about that just go to Google, just go to Yahoo or Alta vista just type it and find out the links and study it, so it will give some motivation and it is rather than only studies it will also give you a background that how this people have walked out in the areas of operation research. Obviously I may have missed something some of the names you can basically read about them later on as you do you search for these pioneering researchers.

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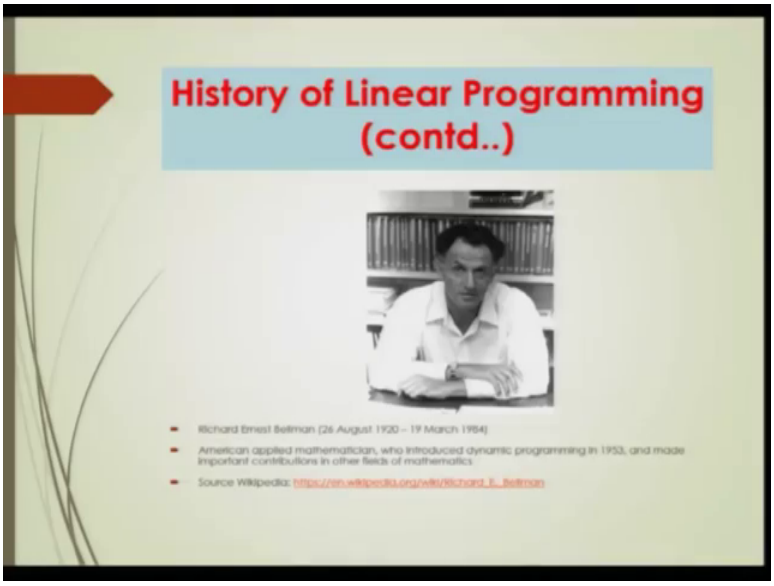

In the area of dynamic programming the person name who's comes up very often is by the name of Richard Bellman, so Bellman has a very classical book in the area of dynamic programming. He was born in 26th August 1920 and died in 19th March 1984, he was an American. So (American applied) he was a American applied mathematician who includes dynamic programming as I mentioned this classical book and he worked in the areas in the 1950s and worked and made significant contribution in the areas of fields of other mathematics.

You can find out dynamic programming would be considered in the area of optimization later on, where research allocation has to be done as you proceed one (point) period of time to the other

and along with dynamic programming if you consider the concept of stochastic programming it would basically be, really be very tough and interesting problems which if solved properly would give a lot of insight in how the problems are to be solved. So sources about him can be found in Wikipedia.

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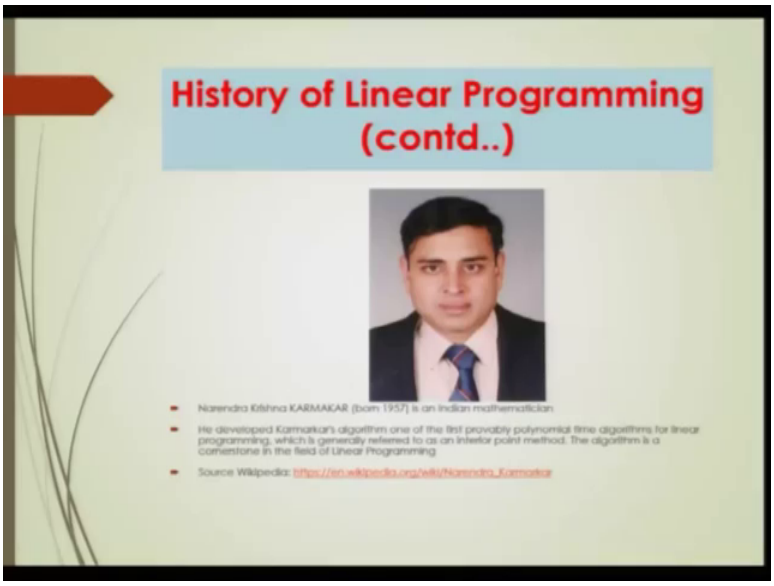

The next person about whom I will discuss is an Indian by the name of Narendra Krishna Karmakar he was born in 1975 and he developed a very interesting algorithm and known by the Karmakar's integer point algorithm. So basically it is it maybe the first polynomial time algorithm for linear programming problem which is basically utilized when you use the concept of trying to find out what is the linear programming solutions, all the computer programs which are used now a days to solve the linear programming and other programming use this concept of (linear) integer point algorithm based on which Karmakar proposed the pioneering work where the search criteria, how the search criteria can be utilized in order arrive at the results. Information about him, so this algorithm is the cornerstone in the field of linear programming and it is utilized very heavily, information about him Karmakar can be found out in the Wikipedia link.

