Data Analysis and Decision Making-3
Professor Raghu Nandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology Kanpur
Lecture 16

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe and this the DADM-3 which is Data Analysis and Decision Making-3 course under NPTEL mock series and as you can see and as you know I would not use the word see this whole course duration is 12 weeks.

Total contact hours is 30, when which converted into number of lecture is 60 because each lecture is for half an hour and we are going to start the fourth week that means we have completed 3 weeks. So but the time you are going to listen to this lecture it is almost the same time where you would have taken the assignment 3rd that means after each week of 5 classes you have 1 assignment.

So we have already completed 3 assignments and as you know this being a 12 week lecture you will have such 12 assignments and after the end of the whole course you will be taking the final examination and my good name as you can see on the slide and it is I will repeat it is Raghu Nandan Sengupta from the IME Department at IIT, Kanpur.

So if you remember we were considering the concept of a basis or a vector and spanning so that means and the later part of the last lecture which is 15th one, I kept repeating the concept of rank of a matrix which is in a very simple sense is the number of such equation which are required to basically express the rest of the equations. So if rank in exactly could be the number of equation (which you have) which is equal to the number of variables. That means that you will have basically unique solution.

Now when which when convert into the concept of vectors, it is basically the minimum (numbers) dimension on the vectors based on which you can span or spread and find out each combination of the vectors which are there. So this is more from the mathematical point of view but when I comes to the actual implementation point of view the simplex method by Danzig it exactly uses the same concept which I am just repeating it.

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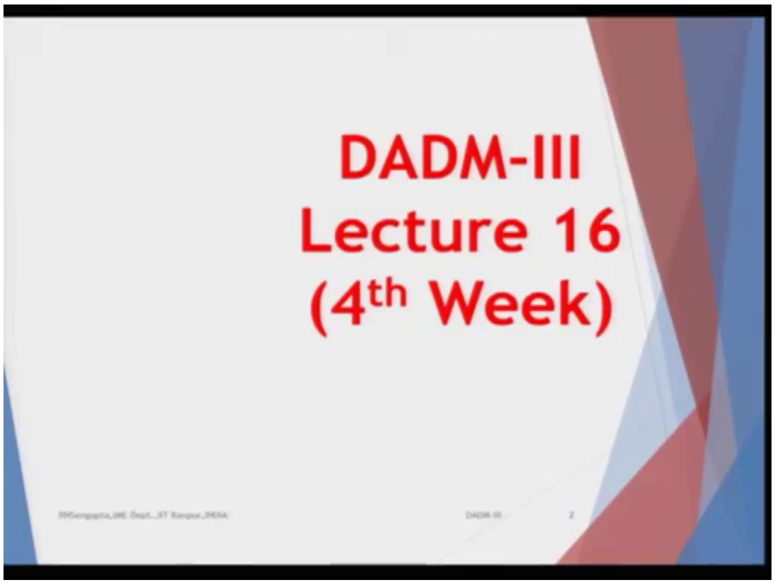

So as we are going to start the 4th week, so the slide show you the 4th week which is the lecture number 16 which is the first lecture for the 4th week.

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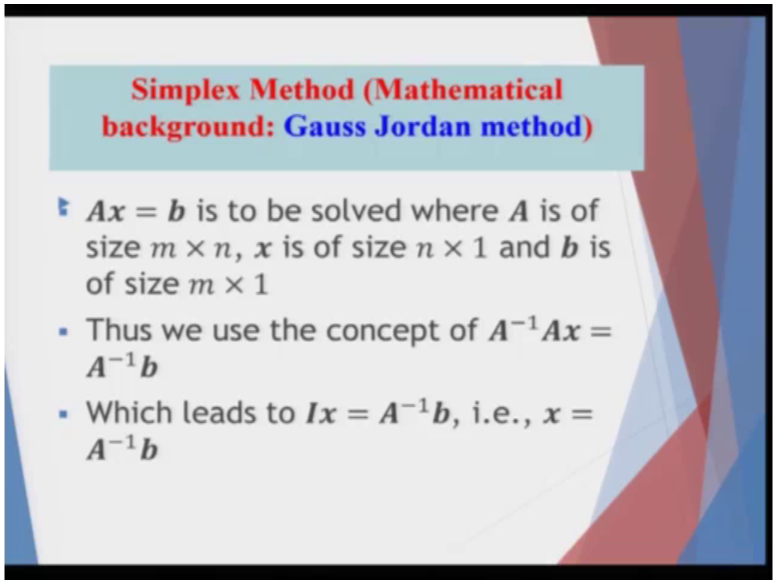

Now what we need to solve actually when you come to the matrix multiplication concept on the basis solution in the simplex method proposed by Danzig? The precursors is you have N number of equation to solve and N number of such variables to find out. So m or n which I am talking about is basically the number of rows which is the number of equations and n the column is the numbers of variables.

