Data Analysis and Decision Making-3
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Lecture 19

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you were ever you are in this part of the globe and as you know this is the DADM-3 which is Data Analysis and Decision Making-3 course under NPTEL mock series. And this total course duration is basically if you consider the time spend for 12 weeks which when converted into number of hours is basically 30 hours in total and total number of classes is 60 considering that each class is for half an hour.

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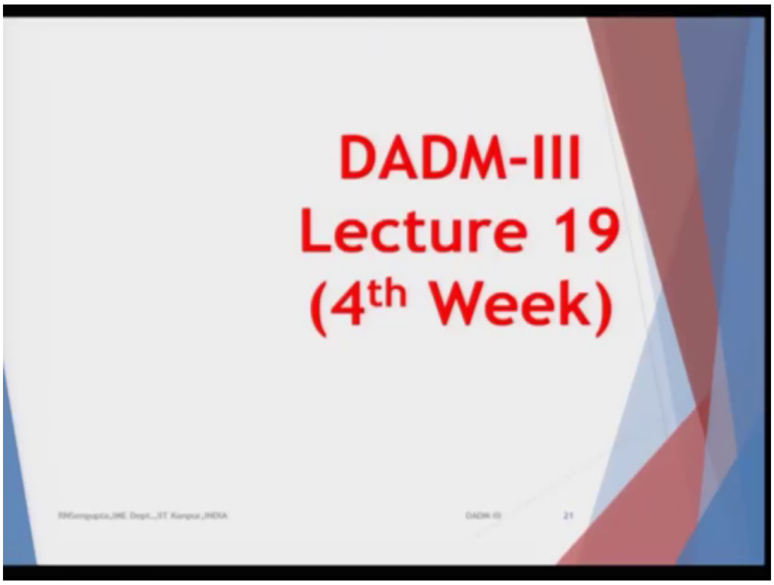

And as you can see from the slide we are on the 19th lecture which is the lecture on the fourth week and after each week we have assignments and you have already completed 3 assignments, so we will be completing the 19th and the 20th lecture and we will go, we will go for the fourth assignment. And into totality as you know there are 12 assignments and after that set the whole course is over you will basically take the final examination.

Now coming back to the last class which is the 18th one, we were discussing that when you have the initial tableau, initial tableau means the initial matrix which you have. So on the top one I am just talking about the mechanical part but obviously in between in when I talk I will talk about the conceptual frame work how you proceed. So on the top portion you basically have the objective function but consider the objective function is basically converted from the right hand to the left hand side that is why the minus and plus sign concept would be repeated time and again.

And in the initial tableau on the top the second (the) the set of values of A would be corresponding to that basic feasible solution from where you want to start your iteration process. And the left most column would be basically the basic feasible solution corresponding to the fact for which the variable are non-zero. Now in the initial problem which we consider which were they were machine 1 and machine 2 with constrain of 8 hours in each both the machines, the initial basic feasible solution was basically the variable slacks and the surplus such that both of them were positive because they were only two.

So both of them were positive which is basically a feasible solution not the optimum one but feasible solution based on which you can start. Now for the maximization and the minimization problem the concept was were very simple, in the maximization problem you will take the maximum negative one because they are on the left hand side so obviously the minus sign would be considered, considering the fact that the maximum negative value would be the one were one addition of that particular decision variable would increase your objective function the maximum.

Similarly obviously you will have the just the diametrical opposite rule for the minimization problem. And once you decide on the entering one you have to also consider the exiting one, exiting one would basically the one which will give you the maximum and the minimum for the ratios. In the sense that if we divide on the right hand side which is the constraint maximum value by the per unit time frame which you are taking per unit because each decision variable which was on the product to be produced in machine 1 and machine 2. So their per addition would basically eat up into the resources which are the constraints, so at what rate they are eating up would basically give you the maximum and minimum value based on which you will decide the exiting one.

Now the point where they the entering and the exiting one basic limits as the pivot element based on that you will slowly try to convert that matrix A into an identity matrix and as you do that you will get the values on the right hand side or the constraints which will give you the final values of the decision variable, whether X or S that is the different question. And the value of Z would basically give you the objective function which is maximization or the minimization problem.

