Data Analysis and Decision Making-3
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Lecture 12

A good morning, good afternoon and good evening to all of you wherever you are and a very warm welcome to this DADM which is Data Analysis and Decision Making-3 course under NPTEL MOOC and as you know this course is for total 12 weeks, contact hours is 30 which when broken down into number of lectures is basically 16 number because each lecture is half an hour. And we have already completed 2 weeks of classes and as you can see on the slide this is the 12th week, 12th sorry, 12th class which is the 2nd class in the third week and 2 assignments and coming back to the main issue which I repeat before each class is basically you will have 12 assignments and one end semester or final examination. And you have already taken 2 assignments so we will complete another 5 go till the 15th lecture and then you will face the or take the 3rd assignments.

So we will discussing about the problem of the Asian Paints having some raw materials, raw material 1 and raw material 2 to produce two outputs paint 1 and paint 2. Similarly in the other example you have two products to make, two raw materials to make two products from machine 1 and machine 2. Now if you remember in the last class I mentioned that how you formulate the decision variables or number of distribution variables will basically dictate how the problem will be solved.

So for the problem when if you for the Asian Paints or the paint factory problem if you take the raw materials as variable X1 and raw material 2 as variable X2 then the problem formulation would be done accordingly but the constrain concept will remains same, how the constrain are basically formulated will depend on what variable you are taking.

If you formulate paint 1 as raw material as decision variable 1 and paint 2 as decision variable 2 your formulation will change accordingly. Now remember here the number of decision variables will dictate what is the dimension of the problem because in the case it will be easy for us to basically draw the diagram and I had shown that, that how the constrains can be drawn using simple linear equations and you can find out the boundary.

In the last slide which I showed the last day in that 11th lecture it was basically how you can find out the overall feasible region inside which you have to basically find out the answer. Now remember one thing, if I keep increasing ,in the same problem, if the dimensions can be changed depending on if you consider say for example there are two raw materials to produce 5 different paints. Now in one case if you consider the paints as decision variables so you will basically have X1 to X5 in that case it is the dimension problem of 5.

In other case if you basically formulate the problem as two decision variables only for the raw materials 1 and 2 then it becomes the 2-dimensional problems where which you can basically plot it in a Cartesian Coordinate on a graph paper. So this I did explain in pretty much details repeating time and again. But we will basically try to solve the problem as it is and then basically proceed accordingly.

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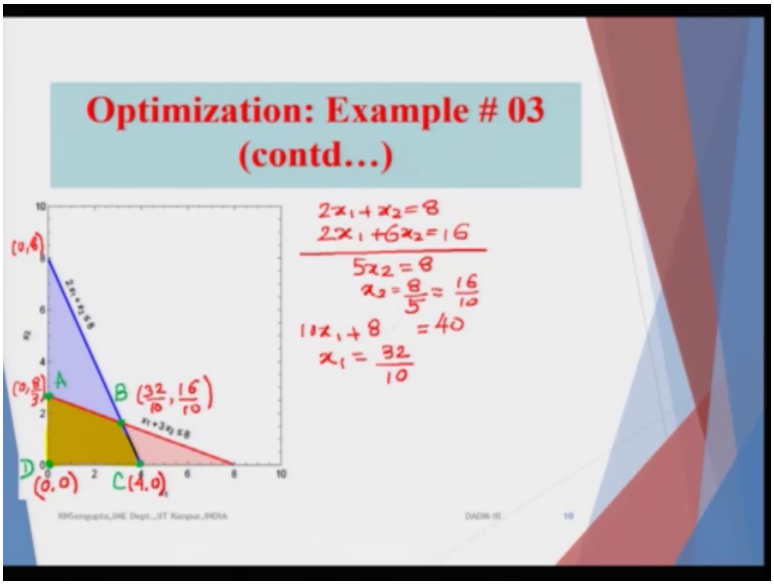

Now when we draw the, if you check the constraints if you remember, so the constraints were in this way, 2 X1 plus X is less than equal to 8 and this 8 is basically the number of hours. So X1 and X2 are the decision variables. In another case it is X1 plus 3 X2 is less than 8. So this is basically machine 1 and machine 2. For product 1 you read, each product needs 2 hours in machine 1 and each product of (variable) of X2 decision variable takes 1 hour in machine 1.