So if m and n are same then the number of and none (can), none of the equation can be expressed as a unique combination of the other, you will have the unique one solution for all the X's. But that would come later so actually what you have is you have in the matrix concept a matrix A multiplied by X who is a vector that is equal to B which is also a vector. So if the dimension of A is m cross n, the dimension of X has to be n cross 1 because it is a vector and the final result is m cross n multiplied by n cross 1 would give you basically m cross 1.

Hence the dimension of B which is on the right hand side would basically be m cross 1. So let me go back what I read and let me go back to the slide, B where A would basically have the parameters based on which you will basically multiply X's values and B are basically the right hand side of the constrains and X as the decision variable which you have or the values you want to find out for the variables.

So Ax is equal to B so this equality which is come out if you remember in one many of the constraints is less than type, greater than type. So those are immaterial because they can basically be brought down into the equality sign using various simple concept of addition and multiplication of variables which will consider as slack value and surplus which will make things very clear to you.

So Ax is equal to B is to be solved where A is of dimension of m cross n, m numbers of rows, n number of column, X is the size of n cross 1 and B is size m cross 1. Thus we are going to utilize this concept. Now this is very important even though I have not written it here it will become clear. Thus we are going to use the concept of pre-multiplying A with its inverse or post-multiplying A with its inverse the post multiplication of A with its inverse is not shown here.

But pre-multiplication and post multiplication the inverse should be exactly equal, so pre-multiplying A or post-multiplying A with its inverse would always lead to the identity matrix I. So once the whole process of multiplication on the left hand side you are doing some calculation you are converting A slowly, you are converting the rows or the column however it is, slowly A gets convert into I.

Now the same process of steps you are also doing on the right hand side because you want to balance the equation like it is just like you balance the weights if you take out something from the left balance weight which you have, then you basically take out the same quantum of same product from the right hand side of the weighing scale, if you want to add you add.

So do you are just doing the balancing act like you have 2 buckets of water and then you are trying to balance, so taking out some water or fluid from the left bucket would also result in the same steps being taken there. So as A is being converted into identity, identity matrix the same operations of the rows and column are being done on B, the reason being that when you are pre-multiplying A with A minus or post-multiplying A with A minus you are doing the same thing on the right hand side.

That means pre-multiplying b with A minus or post-multiplying (A with) B with A minus. So at the end of the whole process step by step which you do, why I am saying step by step? Because if you go back to the fundamentals of matrix multiplication by inverse, what you do is basically find out the determinant and the concept and trying to find out the transpose considering the row and column are fixed such that the end result gives you the A minus.

So each and every step you do that and at the end of the results on the left hand side you have X only pre-multiplied or post-multiplied by I that is immaterial I is one only and the right hand side you have A minus A invert A minus into B. So hence the concept of row and column is maintained and post-multiplying or pre-multiplying would give you the same answer.

So hence when the whole process is done, so (each) X is basically a vector, on the right hand side obviously you would also have a vector so the end of the results would be the first element of x which is x1 would be equal to the first element of the combined combination which you have A inverse, A knot my A minus, A inverse into B. Similar the second element which is x2 would be equal to second element on the right hand side.

So the whole operation would give you the unique values of x, hence those would be the solution based on which you can find out the simultaneous equation. So this concept will be utilized time and again when we use the concept of simplex method proposed by Danzig that is all. The concept of Gauss Jordan concept. So let me read the second bullet point thus when we use the concept of A inverse A that will give me A inverse b and once the whole operation is done I will have Ix which is identity x is equal to A inverse b such x would be given by the elements of A inverse into b.

