Data Analysis and Decision Making-3
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Lecture 13

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe. And as you know this is the DADM-3 which is Data Analysis and Decision Making-3 course under NPTEL mock series.

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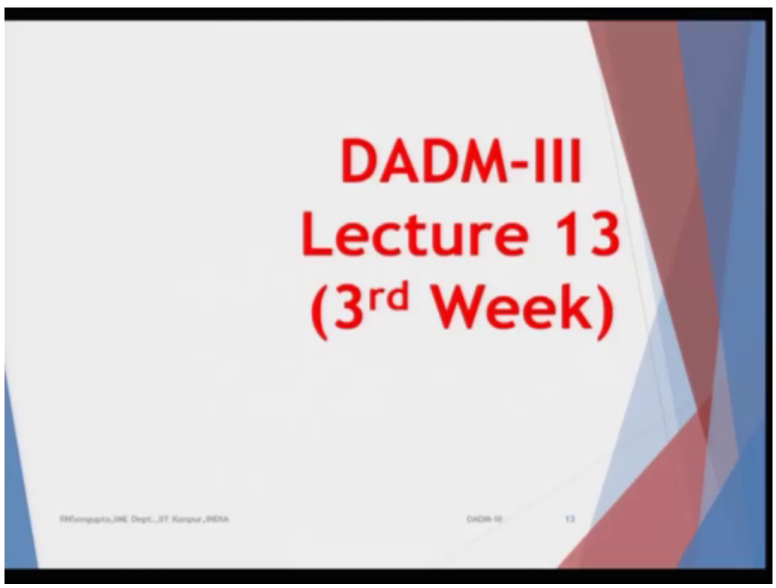

And as you can see from the slide, this is the 13^{th} lecture which is we are in the 3^{rd} week, 3^{rd} class in the 3^{rd} week. And this total course is for total contact hours is 30 hours, which when converted into number of lectures is 60 because each lecture is for half an hour. And the total duration of the weeks is 12 weeks. Now this and as you know that each week we have 5 lectures and after each 5 lecture we have 1 assignment, so you have already completed one assignment, two assignment sorry after week 1 and week 2. And after 12 assignments after each total 12 weeks you will basically be taking the final examination.

So if you remember I was discussing yesterday in a very simplistic way that given an optimization problem whether maxima, minima and the dimensions was such that you can basically utilize the Cartesian coordinate to draw it. You can basically first draw the feasible region which is basically the bonded area correspondence to the constraints which you have. And also remember that in the linear programming case, you will have one of the, two of the constraints considering its the 2-dimension problem that X1 would be greater than 0 and X2 would be greater than 0, greater than equal to 0. And in higher dimensions, if you have more than 2 variables like X3, X4, X5 all of them would be restricted by the fact, that all of these decisions variable should be greater than 0.

Now again one thing I am repeating in time and again here that when you are solving a linear programming problem, there would be instances where the actual problem would give you actually, in practical since the value should be integer. But we are not going to consider that, when you are going to solve the linear programming problem, that will only come up in the case when you solve the integer programming problem.

And point 1, point number 2 is that I also mentioned that when you are solving the linear programming problem, considering the feasible region actually if you want to find out the objective function at each and every feasible point, there would be infinite such feasible points. Hence, the total number objective function to be solved would be infinite number. But considering the simple concept of the corner point solutions you will only consider your feasible search to be the corner points.

So in the problem which we solved, if you remember, I am not going to draw it I am just going to mention it. So you had the points A, B, C, D and D was the basically the origin, A was the point where it was on the X on the Y-axis. And C was the point which you have on the X-axis and we found out that the objective function as it moves outside and basically leaves the feasible region, so that point is basically the maximum point considering is in optimization problem which is a maximization one.

