Data Analysis and Decision Making-3

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Lecture 18

Welcome back my dear friends. A very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe.

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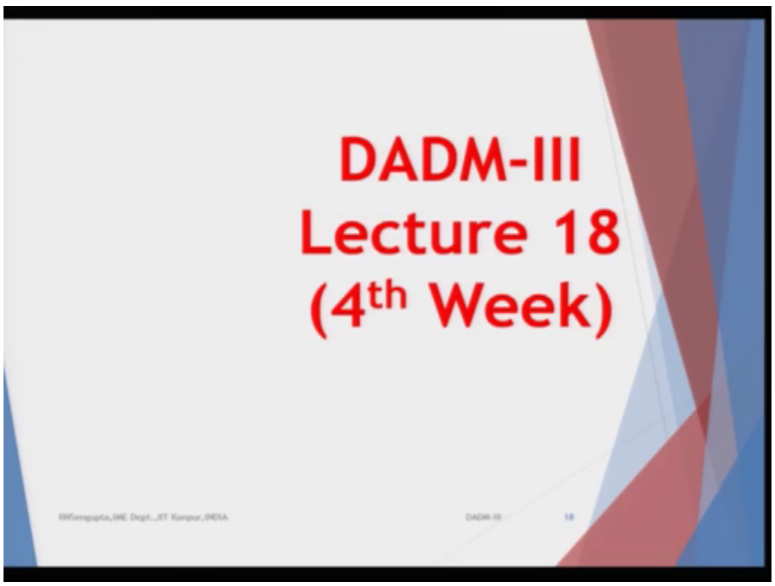

And as you know this is the DADM-3 which is Data Analysis and Decision Making-3 lecture or course, lecture series under NPTEL mock lecture and this course if as you remember and I do mention it time and again is for 12 weeks which is basically 60 lectures and that is 30 contact hours and each week we have 5 lectures each for being for half an hour. And as you can see from the slide we are in the fourth week which is the 18th lecture. So you already taken 3 assignments because after each 5 lecture which is for 1 week you take 1 assignment.

And then after the 20th lecture you will take the 4th assignment and then in totality you will basically have as logic says 12 lectures and at the end of the course you will basically have the question paper, final examination and my good name is Raghu Nandan Sengupta from the IME department at IIT, Kanpur. So if you remember that we were discussing the simplex method, we are going a little bit slow in order to make you understand the basic solutions, the basic feasible solution, basic infeasible solution and from where to start? In which direction to move? So those are conceptually I am trying to clear it time and again.

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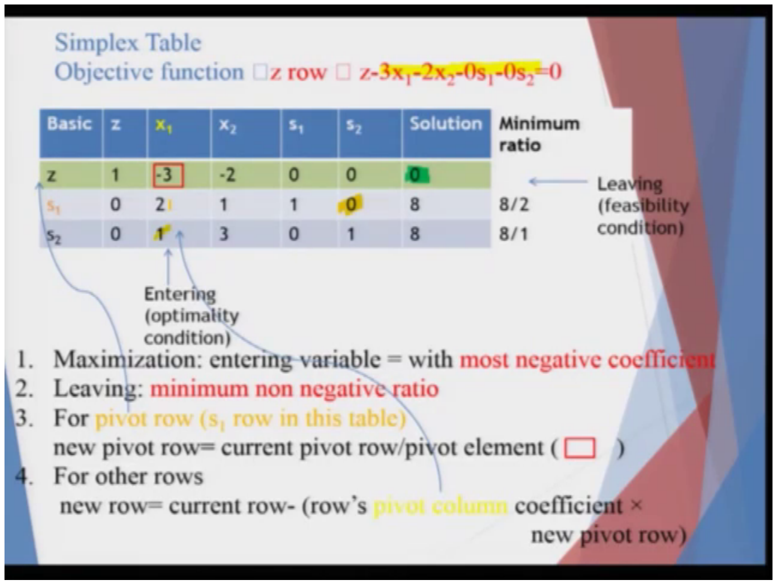

So if you remember in the problem, so you had basically, the objective function Z and it basically has technically two decision variables x1 and x2. Now which means that trying to maximize the problem or minimize the problem would basically entail that x1 will take some value, x2 will take some value but what are these values I am going to come to that later, point 1. Point number 2 is that if you concentrate on the constraint which I showed the last day they would be a slack and the surplus. Slack is that how far it is lagging behind and surplus is how far it is over and above the value.

