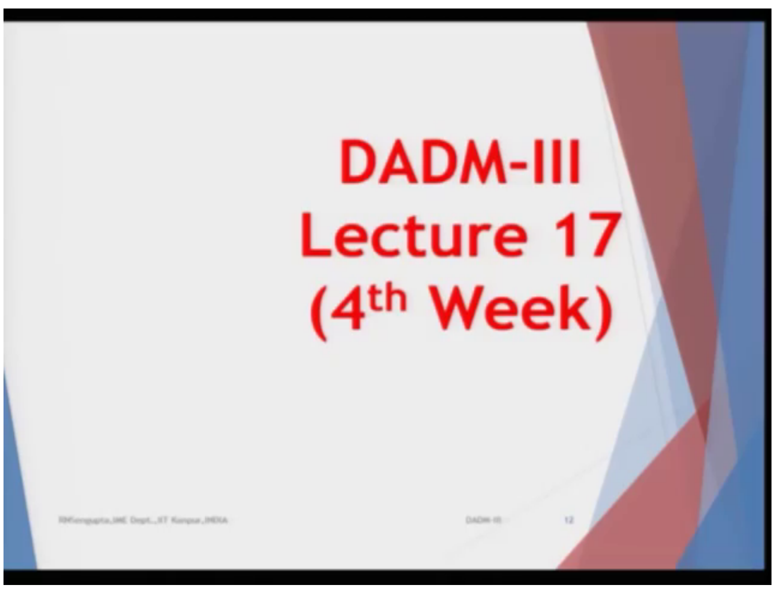
Data Analysis and Decision Making-3
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology Kanpur
Lecture 17

Good morning everybody, good afternoon, good evening to all of you wherever you are in this part of the globe. And once again warm welcome to all the students or the participants who is taking this DADM-3 which is Data Analysis and Decision Making-3 course under NPTEL mock series. And as you know I am I just give you a very brief about 30 seconds introduction about this general idea of the course. It is a to course of 12 weeks and each week we have 5 lectures each being for half an hour. So the total number of lectures is 16 number and the total contact hours goes to 30 and my good name is Raghu Nandan Sengupta from the IME department at IIT, Kanpur.

(Refer Slide Time: 01:05)



And we are as you can see we are in the 17^{th} lecture which is in (4^{th}) second class in the 4^{th} week we have already completed 3 weeks of 5 lectures each and after the each week as you know there are assignments. So you have already completed 3 assignments and total of another 9 to go. So it will make 12. And now obviously at the end of this course you take one final examination.

Now if you remember we were discussing about the concept of simple use of Gauss-Jordan method. And the concept that the basic solution or (feasible) so this basis was basically the concept in the (conc) when you are using the concept of vectors. And how you span the whole space try to find out that each and every vector or each and every so called simultaneous equation, if you are considering from the simultaneous equation point of view which actually we would be doing for problem solving.

If you are considering any vector or any set of simultaneous equations, you will be able to express all other vectors or all other simultaneous equation by a convex combination of the vectors of the simultaneous questions which you have. And when you are considering from the point of view of matrices and if you have then the concept of rank would come. Rank is basically the minimum of m, n or n, where m is the number of rows, n is the number of columns. And as you remember that we have mentioned that these number of, the rank of a matrix would basically be utilized in order to solve the set of equations which I said that it is basically a x is equal to b. And you basically find out x by pre-multiplying or post-multiplying A with its inverse.

Similarly you pre-multiply or post-multiply B with A inverse and. Now when you are going to start the algorithm so if you remember the, it was also mentioned please excuse me if I am repeating it. So the differentiation, the concept of differentiation would give you the maxima or minima but what is the maxima and what is the minima? And From which point we will start to do the search and where we should end? Is it cannot be obtained from the concept of differentiation the concept or calculus.

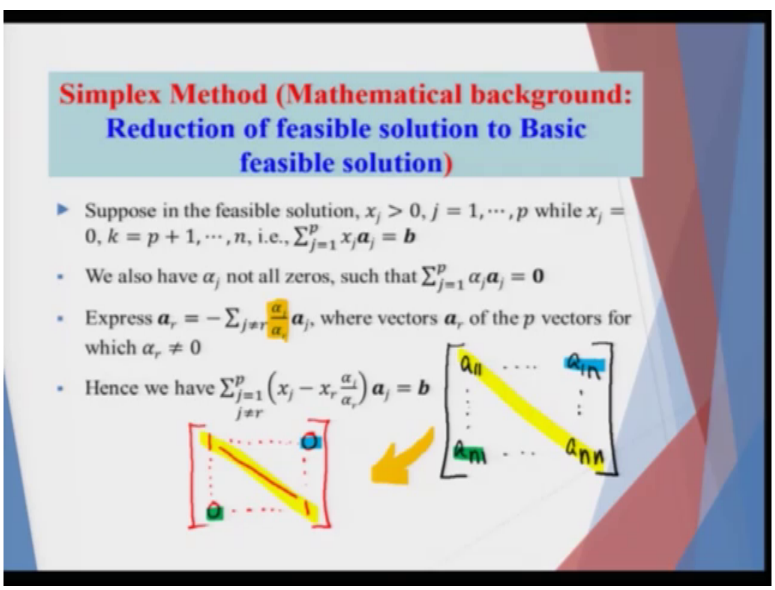
So we have to basically formulate the plan from which point we start and where we end and which direction we should go. And if you remember I have been mentioning the movement direction would be such that each unit of increase or decrease of one decision variable with respect to the cost factor would either increase or decrease your objective function by the maximum amount considering it is the maximization problem or the minimization problem.