Now generally these I am mentioning are the names but generally the historical prospect is based on for or actually came up from the environmental factor, environment means I do not mean the mother nature, basically based on the fact during world war 2, resources have been utilized in the

optimum way to basically produce different things, it can be utilized for producing food for the army, trying to basically utilize them in the best possible manner, in the shortest possible time to produce fighting equipment, to transport people, to utilize the land, the sea and so and so forth in order to optimize whatever you want to do.

So obviously optimization would be faster transport of human beings maybe the maximum number of guns which could be produced or trying to basically minimize the throughput of production for when the ship or a this some this aircraft carriers were being utilize or you want to basically try to fire the this aircraft, this tanks and how this throughput of the tanks can be utilized, how the firing of the tanks could be utilized in the maximum possible extent to basically bring the maximum damage for the enemies?

(So) this I am giving you a very simple examples but the actual background for operation research which came out basically by the demand in nature, by demand in environment how we can, how human beings could utilize that, utilize the resources and during the world war 2 and based on that all the theoretical what came put which I had mentioned. So the names which I have mentioned are not the end of the list, so there are different sets of people who are very famous, you can definitely have a look and basically understand how interesting the field is and my discussion about the world war 2 is also very brief but do check about how during world war 2 operation research had been utilized and there is very good links in Wikipedia study them and that will give you a lot of information.

This information does not mean they are anything to do with direct academic thing, about how you solve the problem but they will give a good flavor how the story has been build up such that it will also motivate you to study more about different concept which we will be pick up later on as we proceed with the course.

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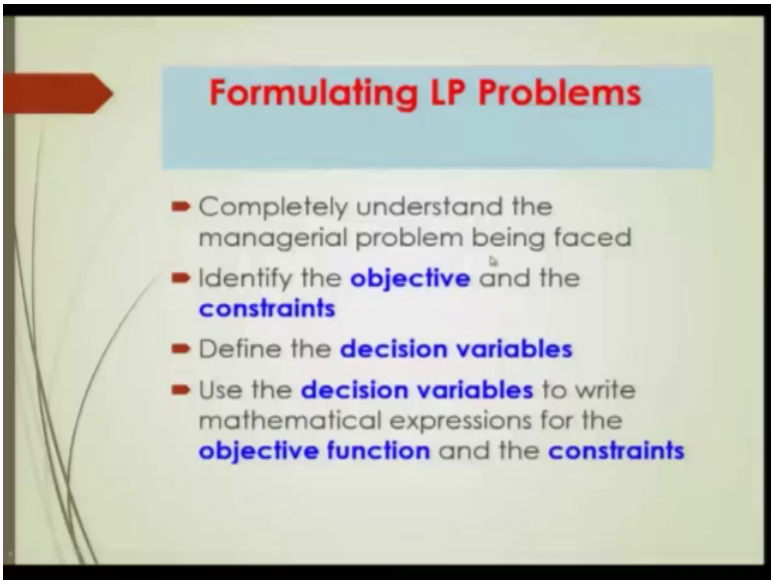

Now formulating in the linear programming ok before I go into the formulation obviously there would be other people coming up in the due discussions, I will mention them as required in the discussion as we consider the concept. So formulating the linear programming the ideas would be you need to completely understand the managerial problem and what the practical situation is and what is being faced in practical in practicality in the application side. Now remember one thing main problem would be to formulate the problem in the right sense such that based on the assumption, based on the practicality we can get the best solution which will give us some hint that what is the best possible solution in the actual practical sense or when we are trying to basically solve the problem.

So we have to understand the problem to the nitty-gritty to the barest possible in the least possible way, in the sense that we go into the depth try to utilize the formulation in the best possible manner and arrive at the results. So you should identify what is the objective and what are the constrains, so the constraints would basically mean that what are the decision variable based on which the decision variables limitation be less than type, can be greater than type, can be bounded between less than type and greater than type.

So once the decision variables are identified we will try to find that how the decision variables go into the objective function, how the decision variables go into the constraints, so whether they are 1 constraints, 2 constraints, 3 constraints you have to basically understand them accordingly.

So will use the decision variables to write the mathematical expression, the mathematical expression would be coming up in the objective function and the constraint but remember one thing once I had mentioned that they are all linear in nature the constraints and the objective function would always be linear in nature.

So once you formulate that you can either solve it in the simplistic sense using the graphical method and in the high dimension sense when you have more than two decision variable you will basically utilize simple mathematical linear concept of simplex method and then later go in to dual problem, primary problem in order to solve the simple area of linear programming. So with this I will end the second class or which is the second lecture in the first lecture in the first week and basically try to understand one or two different examples of different flavors and then proceed with the solutions for the linear programming. Have a nice day and thank you very much.