Similarly products for decision variables 1 and 2 in machine 2 in the respective hours are 1 and 3. So hence it is 2 X1 plus X2 is less than equal to 8. So if you want to find out which side the equation would be you put 0 on to the left hand side of X1 and X2 and you find it is less than 8 so obviously 0 would be included in the feasible region corresponding to the fact that, that line is only applicable. That line means, the blue one which you see here.

Now in that case so the overall area which will be applicable in the case for the variable 1 actually would be some part of quadrant 1, some part of quadrant 2, the whole of quadrant 3 and some part of quadrant 4. When I go into say for example the second equation X1 plus 3 X2 is less than equal to 8, again 0 satisfies this. So 0 would be a part of this equation. If you consider the whole feasible at least the areas corresponding to equation 2 it will be some small part of quadrant ,1 small part of quadrant 2, whole of quadrant 3 and some part of quadrant 4.

Now the other two constraints as I mentioned is the product has been manufactured and they definitely should be integer but we are considering again I am repeating it, they would be we will consider them to be greater than equal to 0 so X1 is greater than 0, X2 is greater than 0, in that case X1 greater than 0 means only quadrant 1 and quadrant 2, X2 being greater than 0 means only quadrant 1 and quadrant 4.

So if you basically find out the overall common area corresponding to the 4 such constraints technically which is equation 1, equation 2, X1 greater than 0, X2 greater than0, the overall area is this one (which I will) which is shown here but again I will mark it, so this is the area. So this is 1. Now the interesting part is this and this can be extended to 3-dimensions. In the 3-dimension one if you have. So consider (then) I am standing in the origin and towards you is basically X1 axis to my right is basically X2 axis and vertically up is basically X3 axis. So consider that are 3 variables, in that case it would be a surface and a cube or cuboid or some space would be there which will basically use the world of hyper planes which would basically be the overall bounded area of this hyper planes which will give you the overall feasible region.

So if it is a cube considering that, so why we will get a cube? In the case if X1 is greater than 0, X2 is greater than 0, X3 is greater than 0 and each of X1, X2, X3 are say for example less than equal to 2 individually then obviously we will have a cube or certain dimension. Then the overall area feasible region would be the all the points, infinite set of points inside the cube but how we

will basically overcome that, that such techniques to find out the optimum solution in minimum number of feasible region, will going to feasible points we will going to come that within few minutes.

Now if you consider the area which I have just marked yellow, so that has how many corner points? 1, 2, 3, 4 one of the corner point is 0, another corner point is basically on X2 axis, another corner point is basically on X1 axis, another corner point is basically the common point which satisfies the equation 2 X1 plus X2 is equal to 8 and X1 plus 3 X2 is equal to 8. Now if you remember we have (diss) we had shown one picture of Karmakar. So Karmakars internal point algorithm I am not going to go to in details, basically it states that your feasible regions is actually infinite number, but your search space, search points should be only be the corner points.

So basically your now search technique would be such in linear programming would only concentrate on 4 points. Now how do you do the search, that conceptual has to be analyzed. Now before I come to the objective function, look at this space. So if I find out the actual points would be I will mark it then again erase it. So let me go one by one. So this point is 0, 0, this point is what? 2 X1 plus X2 is less than equal to 8 and X2 is 0, so X1 is4. So this would be 4, 0. The first (qua) the value is basically corresponding to X1. When I come here, this equation becomes X2 is 0, so X1 becomes 0, so X2 is 8, so this would become 0, 8 and this equation, so this becomes so (though) no, that is not required sorry so even though I marked it.

So this point would become where X2 would be the 0, so it will be 8 by3. It was x, let me check, double check, so X1 is 0, yes 8 by 3 and if I consider this so this is I will mark some the so this is 0, 0, this is 0, 8, 3, this is 4, 0 and this point so let me mark this points, this is Ai will go in a clockwise directions this I will mark as B, this I will mark as C, this I will mark as D, so our search would be only inside A, B, C, D. So let me solve the equation. So you have 2 X1 plus X 2 is equal to 8 and X1 plus 3 X2 is equal to 8.