In the cases of minimization one, obviously we move more towards 0, hence basically you will basically leave the space when you want to solve the problems. So the point at which the minimum value occurs when the objective function leaves the feasible region would be the minimization problem. So in the problem sorry 1 minute, so in the problem which we solved the minimization problem would have been true when the objective function left the point D, which was the origin. So but obviously there would be points also when we consider the feasible region where which would be non-zero corresponding to the fact the origin is not the point when the minimization occurs, point 1.

Point number 2, also I mention that the general concept of trying to solve the problem would be that you start at any 1 point out of this A, B, C, D so this is very easy to visualize .So if we start at the point D, you will basically take the direction to move in the case if it is an optimization problem whether maximum-minimum to find out at which in which direction the rate of change of the objective function with rate of change of any individual decision variable is happening as the highest rate or the lowest rate depending on whether it is an optimization problem to maximize or minimize.

Now this search technique would be utilized point 1, this is point number 2, point number 3 also is that I have very fleetingly mentioned that when you are trying to solve the problem we will try to use the simple concept of Gauss-Jordan matrix solution method, where you will be tempted to utilize the Gauss-Jordan method such that the overall solution when you solve the problem will give you the actual answer based on which you can comment whether optimization problem has been reached whether it is the maximum or minimum.

Now the optimization problem would be reached based on the fact that, when you are solving the maximization problem for the last example, the point B was the case when the objective function was taking the maximum value. So any directional movement from point B either to point A or to point C whichever direction it is, you will find out the rate of change of the objective function would be such that the objective function will decrease, so hence you have to basically stop at the point of B and that will also come out in the problem when you are going to solve.

Okay another thing, in the last example we considered the concept of shadow prices over the fact that 1 unit change in the constraints, so there can be more than two constraints and I told you that for the problem the constraint in the decision variables would basically be changed depending on how you are thinking about the problem and this concept or trying to pick up the idea of which variables to consider and which variables not to consider would basically come from your experience.

So when you are trying to basically increase or decrease, the decrease part I did not consider when you want to (decrease) increase or decrease the constraint either for machine 1 or for machine 2 you found out that the decision the feasible region obviously would change, hence your (search) space would also expand and contract but the corner points in this case it would basically remain the 4 points, so in this case we have basically renamed that with a suffix 1 or 2 or 1 and 2 depending whether we are changing both the constraint 1 and 2. And this idea would be extended further higher dimension also. So let us basically proceed the problem .

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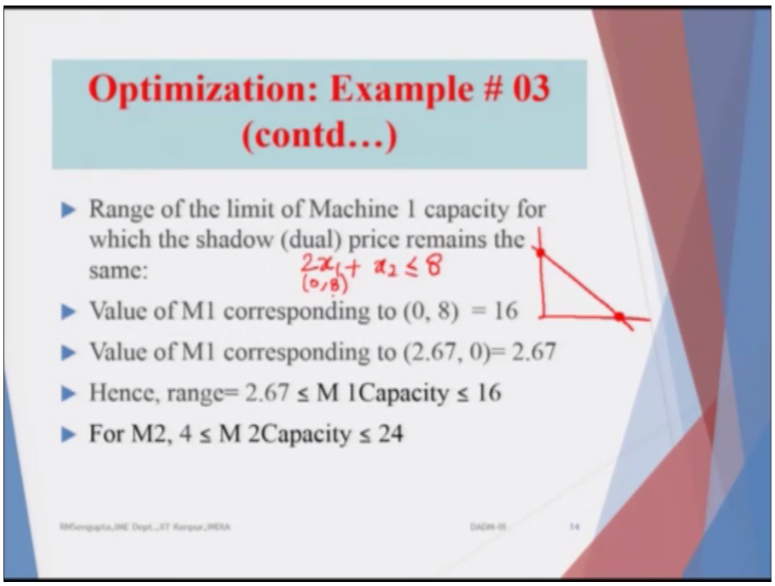

So the range of the limit of machine 1, capacity for which the shadow price or the dual price remains the same would the case when the value of M1 which is the machine 1 corresponding to the fact that it is 0 and 8. So 0 and 8 would basically be the value of 0 would be for X1, 8 would be for X2. So if you put that in the constraint 1 it will basically be 16. So in that case, that whole overall utilization of the machine 1 is being done in the proper manner.