Now the slack and the surplus variables technically would basically (being) be given a sign of plus and minus depending on how you formulate the problem but remember in the actual solution if you want it to be the exact optimal solution obviously, the utilization of the constraint or utilization of the machine or utilization of the raw material resources whatever is given in the constraint should be exact 100 percent such that the slack and the surplus should be 0.

Now when you, so in this problem if you have the slack and the surpluses are basically given by S1 and S2, technically if you formulate the problem when you start off their your objective function will be 3 x1. So you are taking out on the right-hand side, so as you basically optimize you will get the value, this is just a nomenclature. So technically x1 is the first decision variable, x2 is the second decision variable the slack and the surpluses are given as S1 and S2, so in the objective function you will basically have actually the parameter weightages so this 3 and 2 sorry 3 and 2 which is for x1 and x2 are the per unit additional value which you will get by increasing x1 by 1 unit and x2 by 1 unit, so its 3 and 2.

Now similarly if you see the parameter value or the number amount which will be added on subtracted by increasing or decreasing S1 and S2 would be 0. So the actual problem will be, so relieve us at this minus is, minus is you go they are coming from the right hand side, so it is 3 x1 plus 2 x2 plus 0 S1 plus 0 S2, so in case S1 and S2 are not 0 then obviously they will be coming into the formulations accordingly. But obviously 0 value being there their overall effect in the objective function is 0.

Now come back to the constraint, so now how you write the tableau? This is exactly the Gauss-Jordan method you are doing. So your parameter value for Z is 1, minus 3 is fair for the case for x1, minus 2 is for basically x2, 0 is for S1 and 0 is for S2, now when you start when you are kick starting the whole process you will technically consider that 0,0 which is the origin is a basic feasible solution, feasible mark this word, basic feasible solution from where you can start off your iteration process. So at that point x1 and x2 basically are 0, so if they are 0 obviously S1 and S2 have would have some value that does not matter.

But the real fact remains that as Z is a function of S1 and S2 only hence the objective function at the origin value is 0. Hence the solution as you see here where I am hovering my this electronic pen is 0 so I will just mark it with a different color let it be so this is 0. Now come back to the constraints, so if you see the constraints in the constraints Z is not there so obviously I would not mark it, so the parameter value for Z is 0. Now if you see the first constraint the unit value for x1 is 2, so this is 2 here for x1 unit value for S1 is 1, so its 1 here.

Now the slack and the surpluses which you are adding there or subtracting whichever you are looking at for the first equation it will be S1. So S1 comes with the parameter value of multiplication value of 1 and while S2 corresponding to the first constraint is 0 hence it is 0 and on the right hand side you have basically 8 because that was 8 was the number of hours which you are utilizing for machine 1.

Now, so I am not going to come to these left most column, now consider the second constraint, second constraint also does not have any Z so this value is 0, second constraint has a value of 1 for (S) x1so this is 1. Second constraint has a value of 3 for x2 hence 3 that means the 3 units of consumption of number of hours for (S) x2 being produced. Similarly 1 hour being consumed for x1. Now for the second constraint S1 which is the corresponding slack surplus I will be utilizing this word slacks, surplus in the same way but obviously with the signs being denoted that slack is what is lagging here surplus is what you are extra.

So the parameter value for S1 is 0, parameter value for S2 is now 1 because S2 is the corresponding slack surplus for the second constraint. So what is there on the right hand side? Right hand is again 8, so now if I equate the problem the equality sign is there, there is no more less than sign or greater than sign. So the actual equation is 0 Z1 obviously that is a nomenclature value how you basically denote its 2 x1 plus x2 plus 0 (S)1 S1 + 0 S2 is equal to 8. So in the problem if there is no slacks surplus everything is utilized in that case the value for s1 would basically be 0. Hence some value of x1 whatever you are producing 2 into x1 plus 1 into x2 would be exactly equal to 8.