So if this becomes to multiplying by 2 this becomes 6, multiplying by 2 this becomes 16, so the (first equation) second equation minus the first is becomes 5 X2 is equal to 8 so X2 8 by 5 so I put up here multiplied by 5 so is 10 X1 plus 8 X2 is equal to 5 is a 40, sorry-sorry so it should not be X2, so this is already care, my mistake-my mistake, so let me check. This is 5 into 2, 10

X1 plus multiplied 5 so is 8 is equal to 5 is 45 then I go to here, so X1 is equal to 32 by 10 which technically this also becomes 16 by 10. So if I come here, so X1 is 32 by 10, this becomes X2 is 16 by 10.

So if you want to check it, (double check) let us double check it. If you look at here, so it becomes 32 by 5 plus 16 by so its 64 by 10, 16 by 10, 64 plus 16 is 80, 80 by 10 is 8 so this is satisfied. If I go here, second equation, it is 32 by 10 plus 48 by 10, 32 plus 48 is again 80, 80 by 10 is 8. So this satisfies, so I have all the coordinates, now only I can erase this, so because once I click the next button another graph will come here so I do not want to make it confusing.

So coming here now when you do the search there are 4 points only, so its for this problem is very easy, you find on the objective function whatever the object function is, you plug in the values of A, B, C, D separately find out the value of where the object function maximum .You put that value and find out the corresponding values of X1, X2 and your problem is solved. But the problem technically is not as straightforward its easy but not as straightforward because in higher dimension one you will have basically almost huge number of points and in which direction if you would proceed in order to find out that you will get the optimum point or the feasible amongst the feasible point is basically the main criteria.

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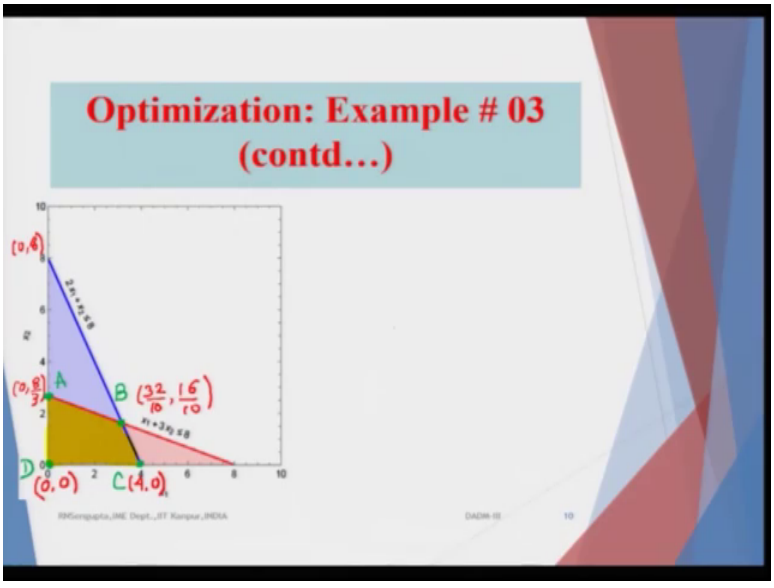

And now consider that say for example I start at 0, 0 my search, why I start at 0, 0? I am going to come to that later I start at 0, 0. Now what problem it is? It is an optimization problem. So in the

optimization problem when you check, what you want to check is that as you change in the objective function initially the values of X1 and X2 is 0, 0 so the objective function value is 0. Now what you will check is that in which direction should you move the objective function such that you increase the values of either X1or X2 considering that only X1 and X2 or if there are more than two number of decision variables consider the N number of decision variables.

Considering that point which of this X1 should you take such that the rate of change of this objective function happens as the highest rate? Because in objective function, so which is a maximization one, so 1 unit change in one of this axis should basically increase the objective function the maximum. So you only proceed in this direction, so if the same logic happens from the minimization point of view obviously you will take that point where the rate of change of that objective function happens in the other direction negative direction the highest.