Now when the value of the, when we take the value of X2 as 0 and X1 as non-zero value that means if I consider the X- axis and Y-axis. So I am considering, so this is the constraint 1, so this value corresponding the fact, so these are the two values .So in one case if it is X1 so this is 2.67 and 0 this value is 0 and 8. So the constraints would be such that the values of the utilization of machine 1 would basically dictate that (how) what is the utilization actually as per the norms the total utilization which you have.

So when I consider the value of machine 1 and machine 2, so let me go back to the slide so it will be easy for me to explain. So it is M1, so I will write on M1 and M2 also. So it is 2 X1 plus X2 is less than 8. I will come back to equation 2 also, so 2 X1 plus X2 is less than equal to 8, so this is 2 X1 plus X2 less than equal to 8. So when we put X1 as 0, X2 basically becomes 8. So the overall utilization would be such that your points are 0 and 8 and this 2.67

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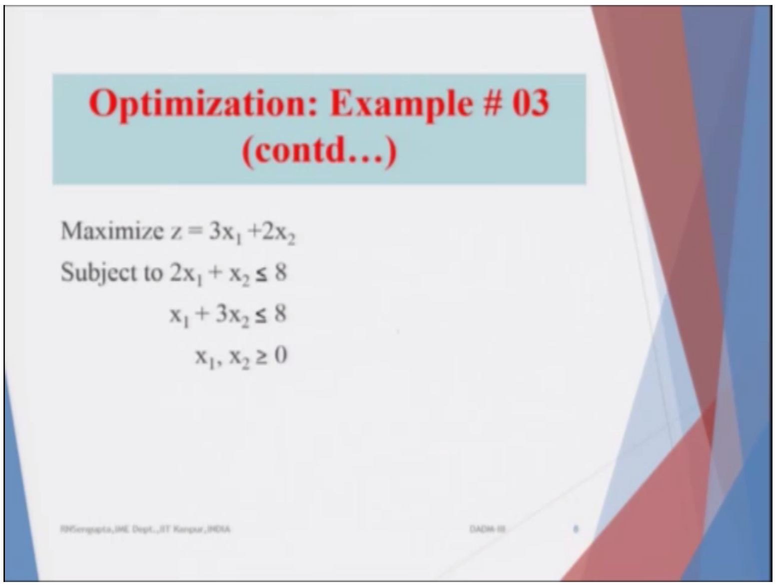

Let me go back. So if you solve this equation, so you will get a value which is corresponding to 2.67

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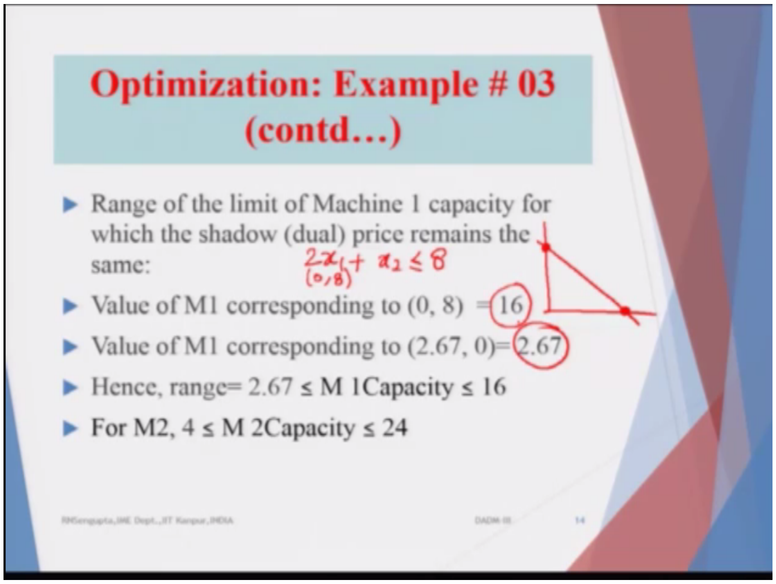

So hence the (utilization) so this is the minimization, minimum utilization which will get from machine 1. So maximum utilization from machine 1 is 60 and minimum utilization from machine 1 is 2.67.