Similarly, if you get the final solution and there is no slack and a surplus for the second constraint also, we will basically have 1into x1 plus 3 into x2 plus 0 into S1, S1 is also 0 plus 1 into S2, S2 is also 0 at the solution is equal to exact 8. Now you have to basically decide that ok another point, so now where you basically start off your iteration process, so iteration process remember our main aim is this, so this matrix if you see is basically the value of a, the x values which you will have so a x is equal to b, x values which you have now is x1 is technically x1 is the variable denotion not x1 what is written there that is not the decision variable.

The first cell in that x may a vector is S1, second cell in the vector capital X is S2, now why it is S1 and S2 because when you start the solution I am considering as I mentioned I am considering the basic feasible solution to start off is the origin. So at the origin S1 and S2 is 0, so if S1 and S2 is sorry X1 and X2 is 0, if X1 and X 2 are 0 then the actual value based on which you will find out the slack and a surplus would be S1 is exactly equal to 8, S2 that is from the first constraint and S2 is exactly equal to 8 which is from the second constraint. So this is basically where we are going to start it.

Now remember one thing, you remember we had mentioned that the slack and the surplus cannot be negative in canceling in the consideration that we are going to start off at a (feasible) basic feasible solution. So if S1 is equal to 8, S2 is equal to 8 it means that criteria that means it is a feasible solution from where we can start off your iteration process. If any one of them was negative we would not have basically started that overall iteration process because that would have been infeasible basic solution.

Now let us consider the whole process accordingly. Now if you see the minus 3, this minus 3 means what? Per addition of x1 will give you an additional benefit of 3 units, if I consider S2 per addition of X2 would add you basically 2 units. So we know the (very) variable which would basically be entering is basically X1 but now, X1 if it enters it has to basically replace one of them so what is that? Now let us consider this if I consider the entering optimality condition would basically be for X1, so now if I am going to consider the exiting one it will be based on a ratio, why the ratio?

Check here, this 2 is the per unit time consumed for (mating) making 1 unit of x1, this one is the per unit for machine 1, this one is basically the per unit time consumed by the (X2) X1 variable considering the second constraint which is machine 2. Now if I take the ratios, ratios technically would be 8 by 2 that means I can increase by 4 units of production of X 1 if I consider1 so which means I can increases by 8 by 1 units. So the leaving variable would be based on the fact that which is going to basically add up to the least concept such that additionally I can keep it increasing to the maximum value of 8 here additionally I keep in keep it increasing to the value of 4.

So if I consider the intersection of the row and the column depending on what is going to enter and what is going to exit that would basically be the cell value based on which I will basically start off the first conversion now this is interesting. If this is minus 3 which means in the long run this whole matrix would be converted into an identity matrix and as I am doing that the value of the solutions would also be coming out accordingly such that this 8 and 8 would be replaced.

Now if they are being replaced the overall calculation which I am doing if you remember I multiplied a minus a, a inverse a into x is equal to a inverse b that means I am slowly now starting of the iteration process in order to convert the actual solution at the end stage such that I would basically have a 1, 1, 1 along the identity matrix and the other values can be 0 and this 1, 1, 1 would be corresponding to that vector of x and that x values would be there on the basic solution onto the left-hand side which is here.

Such that this x whatever I have as a vector would be immediately given by the solution which is on the right-hand side, point 1. And the green cell which I have marked as I am doing the calculation would give me the actual optimum value of Z which is based on the fact which is the maximization on the minimization. Now here, so now whatever concept which I have follow for the maximization I will just reverse for the minimization, why?

When you are trying to find out the dy, dx for the maximization problem or the minimization problem, what you do? You find out d by dx is equal to 0, in the second derivative you try to find out that the rate of change of the function is such that d2 by dx2 for the maximization and minimization problem are just the reverse, I am not considering the inflection point. So let with that let me read the actual procedure.