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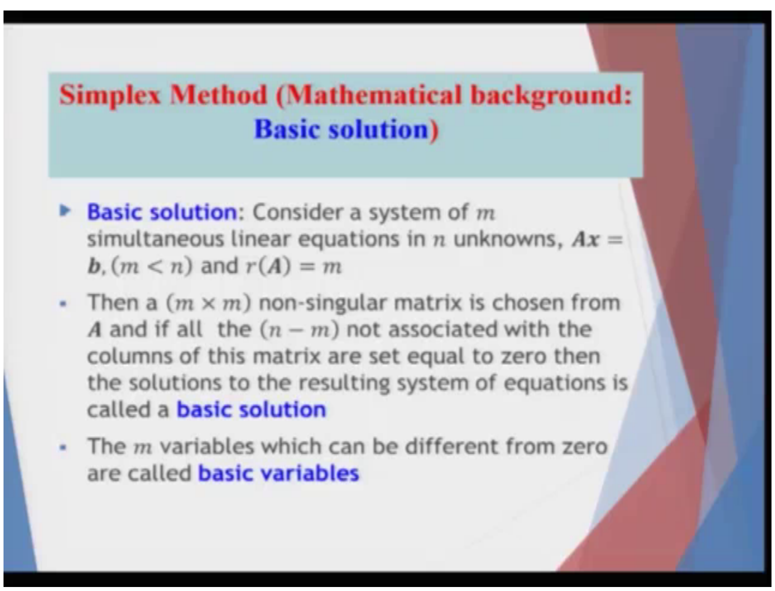

Now you want to start the solution. So that solution based on which will start, the starting point, if you remember in that initial problem I started at the origin. So that would basically its starting point. So that will be consider at the basic solution, now whether the origin or basic solution based on which we are going to start is of one of the corner points in the feasible set that has to be noted down.

Because if the 0 point is not amongst the feasible set then trying to start your problem from the 0 point or the origin does not make any sense. So first you have to check the point based on which you are going to start of your iterative process that means the point depending in from where you will start your slope ascending or descending depending on whether it is maximisation problem or minimisation problem will depend whether that corner point is one of the feasible point in the overall space.

And as you start moving you will all traverse the feasible points only because if you start from (invisible) in feasible point then trying to come back into the feasible region may not be possible that is not doable. Hence we have to basically check the basics feasible solution is a doable one based on which we start the process. So consider basic solution consider the system of m simultaneous equation in n unknowns.

Now this is important to note if I remember I have mentioned that in the last class at some point which is in the 15th lecture that you have m number of equations and n numbers of variables. So m and n matches and all the equations are unique, none can be express for the convex combination of the others you have a unique vector of solution of x1 to x n. Now if m and n are different, so it would mean that either m is more or n is more that will give you two different pictures. So here we will consider that m and n are different, later on we will find out that whether m is greater or less than n.

So consider a system of m simultaneous linear equation in n unknowns, in that case ax is equal to b is the actual equation or format which we have to have. Such in here it says that m is less than n, hence the rank of m is m of a is m that means the minimum. So technically it would be the minimum number of rows and column and it can be proved. So we will consider the minimum size of m or n depending on the convex combination such that it is able to express all the rows and columns by that convex combination which you have. So the row operations and column operations if you remember in matrix (multiplication) this is exactly what we will try to utilize.

So I was trying hint at that time I am sure you have got it. So then what you will basically have is considering that m is less than n, you will take a set of m cross (n) m, m which is in mango, n as in nose. So then m cross m non-singular matrix, non-singular means that the concept of determinant would exist or else you want to find out the inverse and if the determinant does not exist then that is a problem.

So then m cross m is non-singular matrix is chosen from A and if all this rest of n minus (m) n, so n is more, so consider m is 10, n is 20. So you will 10, 10 cross 10 number of variables and equations and expressed the rest 10 by the convex combination of them. So let me read it, if all the n minus m not associated with the columns of this matrix which you have considered as to be independent or actually set to 0, because they are not to be expressed, you consider those variables to be 0 then the solution to the resulting system equation is called a basic solution.

Whether that basic solution is feasible or not we have to check which I mentioned few minutes back. These m, now remember this m has to be unique, it need not be only unique, unique in the sense from where you start and also remember this m is I should not be using the word unique that means only 1 set of m combination which you have there, there can be different sets but the combination of all of them is always m in size.