So the overall capacity utilization if I consider, in one case it would be 2.67 by 8 percentage and in another case it would be 8 by 8 percentage. In one case it is 100 as it is here, another case it is less than 100. So hence, the range of utilization of capacity utilization from machine 1 considering the 8 hours is the case which has been given in constraint 1 is between 2.67 and 60. Now when I go to machine number 2, so let me bring down machine number 2.

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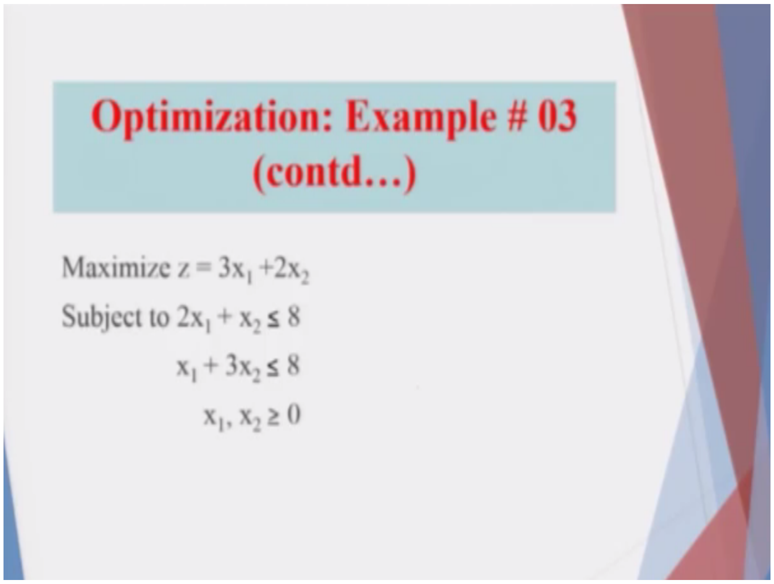

So it is X1 plus 3 X2 is less than equal to 8.

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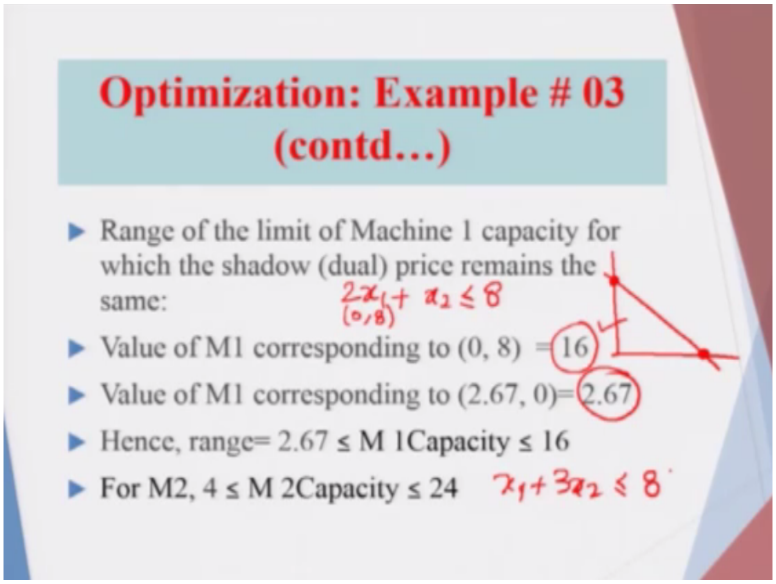

X1 plus, so in this case if I am able to utilize the whole capacity of M2, so in this case the overall capacity maximum and minimum would be given for machine number 2 or the constraint 2 such that we know the range or the limitation of machine 1 depending on the shadow price and the dual price considering that they remain at the same value.