Now consider that, there different type of problems would be there but what is the esssensce which will be considering that whether you have unbounded solution, where that you have multiple solutions or whether you have in-unique solutions, that would depend on how your total iteration process leads to the final answer. So consider this.

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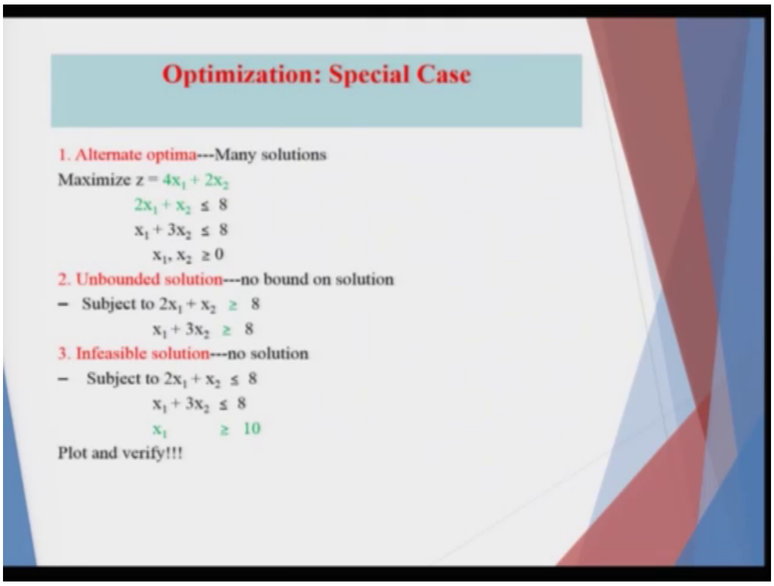

So you have an alternate optima for this (())(4:32) case when the maximization is given by 4 X1 plus 2 X2 and the constraints which are given is basically 2 X1 plus X2 less then equal to 8 and X1 plus 3 X2 is basically less then equal to 8, so (alt) many solutions would basically mean the objective function remains fixed at a certain level, but the basic feasible solution would change. So say for example you are moving along the corner points, whatever the corner points in 2-dimension, 3-dimension that does not matter if you moving from the corner points from the basic feasible solutions.

And that is basically you are trying to optimize, so there would be at may be more than two also but minimum there are 2 corner points for which the objective function remains the same even if the basic feasible the solution, the solution based on which you are getting this value of Z at a fix value may be different. So in this case if you solve I will come to the solution later on if you solve for the maximization problem you will basically have two different corner points for which the objective function is fixed.

And obviously moving between these two point obviously the exiting and the entering variables would give you the information that the objective function is not changing but the corner points are different. Now for the case were there is unbounded solution that means no bound on the solution, considering that the maximization is there is the problem which is given here subject to constraints 2 X1 plus X2 is greater than 8 and X1 plus 3 X2 is greater than 8.

In that case as you keep moving the entering and the exiting one, the number of variables remain the fixed they are not changing, So in this case say for example if you had greater than sign for both of them you will basically have X1, X2, S1 and S2, S1 and S2 depending on how we have been able to portrait them as a slack or a surplus variable depending on the greater than sign or the less than sign. So as you judge the entering and the exiting variables it would basically mean that the combination of the basic solution would basically keep changing and the objective function was also be unbounded and it will keep on increasing.

Because that means you are go for the maximization problem you are moving more away from the origin and moving more inside the first quadrant such that actual objective function basically becomes infinity. Now in the case when basically you have the infeasible solution, the infeasible solution would basically mean that none of the corner points are basic feasible solutions hence the constraints would be such that the overall space based on which you will try to optimize would not be existing.

Now this 3 points we have repeated time and again I will basically come to the problem solution when we consider this.