So we will basically keep moving in this direction along the corner points which is from the edges. And once we reached the optimal solution considering this only one, obviously it can be multiple optimal solutions also how I am going to come to that later. We will basically reach the optimum solution and any further movement along the edge in the other direction you will see that the overall objective function starts decreasing even if by addition or subtraction of the decision variables.

Now when you start the overall objective solution of in the case of the simplex method, you have basically have a basic solution. So that basic solution has to be feasible, it need not be infeasible. Because if it is infeasible that means the point is not inside the feasible state, it is a but it actually gives you some solution but the results are not applicable. Now how you do that from the point of view of matrix multiplication and matrix operations, I will discuss that theoretically and then basically proceed accordingly.

Now consider there are total number of solutions or total number of such corner points is basically n in number. So it is not m, it is n. And you divide this n into two groups, this is arbitrarily, it can be anything. So which one you want to choose will basically depend whether it is one of the basic feasible solution, mark this two words basic feasible solution from where we can start the iteration process.

(Refer Slide Time: 05:46)



So consider that you have one set of x's, x's are those variables which are the decision variables and consider some j's of them. So j is 1 to P, so one set p out of n, for x is greater than 0 and obviously it would mean the rest of the decision variables which is p plus 1 till n are 0. Now if you remember in the problem which had just discussed the last day or this in the last class, we basically took x1 as 0, x2 as 0, x3 and x4 which were the slack and the surplus variables, we put them as non-zero. And we saw that one of them was negative.

So obviously it would mean that that cannot be one of the basic feasible solution because the considering the fact we are (basic) we are considering that all the slack, surplus, actual decision variables are greater than 0 greater than equal to 0 as per the norms of assumptions which you have assumed. Now in this case you will as you said that you will partition I am again repeating partition the n number of decision variables into two sets, arbitrary sets. The first set basically consist of p number of such decision variables. Next considers from p plus 1 to n, this p plus 1 to n are assumed to be 0, decision variables. And this 1 to p are considered to be non zero. So this is what I said.

Suppose in the feasible solution, feasible mark the word feasible solution, x j is greater than 0, I will come to when we solve the problem you will understand what I mean. So this is just a combination of matrix multiplication and division along with the concept of basic feasible solution which you have. Suppose in the feasible solution x j is greater than 0, so this is that is j is equal to 1 to p while x j is equal to 0 for k being p plus 1 to n. Now such that if you put that in question you get all those solution, which is fine, which as I saw in the in the last class. So x1, x2 are 0 and (still) you are still getting the answer.

Now consider we will assume some constants which will mention as alpha suffix j's. And obviously any values of alpha suffix k's even if they are non-zero, the actual multiplication with the value of 0 obviously will get results in the value of 0. And will consider that this alpha j's, j is equal to 1 to p are non-zero such that, when you find out the convex combination is basically leads to 0. Which means technically means that I want to have some set of constants, so this constant would with the operations which will be using throughout and it will keep changing depending on the answer.

Such that we want to have some constants which will be multiplied with this values of the vectors such that the net result of the effect of the vectors which are non-zero would be turned converted into 0 in the sense that we want to start at a point which is basic feasible solutions such that it is inside that, it is one of the corner points in the feasible region. And that is the point from where we start off our solution, point 1.

Point number 2, it will also mean that the (non) this slack or the surplus, whatever it was there in the problem, if there put to 0 and we started from that point and it will give as a good point from where we can start the search for the simplex method. So what we will do is that it is like this, I will I will try to draw it here only. So consider, so this is the matrix and there would be a counterpart also. What is that counterpart I want to come to that later.

So this, this set of points which is written here with respect to what I am writing are related, need not be exact the values would be same or the symbols are the same but the concept would be equal. So you have 1 to 1n, I am considering number, here I consider n rows and n columns that you may think that surprising that it is not because if I add an extra number of rows or extra number of columns it can be made depending on whether m or n whichever is more than the other.

So what I am doing is that so this was a, now I want to convert what think, I want to convert a into an identity matrix I. Because on the left side if I am basically pre-multiplying a, post-multiplying a, with a inverse. Slowly at the end of that it will be converted into I. Now I am doing some operations, as I do the operations on the left side I do the same set of operations as I said you are balancing a weight is being balanced in a balancing scale. So as I am taking out weights or putting on weights, I would do the same thing on the right hand side.

So I am as I am doing the operations on a, I will try to do the same set of operations on b. Such that slowly a becomes I and b becomes basically a inverse b or b into a inverse depending on pre-multiplication or post-multiplication. So what I will do now? This a should look in the end what? I will use a different color, first (would) identity matrix first would be 1, all are 0 here till the last one, all are 0 here till the last one, all are 0 here till the last.

So what you have is this principal diagonal values are all 1. Technically it means this should be converted into 1 and these values, I will use some other color, just for to notation this becomes zero, this become zero, so a1n become 0 and all the value. So I am using a different color scheme to note, make the annotation. So what I do? I should (multiply) I should divide a11 by a11. So if I am doing that and if it I am concentrating on one of the row or the column, I will do the same operations for that row or column accordingly.