For the maximization problem the entering variable with b with most negative coefficient, now mark the word negative, why it is negative? Because we have brought that X1 and X2 from the right-hand side to the left-hand side, so obviously if they were initially positive now they are negative. So negative basically means we are trying to basically give them the idea that the maximum per unit value addition in the objective function would be taking place by that X1 or X2 or X3 whatever it is such that my objective function increases the maximum per 1 unit addition of X1 or X2 or X3.

So that is negative, it is on the right hand side it would be positive. Now the question is if it is a minimization problem obviously it would be basically be the other way now I am going to come to that later. Now the leaving variable would be the minimum non-negative ratio depending on which is living it would basically bring down my objective function the least because objective function is basically trying to maximize, so I will basically try to ensure that the leaving variable would have the minimum non-negative ratio based on which I can proceed. Now if this is the case for the maximization problem, for the minimization problem it will be just the reverse trying to basically see a mirror image that what is the entering variable? What is the exiting variable?

Similarly for the maximization (minimization) problem are just the reverse depending on how you try basically try to do the iteration.

Now where the entering and the exiting variable happens? That is the pivot element, so pivot means? I will basically do my calculation based on that pivot element which is minus 3, so for the pivot row S1 is the row in the table that means technically means S1 would be leaving X1, would be entering the solution. So the new pivot row would be the current pivot row divided by the pivot element the concept is very simple what I am trying to do is that this amount of value which I see so technically if you concentrate on only this, so if here is considered in the final solution consider this, so if S2 is here, I am just thinking about hypothetical example if S2 is here and X1 is here technically it means this S2 would be given by the value which is on the right hand side and if this is X1 the value would be given by the right hand side.

Which means that if S2 and X1 is here the rest of the values of S1 and X2 would not be there which technically means if S2 is here the value here would be 0 because X1 is not there, value of X2 here is also 0, value here for S1 would be 0 and s 2 would be 1 and the corresponding value onto the right hand side is the actual value of S2 because 1 into S2 would be given by this value on the right hand side. Similarly if I consider X1, the X1 value would be given by X1 is 1, X2 value here it will be 0, S1 value would be 0, S2 value would be 0.

So the value here would basically be given by what is the actual value of X1. So if you concentrate you will basically now have a 0-1 values only corresponding to I am again repeat it 0 here, 0 here, 0 here, 1 here, so that means 1 in this place similarly if it is X1 it would basically be 1 here, 0 here, 0 here, 0 here and the actual value of X1 would be here. Now as you are doing it you will this value which was 0 which also change, the final value would basically be given by the maximum value of Z. So our aim is to basically pivot it accordingly such that the convex combination which we ensured in the Gauss-Jordan method are the pivot element will slowly be converted into 1 and the rest element would be ensuring that they give you the exact value of the basic solution based on which you are going to proceed.

Now as you convert this value into 1 depending on which is entering and obviously the corresponding if pivot elements will be converted such that at the end of the day they would basically give you that variable or that parameter which will ensure that multiplying X1 and X2 by that parameter would ensure the exact value which is there on the right hand side. So this is exactly what I am going to do it, so if you remember the Gauss-Jordan method I am trying to again convert, again I am repeating convert A into an i and as I am doing it on the right hand side B gets converted to A inverse into B.

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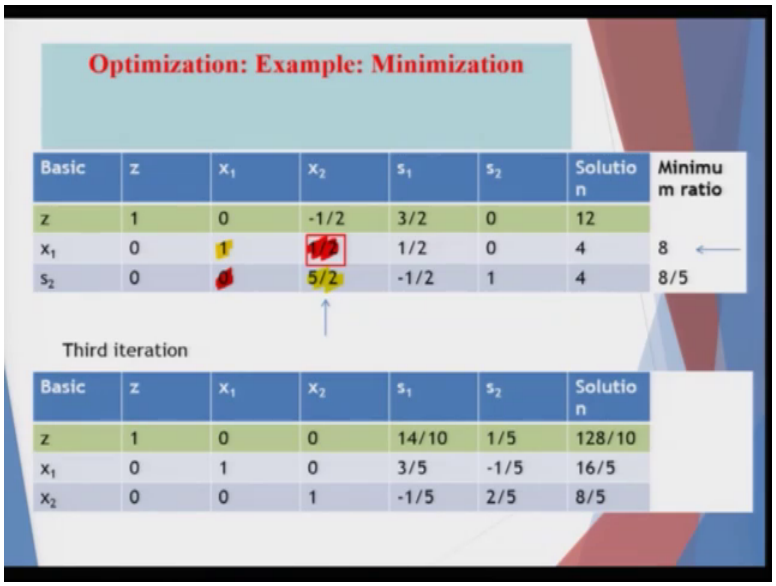