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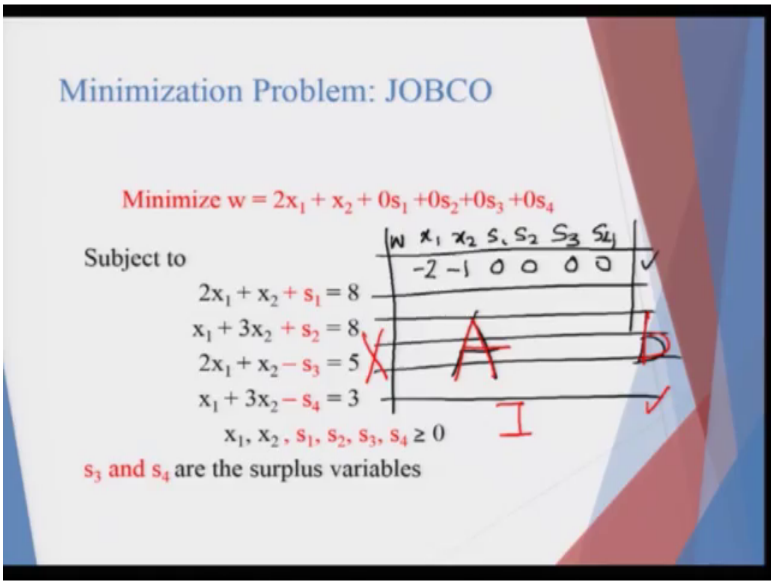

Now consider the minimization problem, So in this case you have a the there are 4 constraints, this 4 constraints corresponding to the fact is that some of them to both of them are greater than sign (or) and two of them are less than sign, we will consider the S1, S2, S3, S4 as the slack an the surplus.

So the positive and negative (())(07:38) would basically means that you are trying to be as your less than time, hence adding to that constrains on the left hand side would make them equal to 8 as in the first two constraints and basically subtracting the value of S3 and S4 will basically as they are greater than sign it will bring them back to the value of 5 and 3 for the third and the fourth constraint.

But as you are doing that obviously your objective function which was initially basically consisting of only two decision variables X1 and X2 would now became 2 X1 which means per unit addition of X1 would give you a cost or a profit factor of 2 units. Second point is 1 X2 it means per unit increase in X2 will increase your profit function for the objective function by 1 unit and S1, S2, S3, S4 which are the slack and the surplus variables as there over all combination or total contribution on the objective function which is 0, hence the each of them would be multiplied by the parameter or the constant value of the 0.

So if you have the initial tableau in that case, so I will basically draw so now remember this is a minimization problem, so if you minimization problem if you take into the minus on the left hand side they became minus but in that case you will consider the most positive variable remember that. So obviously you will have W or Z and the values would be you had basically X1, X2, S1, S2, S3, S4 and the corresponding values which you will have initially would be minus 2, minus 1, 0, 0, 0, 0 and the objective function would be here as already know we know.

Now you basically have to think what is the basic feasible solution as you start, now pause here, if you put X1 and X2 as 0 in this problem then what you will have? You will basically have S1 as 8, S2 as 8, S3 as minus 5 and S4 as minus 5. Now if you remember in the initial cases we have discussed that the initial basic feasible solution should not have any negative slack and surplus. In that case what you can consider is that these points where you have S1 as positive 8, S2 as positive 8, S3 and S4 as negative 5 and negative 3 respectively would not be a basic feasible solution, it will be a basic infeasible solution I am going to come to that later with an example.

So obviously without going to the general details so in this case you will basically have a 6-dimensional problem in the sense you are considering S1, S2, S3, S4, X1 and X2 so based on that you will consider that what is the basic feasible solution and based on the basic feasible solution (you will) what you will basically aim at is that you will start with A and the right hand side you have basically b, let me used a different color, so you will basically have, you will basically have B and the X would be on the left most column.

Then you will keep multiplying and considering the exiting and the entering variables would be based on the fact that is a minimization problem. So in the minimization problem obviously you will look at the least negative and the one going out would basically be pulling down in the increasing or pulling down the objective function in the least possible manner because it is the minimization problem.

So as you convert A into an identity matrix row by, column by column you will basically ensure that the actual X values which will have based on which you will try to solve the problem would be the ultimate values the of axis which will give you the best possible solutions which is the optimum solution.