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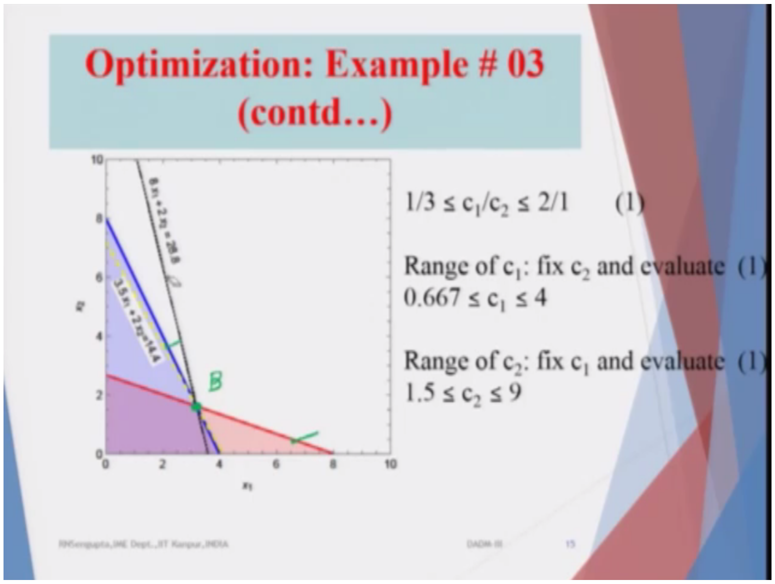

Now, consider the value of the optimization problem as the ranges are changing. So, when we will be considering the range change, so you will basically consider that either the value, consider the its like this. You consider your objective function and the decision variables X1 and X2 to be the point at which you have obtained the optimum values which is this. Let me use a different color. So this constraint was initially true, this one was also true. Now consider that you want to achieve the same value but you want to basically change the combination of the X1 and X2 such that you obtain that values of X1 and X2 obtain the same objective function.

So, what you do is that in point case 1, if it is so I am just basically putting this pointer parallel to the blue line, so what can be done is that I can either rotate it clockwise with the point of rotation being point number so this was basically B, that was the original one, it will keep rotating on the clockwise direction. So what is happening is, the utilization of product 1 which is X1 would start decreasing while the utilization of product 2 which is X2 start increasing will start increasing keeping in mind that the constraint remains the same, point 1.

Point number 2 is that and (it will) I will continue to what range it is feasible, so the range of feasibility would be till what point it basically touches that value which is along the Y-axis .Now for constraint 2 again, which is the red line where which is just below the pen which I am placing it on the slide. So again I can rotate it anti-clockwise and clockwise depending on whether you want to basically increase or decrease X1.

So any increase and decrease of X1 or X2 would be such that the overall objective function remains same, the value of X1 and X2 remains same but to what range they are feasible that would basically come out to what degrees of rotation is possible, degrees means, possibility means, to which level they are feasible, it will be true. Now remember one thing as you do that the feasible region will also change overall. But the overall fact remains the feasible set, the actual values or optimum set where you will do the search considering the corner points remain the same are fixed.

Fixed means, not in values but the numbers are fixed. So say for example if I keep rotating the blue line clockwise. So which means the point of where it cuts or it touches the X-axis which is X1 and touches the Y-axis which is X2 would keep changing. So hence the values of A increases, values of B remains the same, values of C decreases, while value of D again remains the same, so the overall feasible region changes but the corner points basically changes corresponding to the fact that point O, D and B remain fixed.

So when I am basically able to find that? That will give me the range based on which or the ratios based on which I can find. So the ratios which will be utilized to find out the values of X1 and X2 would be or giving the shadow prices I will basically calculate using the values of C1 and C2 would be given between the range of 1 third and 2 by 1 that means, that is what is the ratios per unit range unit values which X1 and X2 basically change keeping in mind the constraint and the objective function.