Now consider that X1 has entered, so if X1 has entered and S1 has left, so here S1 gets replaced, S1 gets replaced by (S1) X1, S2 remains and the values which were there on the top diagonal are exactly the same Z X1, X2, S1, S2 and as I do that look at here carefully after the pivots, the solution is now for 1 unit of utilization of the or the additional of X1 and removal S1 has basic S1 has increased your objective from the value of 0 to 12 which means that I am moving from one of the basic solutions which was the origin to a point which has the coordinates as this.

What are the coordinates? X1 has a coordinate of 4, X2 has a coordinate of 0 because 0, X2 is not there S1 has a coordinate of 0 and S2 has a coordinate of 4 that means I would be at a point for which it is 4, 0, 0, 4 and 0.1. The utilization are going to happen in such a way that they would be utilization of X1 and a slack and surplus would be there accordingly based on which how S1 and S2 are happening. Now obviously it will depend what is the entering, what is the existing one.

I consider the ratios, the ratios would be such that again I follow the same procedure as it has been given that means maximization problem (I am) sorry I went back to this slide maximization would mean the entering variable with the most negative coefficient that means depending on the axis and the leaving would be the minimum non-negative ratios, so if i follow this most non-negative would is basically X2, so X2 will now be a contender to enter the system and the one which will basically be going up sorry this cell would have been here so which is going to leave the system is S2 which is the slack.

Now a question, I will come to the third tableau later on, now you may be thinking that what if the ratios in when I take the minimum ratios and I try to find out the pivot element if both of them are same. So it is possible, if that is possible it means that there are more than one (content) contenders to enter the system and it may be possible there are more than one contenders to leave the system. So if this is the case think from the graphical point of view which means that and obviously the objective function is giving you the same value remember that, so if that is the case which means that they are multiple points for the same objective function to have the same answer.

Which means that the unique solution concept is not there, which also means that you are moving along the edges and two of the corner points are giving you the same answer but the interesting fact is that the corner points have different coordinates, hence many solutions would be there where the combinations of X, the X vector which you have depending on X1, X2, X3 the feasible the decision variables and S1, S2, S3 the slack and the surplus may be different answers even if the actual Z value is the same I will come to that later or not now.

So once X2 is the contender it wants to enter and it will basically be pushing out S2 depending on the ratios so once I do again I (cons) so obviously in this step half this 5 by 2 would basically be the contender or the pivot element I do the calculation convert that into 1. Now see one interesting fact, the moment S in the initial part when moment X1 had been the container and entered the ratio or the value for X1 and X1 is 1 that means slowly I am converting that particular column into an identity one of the columns as per the identity matrix or the value of I.

And the corresponding value on the other cell is 0, so technically if S2 comes(X) and X2 comes and S2 goes so at the end of the day one would remain continuing here, another one would come here and the corresponding values which will we have this and this would be 0. Which means X1 this is 1, so hence the end answer which is there on the right hand side we will see later on would give you the value of X1, X2 this is 0, this is 1 which we will see later on will come out to 1 would be given by the actual value which is on the right hand side and as you are doing the same operations, same operations would be considered for the Z row also the top row and the solutions would basically entail or give you the answer that what is the maximization well that means you have moved from one corner point to the other.