And the right hand side, the values of b would be such, the top value would basically give you the minimization value of W in this case because it is not Z and the other values would be corresponding to the basic feasible solutions as of the X1 and X2 which you have. Now remember one thing, in case if your final solution is only plotted into X1 and X2 which would mean that S1, S2, S3, S4 would all be 0.

Technically it would mean the constraint 1, 2, 3, 4 has been utilized in such a way that maximum utilization is possible from X1 and X2 such there is no surplus, no slacks variables which are left because they are actual contribution in the objective function obviously vanishes because they are all 0 and in the constraint also they would not be any left overs.

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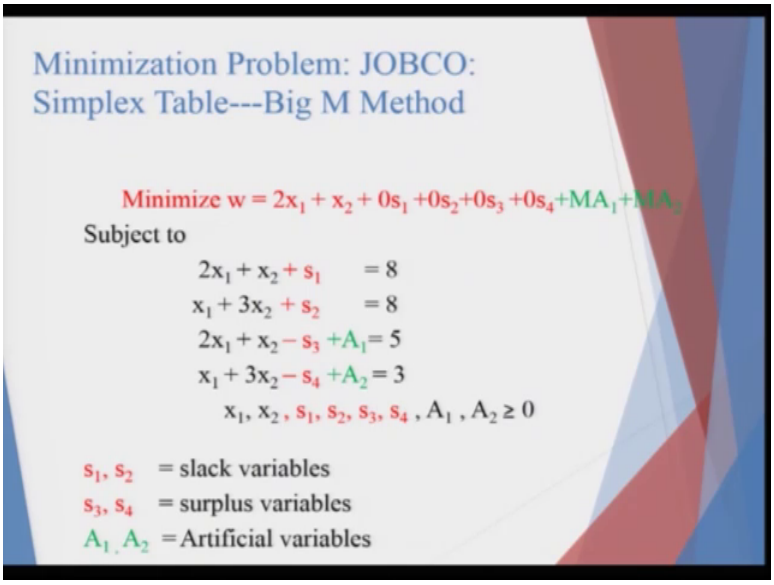

Now, consider that you want to basically solve this problem and you are going to solve this problem in such a way that this initial conditions where the basic feasible solution would have all surplus and slacks as say for example positive. So in order to ensure that we will basically consider the Big M Method, big M method means that means you are adding some, some variables or some values to such an extent so they are so large in value that the overall effect which will have from the slack or the surplus would basically be negated.

So let me consider this problem, so this is the same problem which we had considered, so solution methods would only change corresponding to the fact that the number of the tableau would basically increase in size, how it will increase in size I will come to that later. So in this case you have the minimization problem as the red color shows it will be 2 X1 plus X2 plus 0 S1 plus 0 S2 plus 0 S3 and plus 0 S4 where S1, S2, S3, S4 are slacks and the surplus.

Now in the constraints which you have already seen, the first two constraints had S1 and S2 as positives because if you had (S1) X1 and X2 as 0 then the value corresponding to S1 and S2 would have been 8 because here the constraints are 2 X1 plus X2 was basically less than equal to 8. So adding up S1 would basically make it exactly equal to 8, the second constraint was X1 plus 3 X2 was less than equal to 8 so adding S2 would basically make it equal to 8.

Now the other two constraints was like this, you had 2 X1 plus 2 X2 was greater than 5 and X1 plus 3 X2 was greater than equal to 2, so adding in the negative sense S3 and S4 would make them exactly equal to 5 and 3 but those values would be negative. So in order to overcome that you add the artificial variables which are given by the symbol of A1 and A2 corresponding to the third and the fourth constraint.

Now as you are adding the artificial variables A1 and A2 they would automatically come into the objective function, because as we change the constraint, the technically the objective function would also change in its characteristics even if the effect of those artificial variable and the slacks and the surplus in the long run would definitely be 0. So (your) minimization problem now became 2 X1 plus X2 and S1, S2, S3, S4 had 0 values in their parameters, so it is 0 S1 plus 0 S2 plus 0 S3 and 0 S4 and these values of artificial variables which I will considered to be effect to be very large, such that we multiply them by a very large number M, So it will be M A1 plus M A2 and in the long run their values of A1, A2 should be 0 because their overall effect in the objective function will be 0 considering even if M1 and M2 are very large.