So this is actually what it means, again I am repeating as a11 gets converted to 1, I will be using some multipliers the same set of ratios or multipliers would be utilized throughout this row, or throughout the column considering that you want to basically convert into an identity matrix, considering the row or the column for a. So this ones to go here. So (you) will express ar, any rth one as a minus sign because minus sign is that, if you if you add or subtract the corresponding other elements in the row or the column, the end result should be 0. Because adding minus 2 to 2 would basically make it 0.

So if the initial value in the row or the column for that particular value or the cell is 2, so you will basically subtract to make it 0. So which are not the along such that 0 becomes an element not along the principal diagonal that is given here. So and based on the fact that you have aimed at bringing back the elements which are there in the principal diagonal as 1. So express ar as minus of a ratio, now see this ratio which is important, this one this is important. Now technically if j was equal to r, then obviously that (tech) that ratio should be 1 such that, ar minus of ar would basically be leading to 0, which we do not want, we want 1 to be there. So you will only ensure that one is there in the principal diagonal.

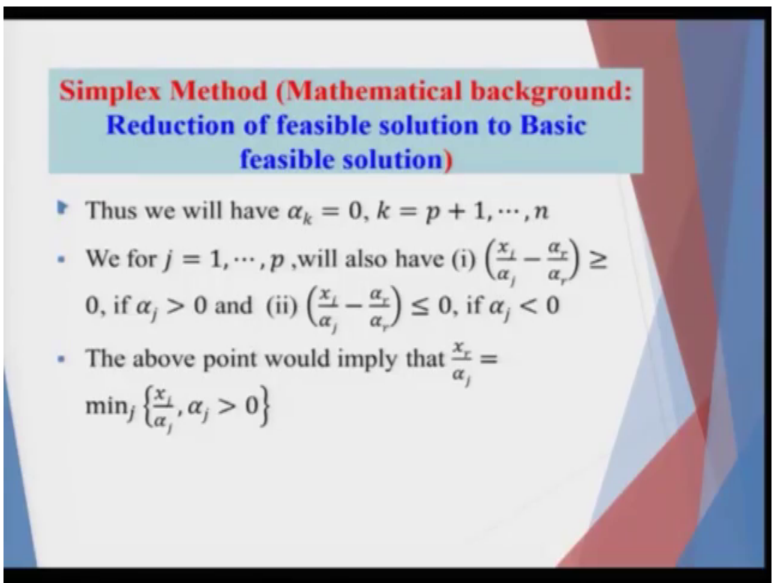
Let me continue reading I maybe repeating few things but please bear with me. So where the vectors ar of the p set of so this I am aiming for those set of variables which are greater than 0. Because as such P plus 1 to n are already 0, so you need not be bothered about them. So where vectors ar of the p vectors are for which ar is not equal to 0 would basically be converted accordingly. Hence as you do the operations on the left hand side you will do the exactly the same thing on the right hand side.

Because you would be (answer) asking, why are we doing that? Because on the right hand side remember b is an element. So consider if I mentioned ok let me let me step back, if I tell you that I have a number 3 and I want to convert it into say for example any other number. So actually what is 3? 3 is equal to 3 into 1. So in the same way what is b? b has been either pre-multiplied or post-multiplied by I an a identity matrix. So what I am doing is that I am converting I with a mirror image. So if this is the mirror, I am basically seeing the picture of a, multiplying it for a minus 1 and converting it into I.

And in the similar way as I do the steps, I do the similar steps on the right hand side and that I is there, it is converted into basically a factor such that it is a minus 1. Hence this is the last point. Hence we would have for all values of j, j is equal to 1 to p. The fact which you (have) they are on the right hand side would basically pre-multiplied or post-multiplied depending on the same set of operations which is there for the left hand side. Again I am repeating weighing balance, take out weight, add weight on the left hand side, you do the same quantum of operators on right hand side such that the balance remains.

Which means a gets converted into I and I which was there on the right hand side which was pre-multiplied or post-multiplied by b gets converted step by step in the same way.

(Refer Slide Time: 17:18)

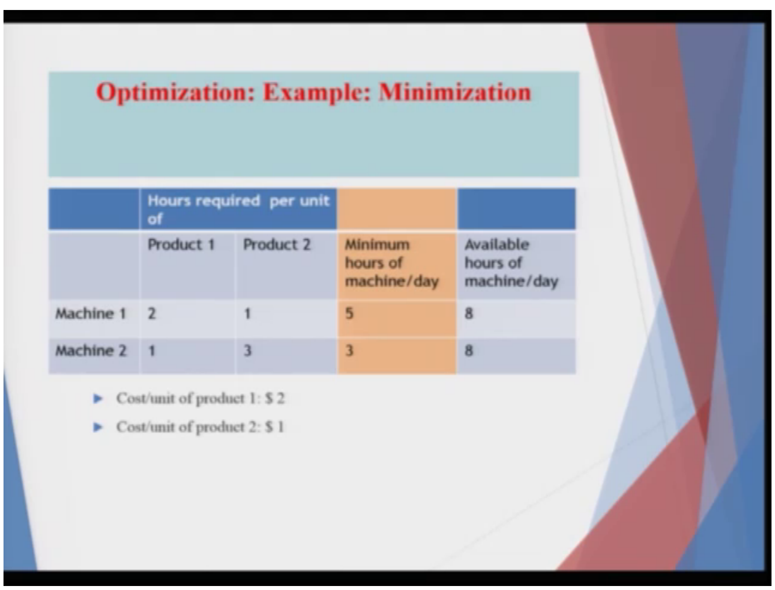


Thus when the whole process ends, obviously remember the for the basic feasible solution the variables which was 0 remains 0, because the multiplying facts which you have for alpha k's, k is equal to p plus 1 into n are all 0. So any pre-multiplication or post-multiplication, addition, subtraction obviously would result in the value 0 because they remain same. Once the whole process ends so technically there were some, there could have been some minus values also or positive values also in the row or the column based on which you are doing the operation.