So you will keep moving in the direction. So say for example I move from D to C, because the consider that, that gives me the point that changing X1 and not X2 basically increase the objective function but the highest when you compare X1 and X2 power 1 unit of change then once you reach C you basically move to D. Considering that now you have basically being able to change both X1 and X_2, once you reach B, consider that the objective function with the maximum you again because you do not know you have to stop and the algorithm has to stop.

So what you do is that you again move your direction from B to A. Considering this is a very simple case of 4 corner points you move from B to A. What will happen now is that, the rate of change of the function would be such with respect to objective function. Because you are changing the objective function with respect to one of the variables, it will happen in such a way that the overall value of the objective function rather than increasing will start decreasing, which means that, that is the point where he should stop and report that objective function and those corresponding values of X1 and X2 which will give you the maximum value.

Now this is the general way the algorithm should work conceptually but there are other integrities and other details inside that which we will come to that later. Now when I am trying to solve this problem pictorially all these details what I said and all these details will be coming up in the theoretical part when you basically have a tableau or a table and basically proceed

accordingly. But in the case when you have the pictorial one, I will basically draw it that is why I have removed the all the simple equations here.

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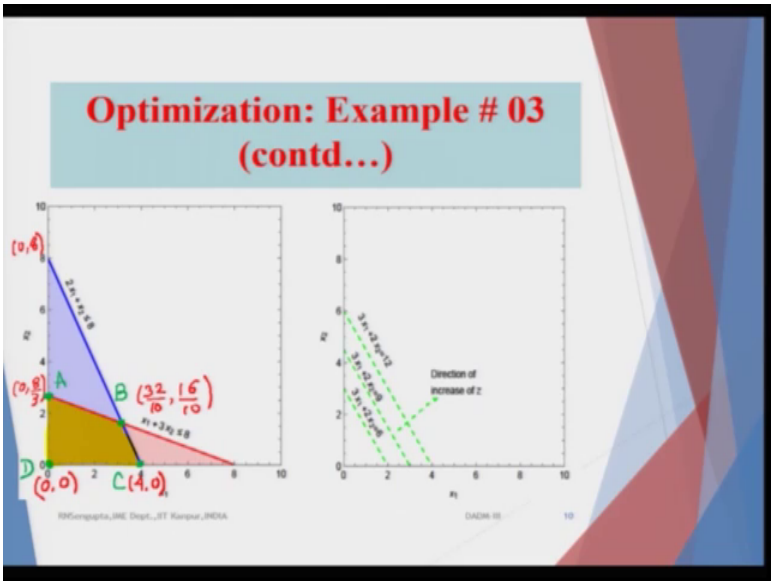

Now the objective function is basically given which is the green line, will merge both this graphs together later on or you can basically visualize how they are merged. Now the objective function has to be drawn because I will try to find out that in which direction the objective function keeps increasing, such that at some optimum point depending on X1 and X2 you will basically reach the maximum point.

So now consider, the objective function is placed like this, why do I place the green line in this is very simple. You find out what is the dy, dx of this line. Depending on the values which you have so if you find out the values technically the equation is given as 3 X1 plus 2 X2 .So considering and that is per unit because if you remember 3 units is coming from product 1 and 2 units are coming from product 2. So if you consider the and that is equal to Z which is the objective function you would place that line accordingly.

Then the Tan of this angle or the points at which X1 and X2 are touched by this line are at X1 is equal to 2. Because in that case obviously you will basically have the points satisfy this equation. So if you place that line I am not going to draw it, so I am just hover on my electronic pen over here. So here this green dotted lines is basically the inclination of the, are the slope and how the objective function will look.

Now what is what you want in the object function is that it should keep increasing, so if it keeps increasing you should move more in away from the 0, 0 and go deep inside the quadrant 1, so start shifting it. But the point is that at one point of time it will leave the feasible region which is A, B, C, D so you find the point at which it leaves the feasible region that will give you technically that would give you the point where X1 and X2 are those values for which the objective function is maximum and in this case you can easily find out using very simple concept that if you see the line, the line would basically leave the point the feasible region at point B hence the point B would give you the maximum value of the objective function.