Now in the case if I fix C2, the rate of change of the function for product 2 correspondingly then the values of C1 range are between 1 by 3 to 4 such that it will give me the values of maximum and the minimum profit which can be obtained. Similarly for the fact, when I want to change C2 keeping C1 fixed the values are 3 by 2 which is 1.5 to 9. And in the higher dimension obviously it becomes that you will have many such points, many such planes which are the hyper planes will be rotated in different directions corresponding to the fact that the change in X1 and X2 or X3 or X4 depending on (how many) whatever the dimensions are would give you the values, how you basically find out the solution.

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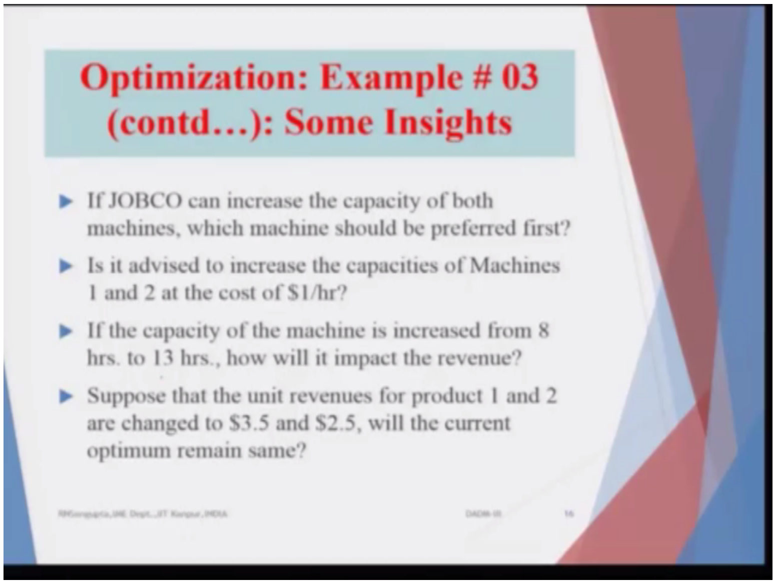

Now the questions would arise, so if this company can increase the capacity of both the machines, so which machine should we prefer? So obviously the question in a very layman terms would be, it would prefer the machine where the increase of unit cost or the total cost producing product (M) X1 and X2 is the least out of these two constraints because there are two machines and the corresponding increase in the objective function is also maximum .

What we will consider is that, objective function value is increasing, cost is also increasing, so (you will) I will basically make a compromise that the rate of change per unit increase in the cost what is the per unit additional in the objective function value in the positive sense. So we will take that machine to be true, so in this case we have two machines so we will only consider two values in order to make the comparison.

We will also consider, is it advised to increase the capacity of both machines 1 and 2 at the cost of 1 dollar per hour? So that would basically mean that as the constraints increases, as they shift more towards away from origin in more into the quadrant 1, both for constraint 1 and constraint 2 considering the objective function is the maximization problem, we will basically try to find out, that how does the value of B change? So in one case, if only constraint 1 is changing, B changes to B1, if only constraint 2 is changing in that case B changes basically to B2 if you have seen that. And if both of them are changing by 1 unit, it will be basically be changing B is changing from B suffix 1 and 2 which I had basically shown on the last day.So we will basically consider that how it will be done.

If capacity of the machine is increased from 8 hours to 13 hours, how it will impact the revenue? So if the capacity is changing that means both the constraints which is machine 1 and machine 2 will shift parallelly by 5 units so constraint 1 also shift 5 units, constraint 2 will also shift 5 units both of them change at the same quantum, so in that case you will basically have different value of A, a different value of B, a different value of C but the value of D will remain the same because that is the origin. So you will basically try to find out that, what is the objective function considering that you have to now do the search at again at the 4 different corner points as the case is.