Now which m you will chose such that you starting basic feasible solution would lead to your optimum solution is also a point but we still consider the initial that the basic solution would be there. So the m variables which can be different from 0 are called the basic variables. So say for example I have x1, x2, x3, x4, x5 as the number of n and out this if that means x1 to x5 which is 5 is n and if I am able to express the set of equation as 3, 3 cross 3 vector I take.

So these 3 variables consider it is x1, x2 and x5 are non-zero, while if I mention x1, x2 and x5 are non-zero then I will consider the variables x3 and x4 are 0 such that vector based on which I will start of the solution would have x1 a non-zero value, x2 are non-zero value, x3 a 0 value, x4 a 0 value, x5 a non-zero value. So based on that, that I will consider a basic solution and start. Now it does not mean that I can also have a basic solution where I can start of the solution can be x1, x2 are 0, x3, x4, x5 are non-zero in that case the vector based on which I will start of my solution with the basic solution can be x1 is 0, x2 is 0, x3, x4, x5 are non-zero, so this can be possible.

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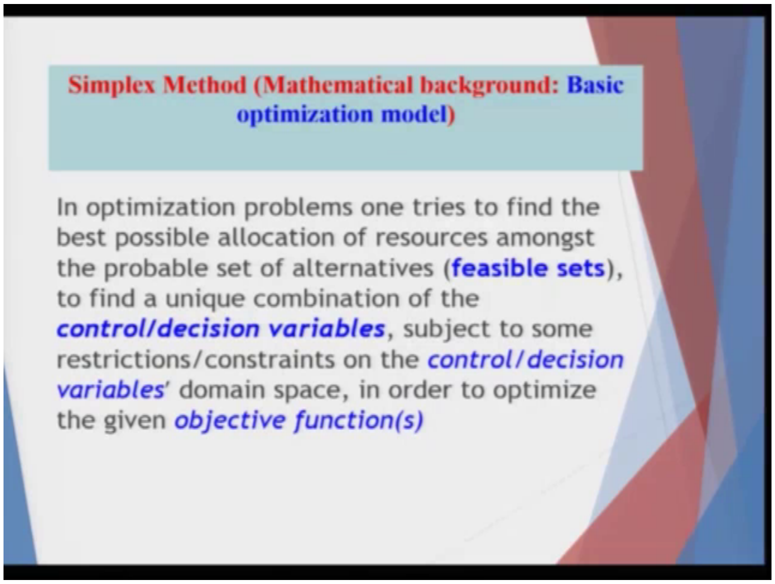

In optimization problem one tries to find the best possible allocation of resources so I will basically now go into the general formulation and then again come back to the solution technique. So is basically I have given you the general field how you will solve that but how the greater than (equal) less than values would be considered and will basically state in a very general format.

So whenever we are solving an optimization problem your (our) main concern is to find out the best (solution) possible solution of allocation of resources. So resources can be say for example (())(15:16) resources can be say for example some food material you want to give to different people or children who are undergoing some malnutrition treatment required or it can be say for example resources you want to allocate to different states in India to build a bridge or you are the CEO of a company and your resources are constraint and you want to build up say for example 5 roads and you are in the civil construction, so that would also mean the resources are there which you want to optimize in the possible manner and that optimization would basically come as a maximization or a minimization problem.

So it means that best possible allocation of resources amongst the probable set of alternatives which are there, so these alternatives (where) which I have mentioned at the corner points. The whole set is the feasible region, so the corner points would be the search set where I will basically restrict my search. So they would be the feasible set and we want to find a unique combination of the controller of the decision variables subject to some restriction and constraints on the control variables.

Like say for example 2 x1 plus 3x is less equal to 8. So in the problem of trying to solve for the machines, machine utilization is maximum in a day is 8 hours or one person cannot work more than 8 hours. So in that case the constraints would basically specify that 8 hours is there how you will basically try to utilize the machines in the proper manner to get the (optimums) optimal solution.

So you have restriction and constraints on the control decision variables in the domain space in order to basically optimize a given objective function or a set of objective function considering it is a multi-objective problem. So this is the statement and then let me write down the problem in a very simple manner.