So the big M method actually means that you are basically adding up a huge amount of value on to the artificial variable such in the, they are artificial so such that in the long run their overall effect when the problem is solved basically became 0. Now when you, when you start of this problem you will basically have S1 and S2 as the slacks because you are adding them, S3 and S4 as surplus because you are negating them that trying to reduce this third and the fourth constraints and A1, A2 are the artificial variables which have been added to the third and the fourth constraints.

So the tableau when you look at would basically be accordingly, so the first equation would be 2 X1 plus X2 plus S1 plus 0 into S2 plus 0 into S3 plus 0 into S4 plus 0 into A1 plus 0 into A2 is equal to 8. Second one would be I am not writing it I am repeating it again, So the first would be 2 X1 plus X2 plus 1 into S1 because that is already there plus 0 into S2 plus 0 into S3 plus 0 into S4 that completes all the slacks and the surplus, then the artificial variables will come which will be 0 into A1 plus 0 into A2 is equal to 8.

Similarly in the same way and same logic, the second constraint would be X1 plus 3 X2 plus 0 into S1 because S1 is not there plus 1 into S2 plus 0 into S3 plus 0 into S4 plus 0 into A1 plus 0 into A2 is equal to 8. Similarly the third constrain would be X1 plus (3x) 2 X1 sorry, 2 X1 plus X2 plus 0 into S1 plus 0 into S2 because S1 and S2 are not corresponding to the third constraint continuing reading it will be minus S3 plus 0 into S4 plus A1 plus 1 into A1 plus 0 into A2 is equal to 5.

And finally the last constraints would be, the fourth constrains would be X1 plus (3 into) 1 into X1 plus 3 into X2 plus 0 into S1 plus 0 into S2 plus 0 into S3 that means S1, S2, S3 corresponding the first, second, third continuing reading it would be minus 1 into S4 plus 0 into A1 because A1 is basically corresponding to the artificial variable in the third constraint and finally it would be plus 1 into A2 is equal to 5. So when you have this actual basic feasible solutions, basic feasible solutions basically would be a vector consisting of X1, X2, S1, S2, S3, S4, A1, A2.

So all the combinations which will have would basically give you the feasible solution, some would be basic feasible solution and rest would be basic infeasible solution, So we will basically take one of the basic feasible solution and basically proceed in the same direction that for the maximization problem we will consider the most negative and the minimization problem will just do the 180 degrees apart we will take the other direction and then the exiting one also would give you the pivot element.

Based on the pivot element will basically slowly be converting that matrix A into an identity matrix, as you do that the value of the b on the right hand side would give the actual values at the optimum point and it will also will give you the value of Z or W depending on whether is an maximization problem or the minimization problem.

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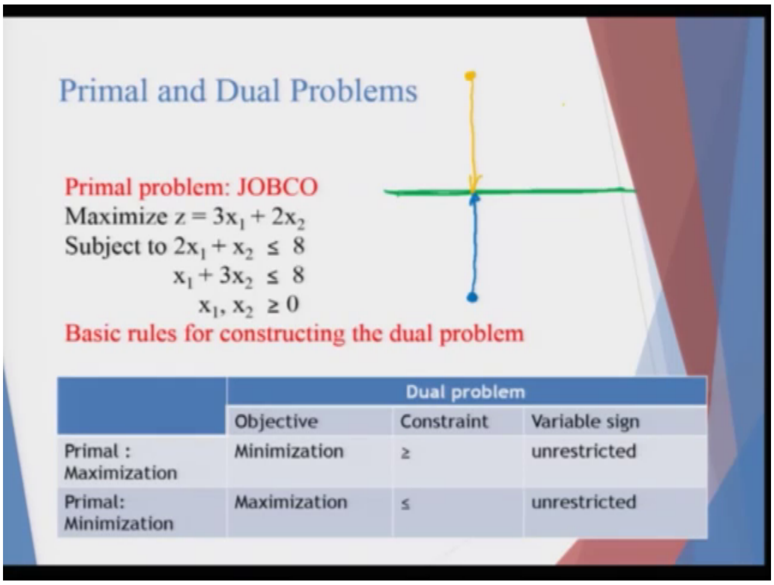