What I said about moving from Direction D to C, C to B, B to A or in whichever direction would basically be the conceptual framework of the algorithm based on which you are going to work so this is the situation.

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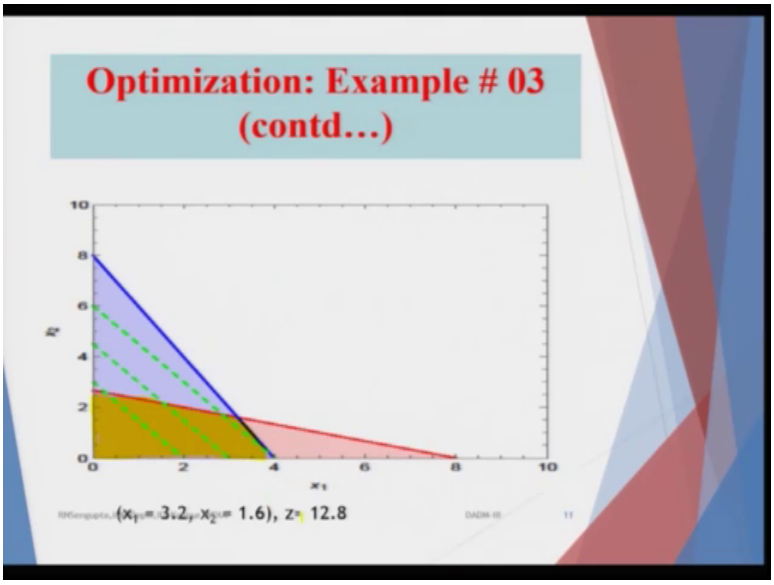

So you have the green, the overall region which is given the I am sure all of you have understood but still I will highlight. So this is again I said A, B, C, D and this is the point for which X1 is 3 by 2 and X2 is (1 point) 3.2 and 1.6 so the objective function is to 12.8. So it means that you are going to produce 3.2 number of product 1, 1.6 of product 2 and the overall objective function value will be 12.8.

Technically the answers of 3.2 and 1.6 are not right because they cannot be decimal but will still proceed along with this point but remember this problem would have been perfect if it was a

paint problem because in that case you can produce any decimal values in liters, milliliters whatever microliters whatever it is about the paints and hence will consider X1 and X2 as the decision variable for the paint problem to be continuous.

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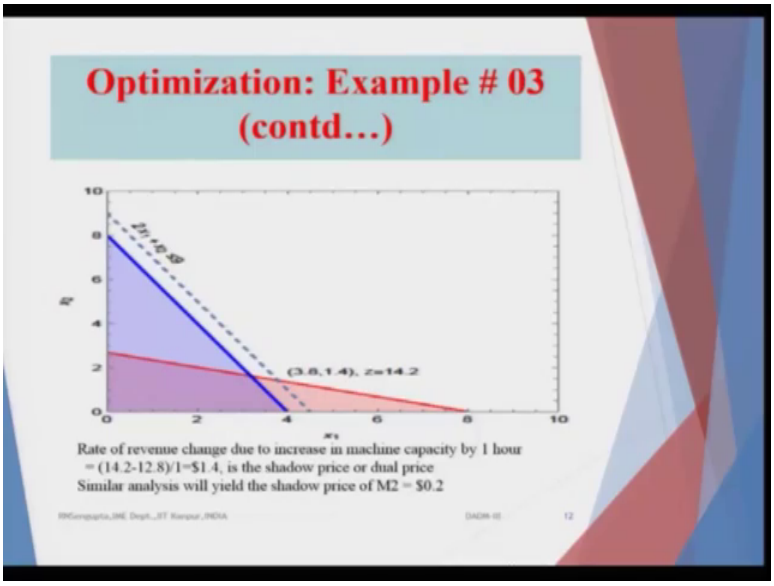

Now consider the problem in such a way that the rate of revenue change due to increase in machine capacity basically happens is basically 1 hour. So which will mean that if I change the equation.