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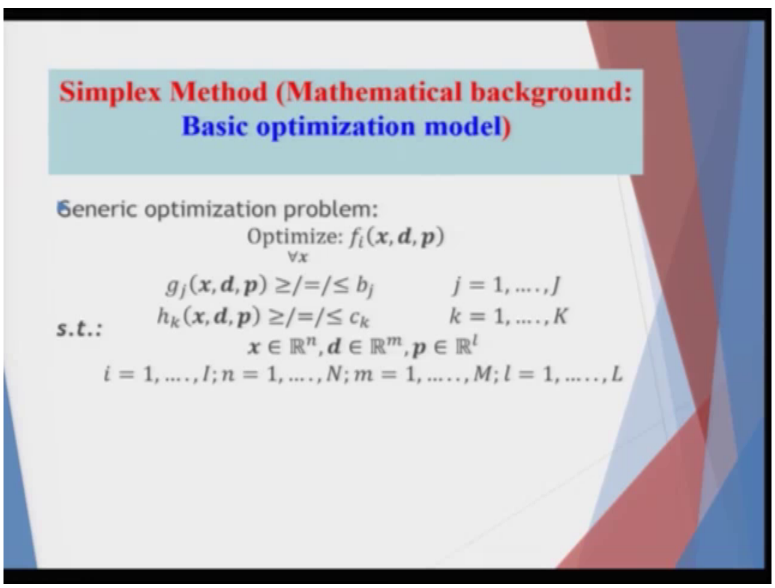

The equations looks does not look ominous they are very simple. So our general optimization problem, so this variation of the problem will be occurring later on. So one can be simplex method, one can be 0-1 programing, one can be integer programing, one can be goal programing, one can and then we can bring the concept of time basically solve it as dynamic programming, it can be stochastic programing considering the distribution of the variables which are there or the parameter which are there, then one can be mixed integer linear programing.

So there can be different combinations, so I will consider you want to optimize some objective function and if i, if you see i here, if i is 1 it is a single objective function based on which we are trying to solve the first set of the problem. If there are more than one i's you will basically have f1, f2, f3 that means you will basically have a multi-objective problem if it is 2 objective it will be a bi-objective problem.

Now on the constraints sides I will basically differentiate here I am trying to differentiate but in the later part when we solve the linear programming there would basically be only one set of constraints. Some of the set of constraints, why I am differentiating that? I will understand later on when we do the probabilistic problem. So you have once J number of sets of variables which can be a greater than type, less than type, equal to type and you have g Js depending on x with the decision variables, d is the deterministic parameter.

We are not going to consider the deterministic parameter for the initial major part of the lectures, p is the parameters which are non-deterministic again we are not going to consider the set of non-deterministic parameter in the first huge number of set of lecture which you have and b is basically bJ is basically the right hand side of the constraint which you have.

So initially this J number, capital J number of constraint which I have initially we will consider them deterministic but later on we can change them to the probabilistic also such that we will be able to solve the reliability part and the robust optimization problem as such. The second set of constraint which is capital K in numbers, small k is equal to 1 to k are the deterministic constraints.

So I am trying to differentiate the deterministic and the probabilistic in this simple formulation, but when we solve the problem we will do away with the probabilistic part. So you have capital K number of constraints again you have x as decision variables, d as the deterministic parameter values and p is the probabilistic parameter value in this second set of K, capital K number of equations we will consider that the probabilistic part is not there, hence p may be 0 such that we have deterministic constraints.

Again it is greater than, equal to, less than c k, we see is basically the right hand side of the equality, equation which you have. Here we will consider x to be of dimension n that means you have n number of variables to solve, decision variable to solve, d the deterministic variables (in) would be in the m dimension, this m and n has nothing to do with the m and n which I have consider in the matrix multiplication and p the parameter which are non-deterministic actually later on but still in this set of problem we will consider them to be 0 in the case when you are considering the deterministic problem, L is the dimension of p.

Here I is equal to 1 to I, capital I depending on the number of such objective function which you have. Small n is equal to 1 to n, small m is equal to 1 to m and small l is equal to 1 to capital L.