So what you will do is that you will basically (multi) pre-multiply, post-multiply with the value of minus to bring them all with the greater than 0 sign. Now what now what you do is that as you do that technically, now remember when you basically complete the same operation step by step you will look at the value. Now the question okay, let the question would be that when I want to consider the basic feasible solution I have obtained it. Now what next? You will take that value, now if you remember I said that you will move that direction where the uphill is most such that the gain it is maximum considering the positive value and the downhill is most in the negative sense such that any one unit drop in one of the variables will give you the minimum the fastest rate.

So you will consider that as your benchmark based on which you will consider the minimum or the maximum. So the hour point would imply that you would basically take the minimum of that with a negative sign obviously it will be maximum, such that you ensure that the movement along the direction later on we will see that the movement along the direction from the basic feasible solution would be in that direction where the addition or the subtraction or the objective function is happening the most or the least depending on what type of problem you are trying to solve. I will show this problem from the point of view of a simple optimization problem and it will be replicated later on also.

(Refer Slide Time: 19:30)

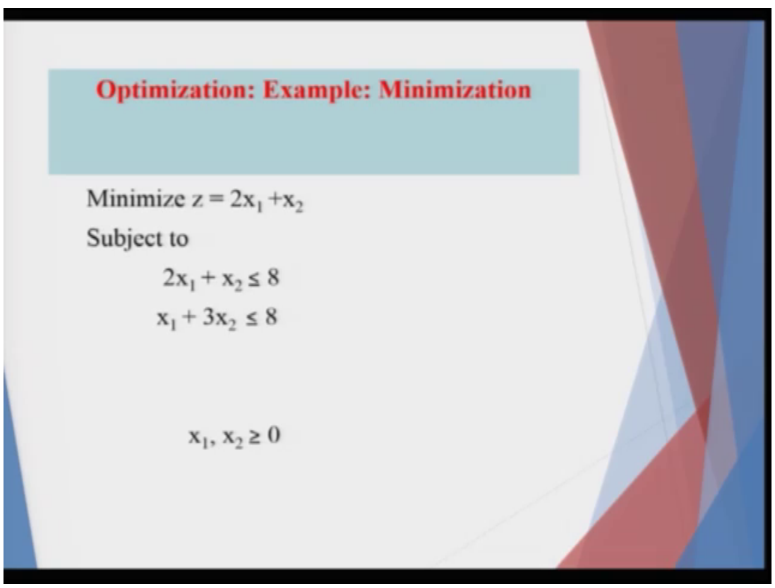


	Hours required per unit of			
	Product 1	Product 2	Minimum hours of machine/day	Available hours of machine/day
Machine 1	2	1	5	8
Machine 2	1	3	3	8

Consider product 1, product 2, so it would not immediately make sense once we solve the problem you will understand. Machine 1, machine 2, hours required per unit is basically 2, 1 for product 1 or 2 for product 1, 1 for product 2. Similarly for machine 2 it becomes product 1 is 1, and product 2 is 3. Now on the same hand, this is the same problem, minimum hours of the machine per day which you can basically have to utilize is 5 and (number) minimum hours of machine per day for machine 2 is 3. And available number of hours for machine per day is 8 and 8. So obviously that minimum need not be possible it could be 0 also. But you have to basically utilize some amount of hours for machine 1 and machine 2.

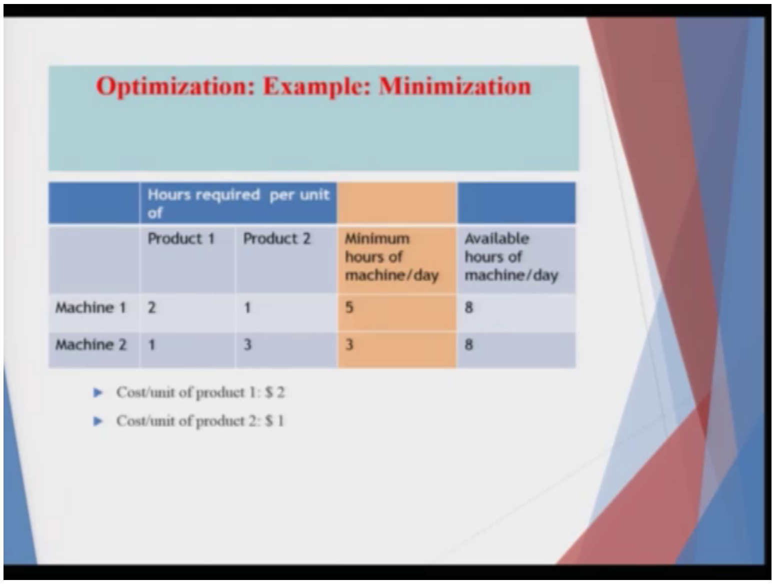
Cost for the products are given, so the cost are basically for product 1, 2 units or 2 dollars, 2 rupees, 2 yens, 2 dirhams, 2 Canadian dollars, whatever it is. And the cost for unit for product 2 is 1. So that means if you add, subtract product 1 by 1 unit you add plus 2 or minus 2. Similarly I will be using the word plus. So but with the negative sign would always remains that is a negative. And the cost per unit of production for 2 would be basically 1 that means addition and subtraction gives you 1.