Suppose considering the unit revenues for product 1 and product 2 are changed from 3.5 to 2.5, so the corresponding values in the objective function in the constraint would also be revenues, revenues are changing that means it will have an impact on the objective function ,so hence it can considered accordingly and we can make the decisions. So now these questions we will try to answer. So this general formulation will be considered here.

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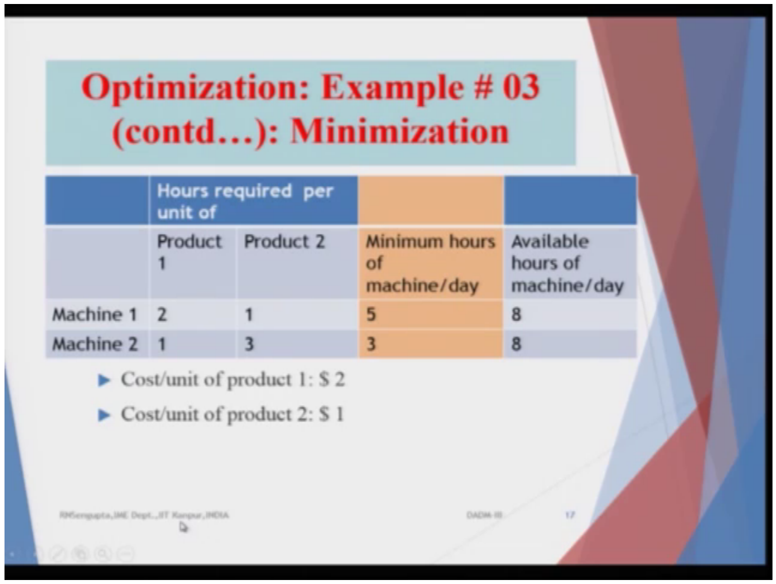

So consider, so let us consider, (will) now we will basically try to solve it using the matrix multiplication. So matrix or Gauss Jordan method or the matrix concept, so here I will basically pause for 2 minutes and then give you the idea, so let me.

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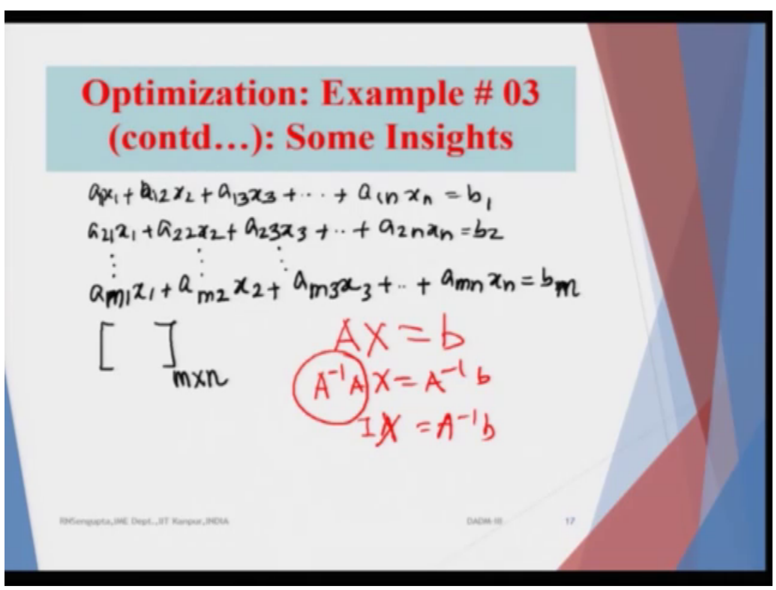

So I will come to this table later on first simple concept of the, let me make this slide beforehand so it would be easier for us, just that is great, ok. Now what does general solving of a linear equation mean? So what we have is, we have consider we have ax1, so we have equation ax1 plus b a11 plus a12 x2 plus a13 x3 plus dot-do- dot plus a1. So this is the actual equation we will use and I will write here so a1n xn is equal to b1 so this equality need not be true, it can be less than, greater than whatever it is. Second equation is a21 x1 plus a22 x2 plus a23 x3, a2n xn.