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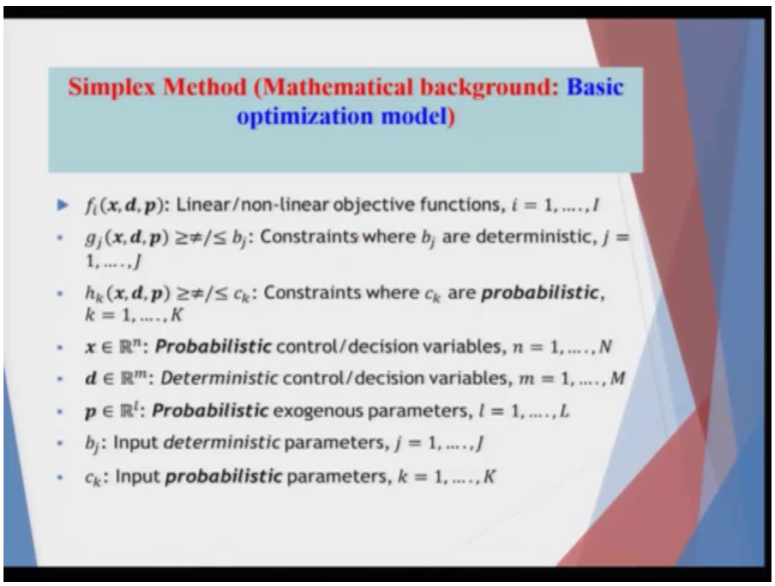

So what are these variables? If I, I am not talking about what is inside x, d, p are the decision variables, deterministic parameters, probabilistic parameters is a linear and non-linear objective functions of capital I in number. So we will consider the linear part with one objective function for the simplex method, g js again function of x, d, p variables, deterministic probabilistic parameters are constraints where bj are deterministic in nature. The second part ok, I basically swapped that explanation of the deterministic and probabilistic one.

H k's would basically be again the equation on the left hand side, which basically is the function of x decision variables, d deterministic parameter, p probabilistic parameter and the constraints would be (equal) equivalent to c k which are probabilistic in nature and they are capital K in numbers. Similarly is for the first set of constraints is capital J in number and as I mention x, d, p are the probabilistic control variables I am mentioning the time and again please bear with me, d is deterministic control variables and p is the probabilistic exogenous parameters where the size of x, d, p which depend on the dimension is capital N, capital M and capital L, bj is the input deterministic parameters which is very apparent from the explanation which I gave and c k is the input probabilistic parameters.

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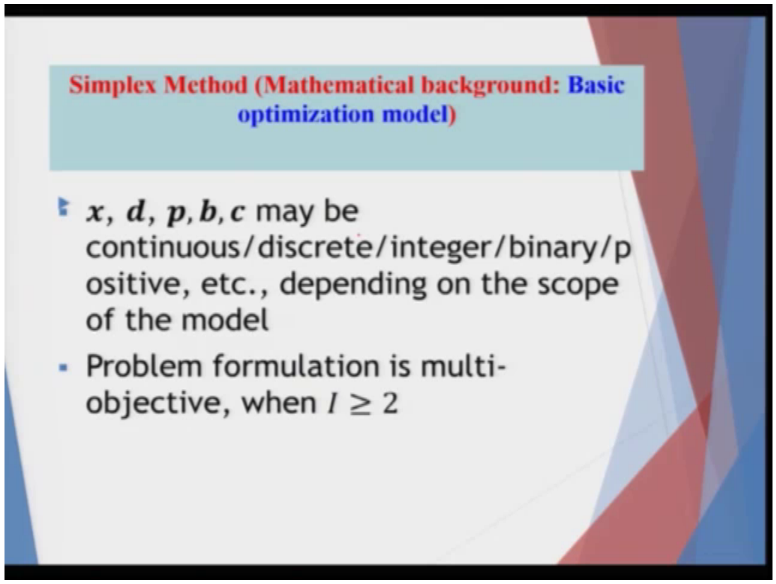

Here x, d, p, b, c may be continuous, discreet, integer, binary, positive, whatever it is depending on the scope of the model. For the case of the linear programing we will consider the continuous one and positive integers if you remember x1 and x2 is greater than equal to 0 and x1 and x2 are continuous for the case if I mentioned I am mentioning time and again. If it is a problem of solving trying to find out the amount of production of paints it was continuous but if you are trying to find out the number of trucks which will be utilize to transport goods or the number of factories which will be build it is integer in nature or the number of boxes would be transferred, would be integer in nature it can be binary.

Build a factory, not build a factory, build the road, not build the road so it will be 0 and 1 and I said an integers obviously give the answer, binary I would say (again) give the answer positive would also be applicable. The problem formulation would be multi-objective if i is 2 or more.