(Refer Slide Time: 20:59)



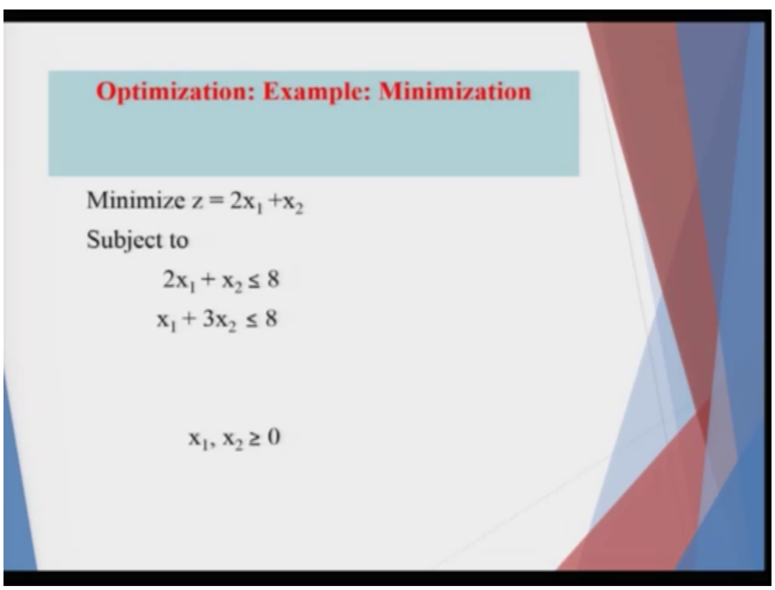
Now the minimization problem is written here.

(Refer Slide Time: 21:04)



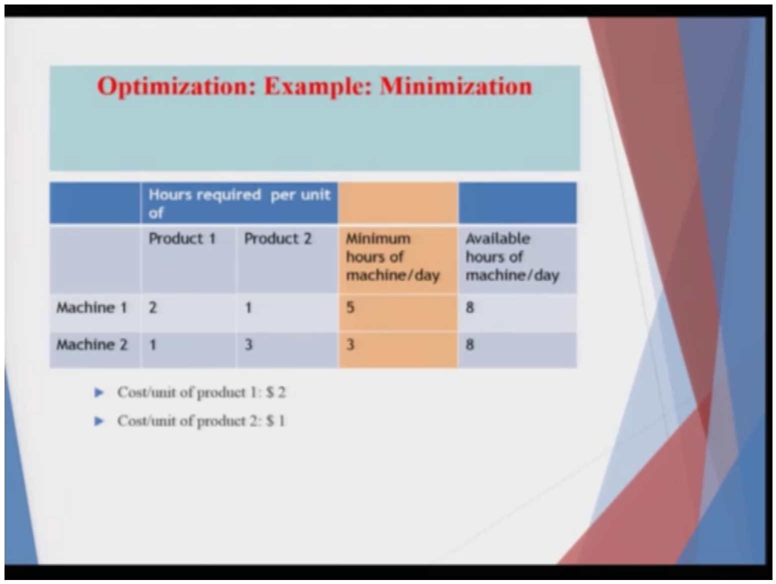
So let me come here, so what you have is cost of product 1 is 2. So it is 2 x1 plus 1 x2, I will consider x1 and x2 as the decision variables for product 1 and product 2.

(Refer Slide Time: 21:17)



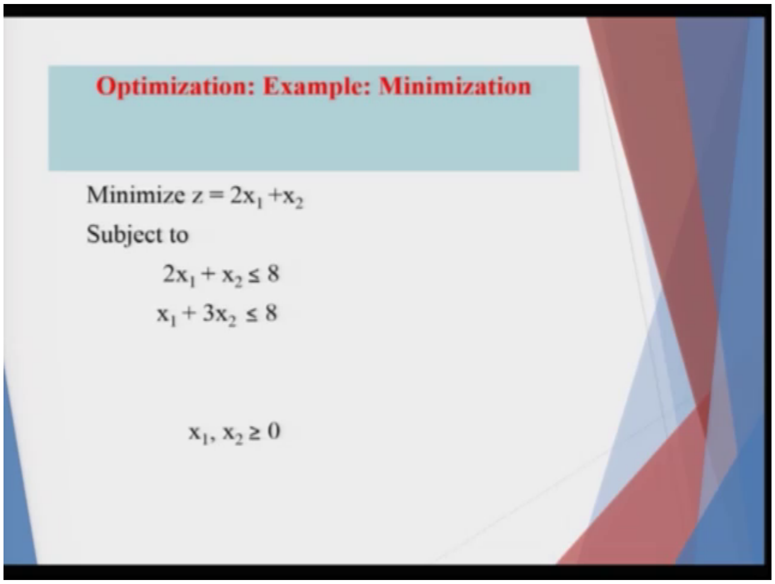
Now I will switching between these slides in order to bring the resemblance of what you have in basically the constraints. So if you see the first constraint is 2 x1 plus x2 is less than equal to 8, 8 is what? 8 is the number of hours. What is 2? What is 1? Because x2 is multiplied by 1, it means the total amount of utilization on the number of hours which you have basically from for machine 1 (pro) and corresponding the product x1 and x2 which is 1 and 2.