So let us do the calculation, so if you do the calculation please-please follow the simple Gauss-Jordan pivot element based conversion of trying to basically ensure that you are slowly converting A multiplied by A inverse into an identity matrix. Once this is done X1 was already there so it stays X2 enters and pushes out S2, so now the basic solution is so you have moved from one a corner point where the actual solution if you do the calculation X1 is 16 by 5, X2 is 8 by 5, S1 is 0, S2 is 0 that means we have now moved from the initial corner point which was 4, 0, 0, 4 which was in this solution with the solution of 12 to the case when it is now 16 by 5, 8 by 5, 0, 0 and the objective function has changed from 12 to 128 by 10 which is 12.8.

Now remember a very interesting fact when you find out the answer, so okay now whether you should proceed further that is also the question. So now ask the question whether there are any contenders for the exiting variables or entering variable. That means the most negative one would be there, so now if you consider the very values which you have in the basic solution it is basically giving you the value of Z is 128 by 10 and the slack and the surplus corresponding to that are such that if you find out the variable values of Z based on the factor and what is X1 and X2 and you put them in the original equation, there the parameters values based on which you are trying to find out S1 and S2 are technically 0.

So it will be 0 into 14 by 10 plus 0 into 1 by 5, so the end result is 0 and the actual value which you would have is basically X1 which is 60 by 5 into the amount which is going to come out for each and every additional value of X1 plus 8 into 5 multiplied the additional value which will come out for adding one of X2 and the value is about 12.8.

Now check here, if you put in the constraint so in the constraints, so there are two constraints with the value was 8.

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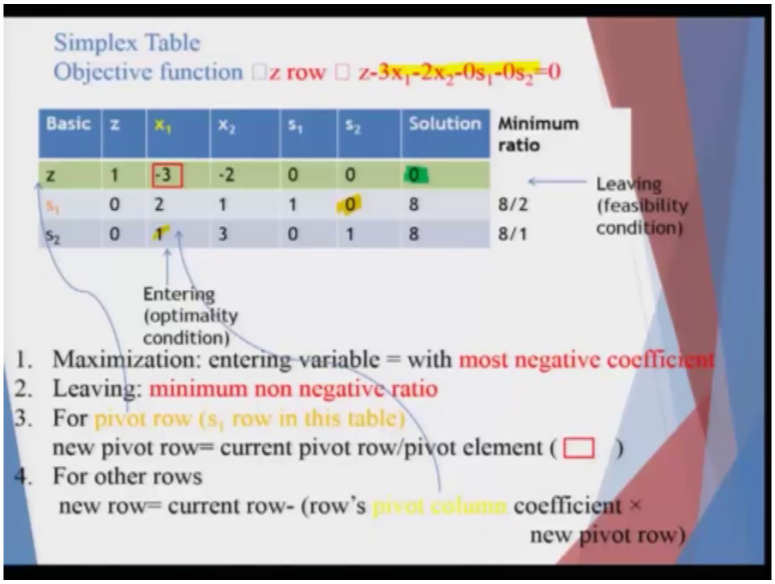

If you put it back the actual values which would have is yes so this was basically 2 and 1.

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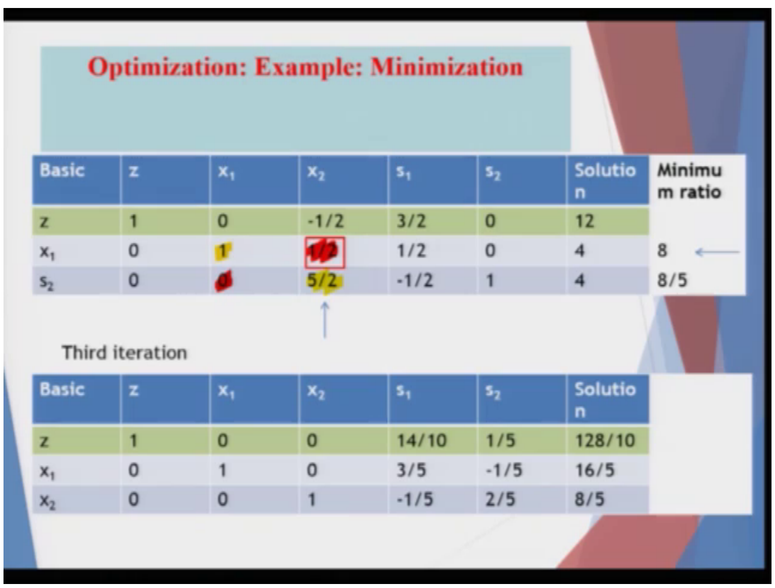

So, if I go back to this if I consider this value of 2 into 1 the utilization would be 2 into 16 by 5 plus 1 into 8 by 5 that if it is exactly 8, which means that total utilization is there for machine 1.