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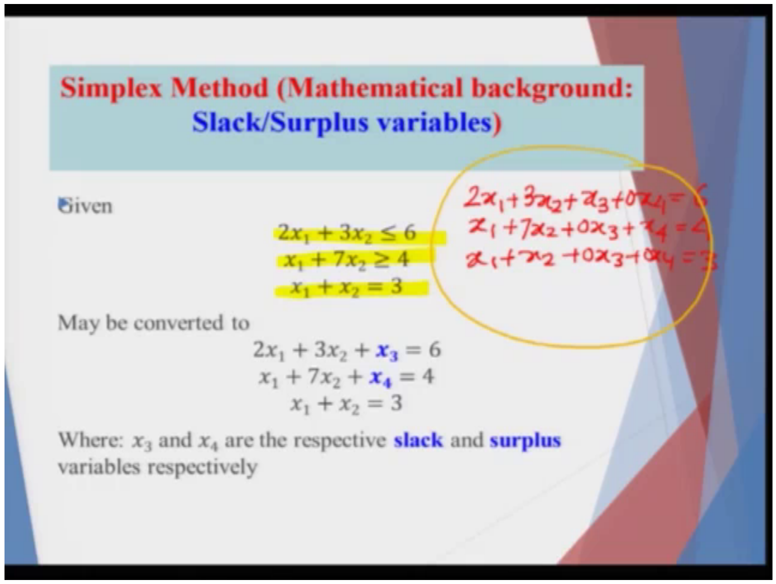

Now, we will basically give the concept of a slack and a surplus and why slack and surplus? Will be applicable you will understand it very simply. So this is, from the solution technique. So consider it has nothing to do with objective function consider the constraints are given as 2 x1 plus 3 x2 is less than 6, x1 plus 7 x2 is greater than 4 and x1 plus x2 is equal to 3. Now if you see the first constraint and the second constraint the left hand side is lower than 6 or equal to 6 depending on some combinations of x1 and x2, but not for all.

For some apart from 1 for the rest of the values the left hand side would always be less than 6. So some amount has to be added which we will consider as the slack such that it when is added to the first equation would make the equality hold true. So this x3 would also change its variables such that depending on x1, x2 and x3 by equality can be (hold) held for the first equation.

Similarly when I have the second equation is greater than 4, I am basically need to add a surplus such that the variable which is x4 would basically bring the equality in the second constraint so there would be some values for x1, x2 and x4 such that equality would hold true. Now look at it very carefully, if you had an equality sign in the first equation, second equation and third equation was not there, so your actual equation would be 2 x1 plus 3 x2 is equal to 6, x1 plus 7 x2 is equal to 4. You solve the problem get unique solutions x1, x2 and your simultaneous equation is solved and you are happy.

Now consider these inequality signs comes, so the moment you want to solve them using the equality 1 you are need to add as I said x1, x3 and x4. Now when x3and x4 are added to the first and the second and not added to the third one actually what the third one is doing and how you formulate would be? You will basically have 3 equations, first equation corresponding to I will just mark them, so this would be the first equation, this would be the second equation, this would be the third equation. So m would be 3.

Now let us add the slack and the surplus, so if I consider the slack and the surplus for both all the 3 equations (x1) of first, second, third. So the first equation is actually converted into 2 x1 plus 3 x2 plus x3 is equal to 6. Second equation when you add the slack or the surplus to convert the greater than equal to sign to equal to sign it will be x1 plus 7 x2 plus x4 is equal to 4 and the third one which I have not written actually when this surplus and slacks are added in order to make sense in this equation would be actually x1 plus x2 plus 0 x3 plus 0 x4 is equal to 3.

So the number of n in this equation would be 4, n is 4, m is 3. So if I have the actual equations written in front of me I will just write it in the slide (side) and make you aware of that. That is what is the matrix based on which you will start the solution. So I am using red colour, so it will be 2 x1 plus 3 x2 plus x3 plus 0 x4 is equal to 6. Second question becomes x1 plus 7 x2 plus 0 x3 plus x4 is equal to 4, and if I come to the third equation it will be x1 plus x2 plus 0 x3 plus 0 x4 is equal to 3. So if I consider the whole set of equations, so it will basically have 3 as m, 3 equations, 4 as n, 4 variables.