(Refer Slide Time: 21:51)



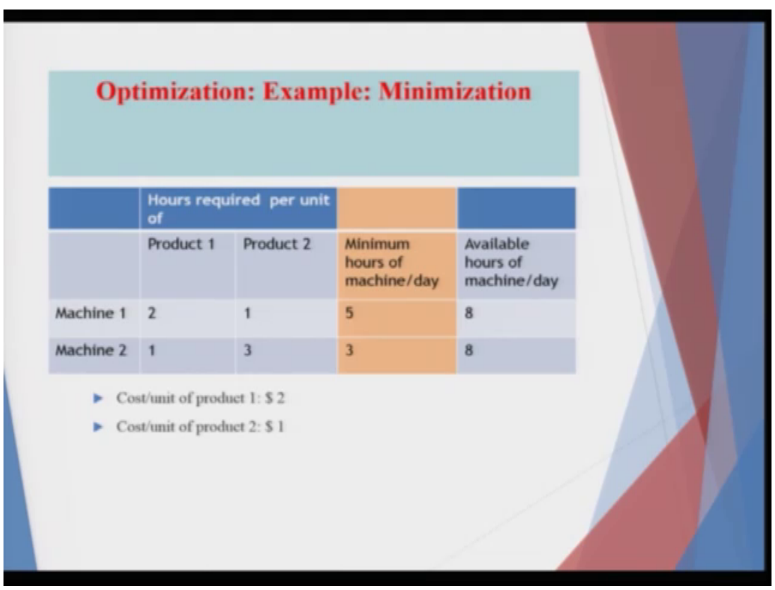
So if you see product 1 requires 2 hours, product 2 requires 1 hour. So obviously it will be 2 x1 plus (2) 1 x2 is less than equal to 8. Similarly if I go to the second one it will be 1 x1 into 3 x2 is less than equal to 8.

(Refer Slide Time: 22:07)



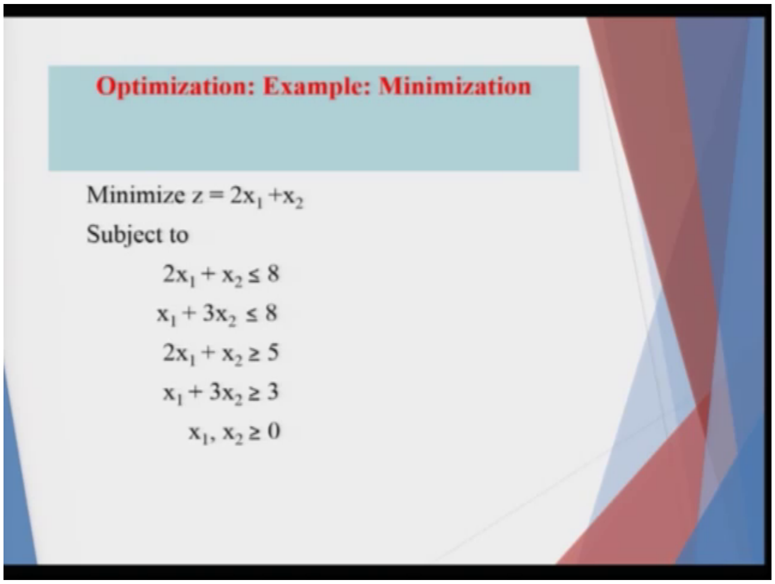
Let me see, yes, it is matching. So you are assured that the set of constraints which you have corresponding to the hours is met. But stop here, there is some extra thing is also mentioned so let me check.

(Refer Slide Time: 22:52)



So extra thing is mentioned is of the color scheme which you have put in the minimum number of hours. So let us see if it is being done. And obviously the fact that x1 and x2 is greater than equal to 0 would also remain.

(Refer Slide Time: 22:36)



So this is the last equation, so there is some space here, what is that going to come let us see. So the first equation basically means 2 x1 plus x2 is greater than 5. Now if you remember we had mentioned the utilization of machine 1, minimum 5 hours, utilization of machine 2, minimum 3 hours. Does it meet the criteria which you have put for the constraint third and fourth? Let us see. Yes, it meets because if you produce two units of product 1 sorry-sorry if you produce x1 units of product 1 utilizing 2 hours and x2 units of product 2 utilizing 1 hour.

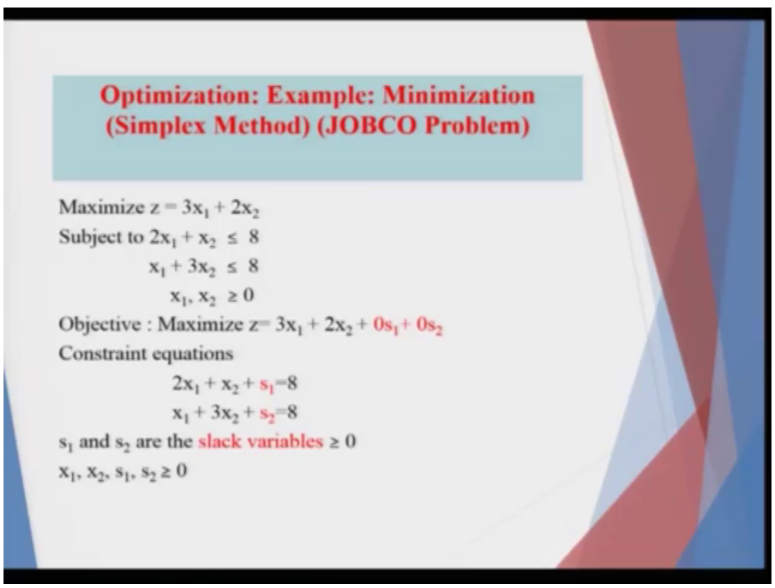
The total sum has to be greater than or equal to 5. Similarly for the last constraint before x1 is greater than 0 and x2 is greater than 0, it says that if you use x1 units of product 1 and x 2 units of product 2, the total number of utilization hours will be x1 only plus 3 x2 because you utilize 3 hours per unit production of x2. So that is greater than 3. So now the problem in the constraints are written. Now pause and see. Let us go back to simple algebra which we have been discussing.