	Dual problem		
	Objective	Constraint	Variable sign
Primal : Maximization	Minimization	≥	unrestricted
Primal: Minimization	Maximization	≤	unrestricted

Now there would be a considering the problem I had mention that very briefly but I will come to that, so whenever you are solving a problem like we were looking at an mirror image, say for example the mirror is in front of you and consider that so technically we know from basic physics is nothing to do with the optimization problem which I am considering,

considering from basic physics that you as you are a real object and the image which is forming considering is a simple plain mirror is basically virtual image.

Now as you start moving towards the mirror, so the virtual image also starts moving towards you. So consider in this way that you want reach an optimum point, so you are trying to basically consider the problem as the maximization problem, So trying to maximize the problem also means that you are slowly increasing the values of X1 and X2 and the Z value is also increase. But let us consider the problem on other scale that means you are looking from the negative direction.

So minus (())(20:17) of a maximization problem it actual means a minimization problem. So now what you will try to do is that and why this is done it will be very clear depending on the number of constraints and the number objective functions which you have. So now consider that if you have basically a maximization problem what you will try to do is that you will try to basically convert it using simple a converting the constraints into a decision variable and that decision variable number of decision variable into a constrain, you will convert that primal problem, primary problem into a dual problem.

And trying to basically solve the maximization problem in the primal problem would also basically mean that you are trying to solve the minimization in the dual problem which means that at the conversation point the maximization of your primal problem would be equivalent to the minimization on the of the dual problem which is minimization and if you consider the other way around, if you are trying to minimize a minimization problem and trying to basically find it out then there is dual would basically be the counterpart which is the maximization of the dual problem.

So technically what we are trying to do is like this, so consider this is the line, what are the objective function is? And I want to reach here, so I can basically reach here in 2 way. In the maximization problem I start from a lower level, from a lower level and keep increasing and changing the decision variables such that at the optimum point I reach the maximum and stop my solution.

From the minimization problem which is the counter part I will basically start from the other side and try to decrease the objective function considering the combination of decision variables, now remember the decision variables which are there in the objective function or decision variables which are there for the maximum problem need not be the decision variables which are there in the minimization problem. So in the minimization problem I will basically consider other set of decision variables but they have relationship.

I will basically decrease my objective function try to reach such that at the optimum point the objective function of both of them would be the exactly the same, point one. And the point at which I will obtain that objective function maximization and minimization would be the same that means the optimum physical point or feasible point or the corner point where I will basically stop by optimization problem would be the same.

Now what is actually what would happen is that my number of decision variables would be considered, the numbers not the exact what we are taking that is not the issue. The number of decisions variables which we have in the maximization problem would be consider as the constraints and the number of constraints we have for the maximization problem would be considered as the variables. Now you just reverse that when you are taking the minimization problem the exact concept remains also the same.

That means the number of decisions variables now becomes the constraints and the constraints now basically becomes the decision variables, why it is done? Is that consider you have a optimization problem where the number of decisions variable is very large but the constraints is very small or on the other hand consider the number of decision is very small but the constraints are very large in number.

So trying to solve a problem where number of constraints is very large or the number of decision is very large may be difficult to solve when you are trying to basically do the problem mathematically such that converting that into a dual would be easier for you to solve conceptually which means that when you start the tableau in one case the tableau would be M cross N whether M size may be very high and N may be very small and in another case the N size may be very high and M size may be very small depending on how you basically tackle the problem.

So in the primal maximization problem if your objectives is basically to minimization one, the primal minimization problem would basically be the maximization problem which is the counterpart, point one, Another point we should basically measure is that if the constraints in the dual problem, the primal problem are greater than sign then the constraints would just be the reverse that means greater than sign will be replace by less than sign and less than sign would be replaced by the greater than sign.