Let us consider is am1, so m and n are not the same. If they are equal then well and good and we will try to make them equal for some reason I will come to that later. Again the equality sign need not be true, less than, greater than whatever. Now look at these equation, so that will remind so here I will be basically talking more of the basic mathematics which I am sure all of you know.

Consider you have an equation 2 X1 plus 3 X2 is equal to 5 and in another case you have basically 5 X1 plus 10 X2 is equal to 10. So what you will do? Use both the equations, I am not going to write it, I am going to basically write it when the actual part comes, so you will use both the equations, first case you eliminate, multiply some of the parameters for either equation 1 or equation 2 and then eliminate either X1 and X2, find out X1 or X2, utilize that X1 and X2 to find out the other thing.

(So given X) first we find out X1, we find out X2 then give or else given X2 we find an X1 and in the just to double check we plug them in both the equations, check whether the values are being satisfied for both equation 1 and equation 2. Now when we solve the problem it may be possible that one equation is the multiple of the other. Consider is X1 plus 2 X2 is equal to 4 and in another case you have 2 X1 plus 4 X2 is equal to 8. Which means, the second equation is basically being given by a convex combination of the first, so which means that the number of solutions which you will have for X1 and X2 to satisfy that equation would be infinite.

Now when we utilize this concept, we are aware that there is a concept of rank, matrix .So once we have the set of equations which is in front of you, which has basically a11 to a1n in the first row, a21 to a2n in the second row, am1 to amn in the last row, so you will basically have 1, 2, 3, 4 till n this 1 minute, so this my mistake, wait-wait-wait, I think should be (here 1) 1, 2, 3 yes sorry so there would be basically m rows and there would be technically n columns. So if I consider so here m rows and n columns.

Now if you want to express that set of equations when you are solving a set of simultaneous (linear) equation what we do, we write it as I will use a different color, so this black color and the red color will may or whatever other color I am trying to utilize that will make the idea much clear. So we use A, which is the matrix corresponding to the A values which you have, you have X which is the vector not the matrix, vector corresponding to the X values which you have is equal to B which is the vector corresponding to the B values which you have.

Whether its a column vector (or the) or a row vector that would basically depend on the dimension of the problem. So in this case A is basically of dimension m cross n then in that case X would be n cross 1 such that the overall result which we will have by multiplying A and X would be m cross 1. So m cross n would be for A, n cross 1 would be for X and m cross 1 would be for B, so how do we solve it? Using simple, very simple concept of matrix multiplication, what you actually do is that you either pre-multiply or post-multiply A by its inverse.

So if you pre-multiply and post-multiply which we will consider by the concept that pre-multiplication by A minus or post-multiplication by A minus would give you the identity matrix which is I, so as you keep doing that step by step you also do the same thing on the right hand side, so if you are pre-multiplying A by A minus, you will also pre-multiply B by B minus on to the left hand side. So hence the end result which you will have would be this, which means the overall result of pre-multiplying A with A minus will give you the identity matrix.

So you will basically have I into X is equal to A inverse B, so when you find out the actual values which are on the right hand side they will give you X. So what you are doing is that you are converting A step by step into identity matrix and you do the same level of (multiplication) pre-multiplication and post-multiplication in the row and the column, on the right hand side which is on that equation such that you basically get the values of X.

So this is the basic thing, you will basically have a starting matrix and will basically convert that A and will basically convert that into I, which is the identity matrix which will basically be the one along the principle diagonal as you know and as you are doing the same multiplication, same operations on that matrix, you will do the same thing on the right hand side. So you will basically, at the end of the day arrive at the value of X which will give you the actual value that will give you the optimum solution.

Now as you are doing it you are also doing the same thing to find out what is the objective functions, so at the end of the day when basically A becomes I the actual results will give X and along with the objective function which we intend to get. Will come to the problem later on and in the next class and have a nice day and thank you very much.