So how many such equations are there? There are 4 equations. How many such constraints are there? How many such variables are there? There are 2 variables. Now the first question would come immediately into anybody's mind is does this constraint give you some infeasible solutions from the matrix concepts? Which means that, are they redundant? That means are some of this equation which are given being able to we are able to express them as convex combination of the other variables, point 1.

Point number 2 is that if we go into the concept of solving it using simplex method. Do we meet many of the criteria of the problems which we have faced in? If you remember in the one of these classes in the last three classes I mentioned that there can be Z value can be an infinite, arbitrarily large. Similarly x1 and x2 can be arbitrarily large. There can be cases where one of the x's or more than one of the x's are not arbitrarily large, they are some fixed values. But corresponding to the fact that the other x's are arbitrarily large, Z can be infinite or arbitrarily large.

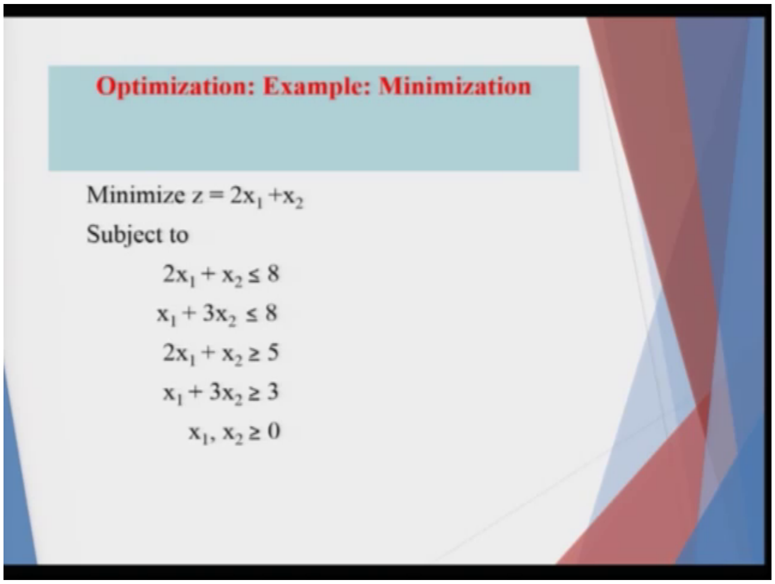
The third case I showed that in case say for example, few of the x's are arbitrarily large still you can have the case when Z is finite. For the Z value at which was 4. Then we also saw that the constraints are infeasible, that you cannot have such constraints. And also the constraints are feasible but you would not have the space or you would not have an answer which is feasible. Because all of this facts, the last 2 facts were important to the case, where they were the feasible region would basically a null set. Null set with no elements considering the fact that x1 and x2 are greater than 0, so let us see and come back to this (form) issue.

(Refer Slide Time: 25:46)



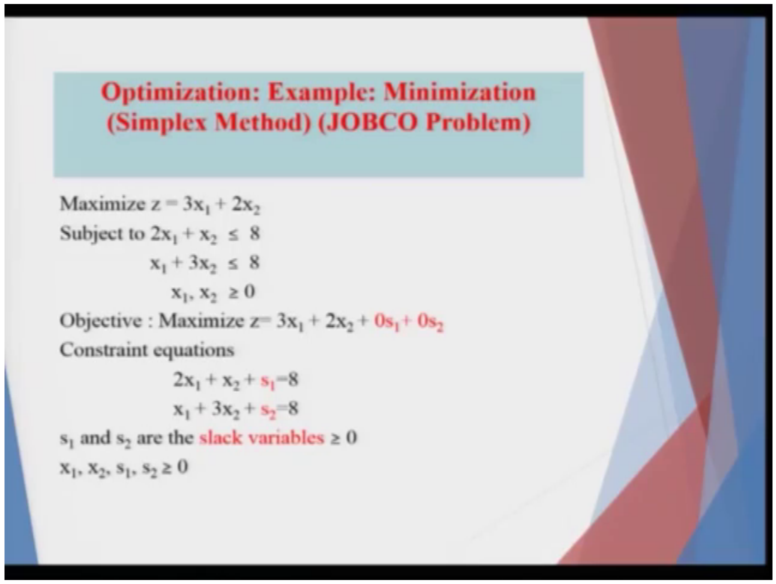
Now what we need to do is that we need to basically add the these variables in such a way that we will be ending with some initial basic feasible solution. So let the problem being stated.