And the variables of the signs would also change in the case when they are greater than equal to 0 or less than equal to 0 basically they would be dictate that what is the sign of the variables, So in case if you are reading along the row the primary problem with the maximization problem so in the dual case it will be a minimization problem. The constraints the greater than sign and the variable sign if they are greater than greater than equal to would basically be converted into unrestricted, unrestricted means that both X and X can be greater than zero and less than zero.

In the primary problem and in the minimization problem the max the counterpart of the dual problem would be maximization one, for the dual problem the less than sign gets replaced by the greater than sign (and less than) and greater than sign replaced by the less than sign and again the variable sign would basically be now unconstraint or unrestricted depending on how you have basically been able formulate the problem.

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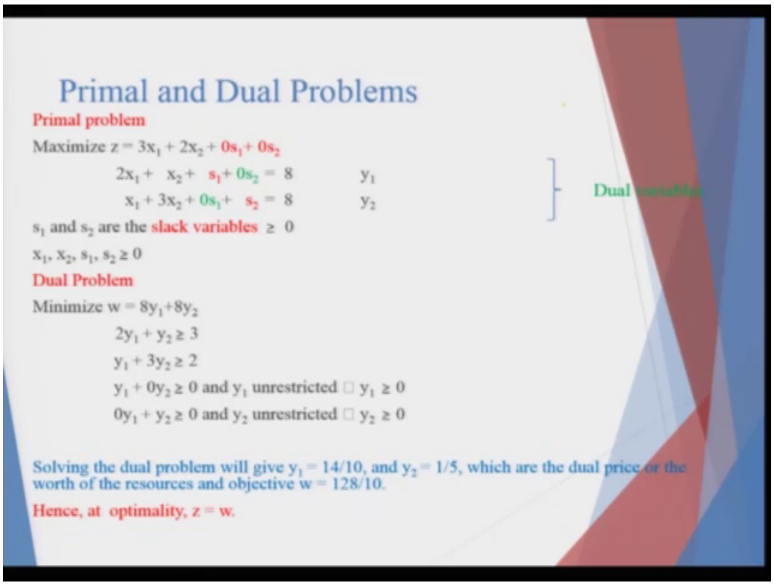

So let us consider the problems accordingly, So consider the primal problem is there, solution techniques again they remain the same, only the primal dual problem are being consider such that considering the number of constraints and number of variables are there one is preponderancely high or the value is very high then the primal can be converted into a dual and vice versa, that means if I am looking at the mirror and I am basically proceeding to that the objective function if it is a maximization problem it will keep increasing and from the other side it will basically keep decreasing till the point where they merge and the value exactly became the same.

So the primal problem would basically have consider it is maximization problem and the variables are X1 and X2 we have not consider the surplus and the slacks, so your objective function is 3 X1 plus 2 X2 you want to basically maximize it and the constraints are of this type, it is 2 X1 plus X2 is less than 8 and X1 plus 3 X2 is less than 8. So in this case as you add up these slacks and the surpluses it actually became like this.

Your actual problem basically became to objective function became 3 X1 plus 2 X2 plus 0 S1 plus 0 S2 because this are the surplus and the slacks and in the constraints you have 2 X1 plus 1 X2 plus 1 S1 because that is the slack and the surplus depending on whether is greater than or less than sign corresponding to the first constraint and the second constraint has a slack or a surplus of S2 that is why it shown green in color.

So it will be 2 X1 plus X2 plus 1 S1 plus 0 S2 is equal to 8 because you are trying to balance both the side of the equation and the second constraint would be X1 plus 3 X2 here S1 is not there so it will be multiplied by 0, so it will be plus 0 S1 and the last variable which will be there for the second constraint will be S1, 1 into (S1) S2 sorry 1 into S2 that is the slack surplus for the second constraint is equal to 8. And here you will basically consider that as you are converting the primal and dual problem you will basically converting the constraints into the variables and the variables to the constraints.

With this I will end the 19^{th} lecture and continue more discussion on the primal dual problem and the formulations later in the 20^{th} class, have a nice day and thank you very much.