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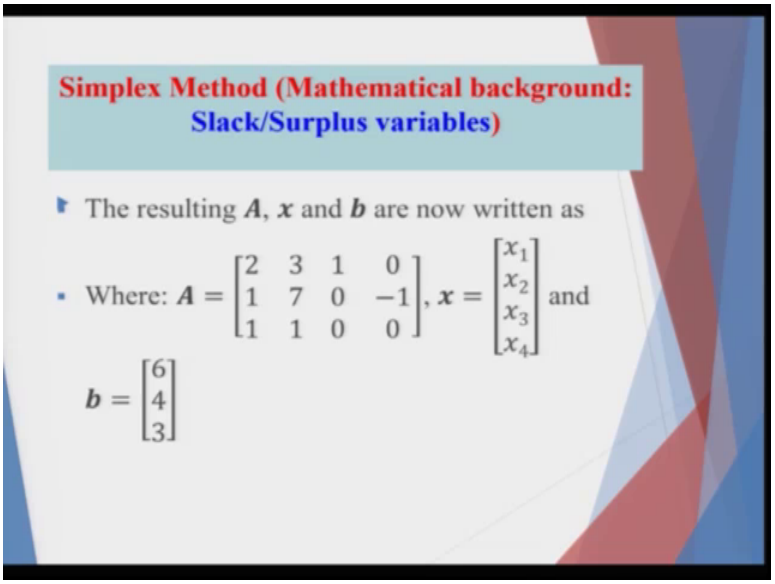

If I write the resulting x, A, x and b are now written, the matrix A would have basically 2 as the value which is multiplying x1, 3 as the (value) variable which is multiplying x2, 1 and 0 are for x3 and x4. Similarly for second equation the values are 1, 7, 0, minus 1. Minus 1 because when you are converting into the greater than sign and the less than sign you have to bring the equality, and the third equation is 1 1 0 0, 0 0 being for x3 and x4 so the b values are 6, 4, 3 and if want to solve the problem consider I can do it like this

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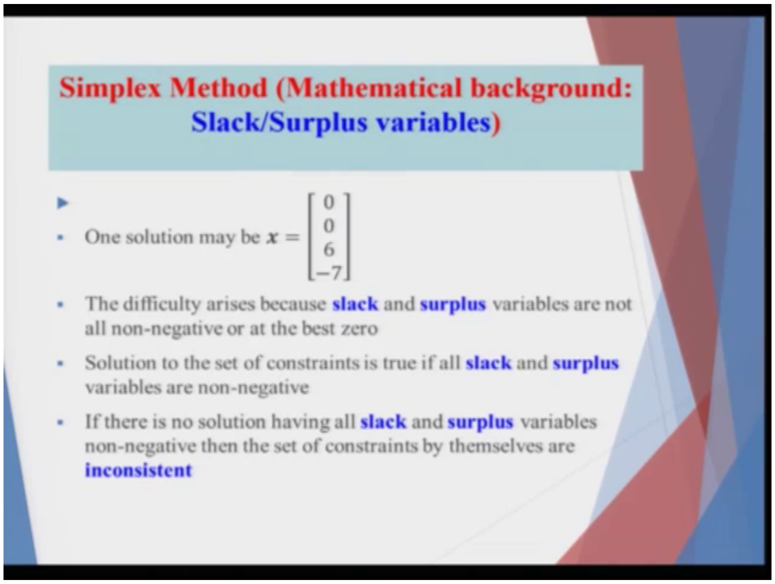

One solution can be 0, 0, 6 minus 7 that means the values of x1, x2 is 0, slack and surplus values are given as 6 minus 7. So the difficulty arises because slack and surplus variables are not all non-negative or at the base 0. So they have to be non-negative, in order basically give you the impression that the basic solution is true.

Solution to the set of constraints is true if all slack and surplus variables are non-negative here one is non-negative, it is negative. So if there is a solution having all slack and surplus variables non-negative then only the set of constraints by themselves are inconsistent hence you would not be able to get the actual solution to start with. So with this I will end the first lecture in the fourth week which is the 16 lecture and continue discussing more about the constraints and the feasibility of the space and then how we will use the simple Gauss Jordan to solve the problem. Have a nice day and thank you very much.