(Refer Slide Time: 26:05)

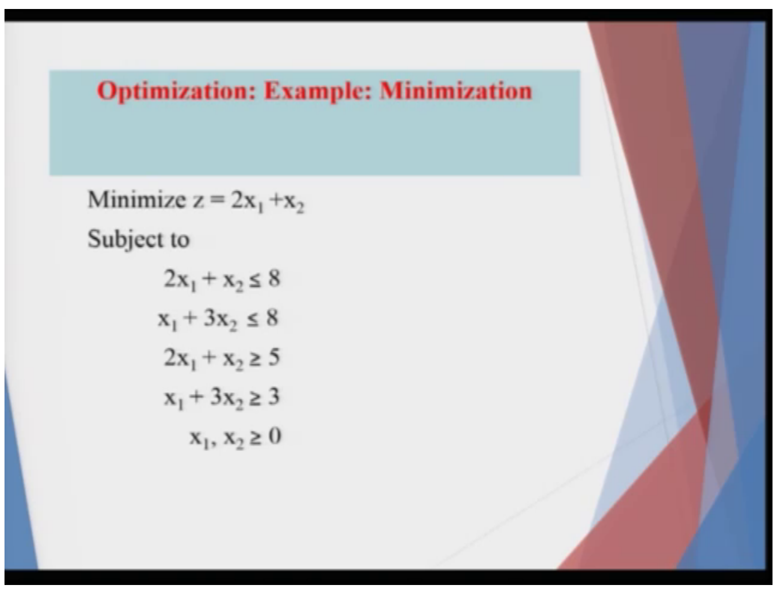


So you have, so in this case you had the problem. So let us consider this the third and the fourth constraints are not there.

(Refer Slide Time: 26:15)

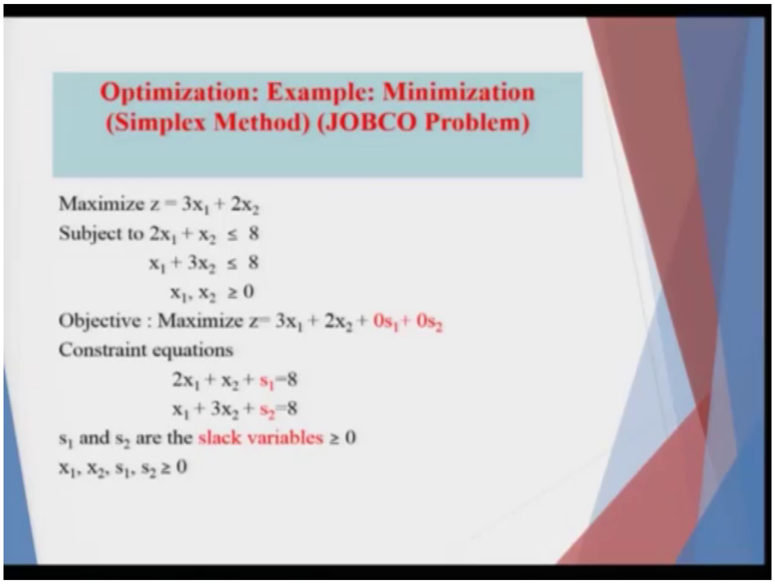


(Refer Slide Time: 26:18)



So it was 2 x1 plus x 2 is less than 8, 3 x1 plus 3 x2 is less than 8.

(Refer Slide Time: 26:24)



It can be expanded do not worry we will do that. Now they are all less than sign. So if there are less than sign when you are going to solve it using the matrix multiplication you want to basically add the slack variables. So let the slack variables here, there you consider x3 into x4, we will use a symbol of s, s1 and s2.

So the first slack for constraint 1 would be s1, the second slack for second constraint would be s2. And the corresponding values of x1 greater than 0, x2 greater than 0 remain. But also remain remember the slacks we will consider would also be applicable for the fact that they are also greater than 0. Which means if you had 2 variables and you are adding one slack, what you are doing is that you are now going from a 2-dimension problem to 3-dimension problem in the corresponding solution would be obtained in the exactly similar way if we had a 2-dimensional problem.

Same movement from corner points to basic feasible solution to one of the corner points and take that direction where the addition or subtraction from the objective function is maximum. So now as you add the slacks in the constraints. Similarly the slacks would be added in the objective function. Now see here very interestingly actually we want the slacks in the long run to be 0, why? If equality sign comes for constraint 1, constraint 2 which means that we are able to utilize all the constraint to the maximum possible extent without any loss and obtain the maximization problem.

Hence when I formulated the maximization problem, it will be 3 x1 plus 2 x2 is there plus the slacks if they were non-zero, they would definitely come, that means they would pulled it down or pull it up depending on whether you are looking at the slack or the surplus. I am using the word surplus also here. So it will be 3 x1 plus 2x2 plus 0 into s1 plus 0 into s2 as the objective function. And when you add the slacks and the surpluses, in the slacks in this problem, it will be converting the first constraint into 2 x1 plus x2 plus s1 is equal to 8. And the second one would be 3 x1 plus 3 x2 plus s2 is equal to 8.

Now what we have now look, m is 2 there are 2 equations, n is 4 now because first one is, so x matrix which will have would have 4 elements. The first element is x1, the second element is x2, third element is s1, fourth element is s2. So if I put x1 and x2 as 0, s1 and s2 would be corresponding values which you will obtain. So you will ask yourself whether that is a basic feasible solutions to start the solution. Because I do not know whether it is optimum or not optimum I want to start my iteration process.

So how you start the iteration process, I will consider that in the problem. But now initially you had a 2-dimensional problem. Now you have converted it into a 4-dimensional problem you will have matrix accordingly and proceed to solve the problem accordingly. Thank you very much and have a nice day.