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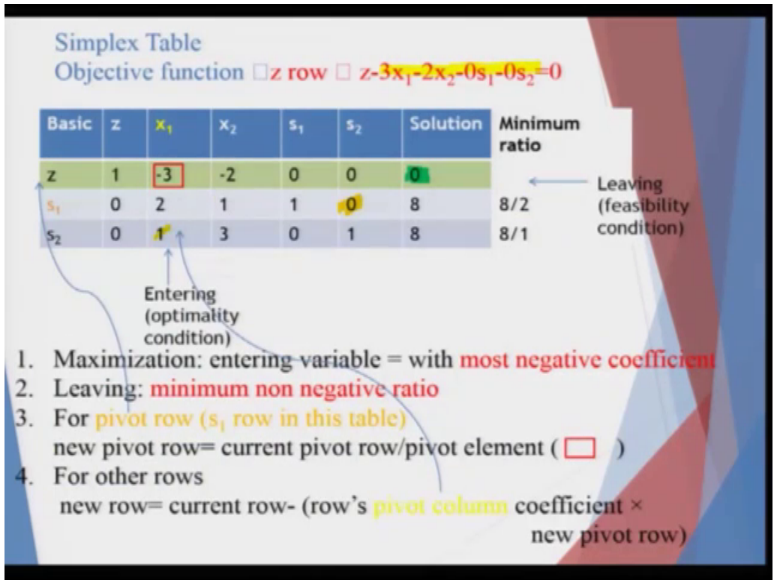

Similarly for the second constraint it will be 1 and 3.

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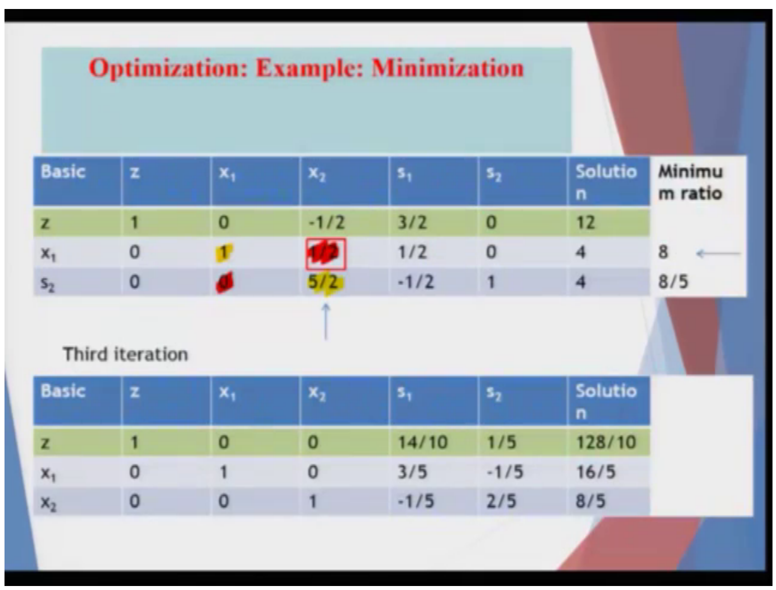

So it will be 1 into 16 by 5 plus 3 into 8 by 5 it is exactly equal to A that means total utilization of machine 2 if not obviously the slack and the surplus would be there. Now if I consider that whether you want to move I have already pointed out there is no negative values that means there would be no entering one and obviously there is no exiting one which means that we have reached the optimal value based on which you can say the objective function has reached its maximum value but it may be possible some of the slack and the surplus may not be 0. So with this I will end this lecture and continue discussing more details in the 19^{th} lecture have a nice day and thank you very much.