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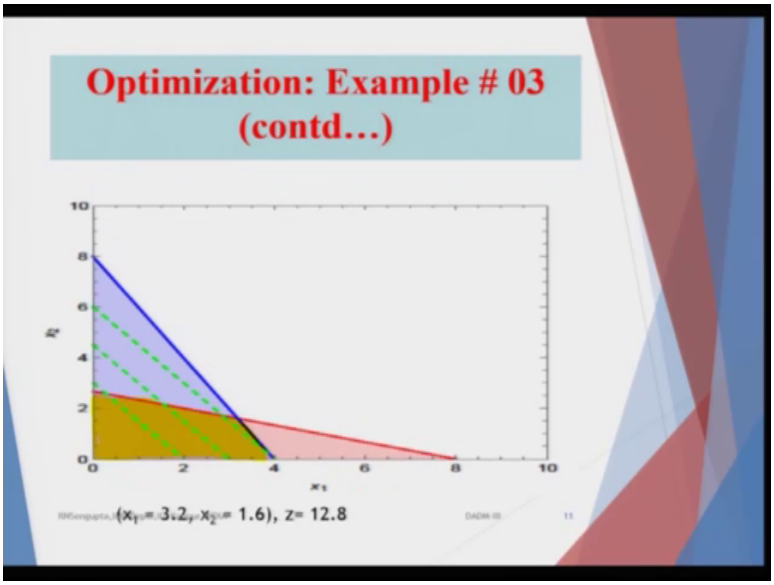

So the values which were there was 3.2, 1.6.

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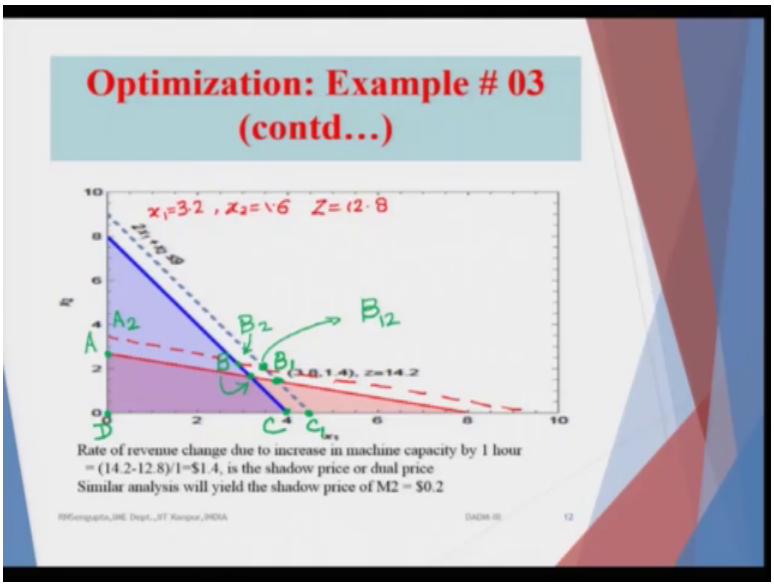

So let me write it down and this value was 12.8 let me write it down also. Now if 1 hour change in the machine capacity is there, so what is the change we are expecting? Is that the value which you have initially 12.8. So the value change which will have for 1 hour change in the capacity would be 14.2. So hence the increase which is happening for 1 unit change is 1.4.

So we are basically adding 1.4 values amount due to a capacity change of 1 hour. So that means 1 hour change basically is meaning that it is increasing from 8 to 9. So if you basically put the equations, so if you remember equation 1 corresponding to machine 1. So what I am trying to do is that, change the capacities of machine 1 and machine 2 separately or you can basically take them together.

So first consider machine 1 is changing. So if machine 1 is changing, so obviously the equation becomes 2 X1 plus X2 is less than 9. So in that case what happens A remains same, B changes which is now let me (draw) mark it. I would not mark the values this is A, let me use the green color because it will be easily distinguishable by you. So this is A, this was initially B, this was initially C and this was initially D. Now what happens is now A remains A, B goes to B dash, C goes to C dash, let me use not dash, let me use 1-2. So it will be easier for us to, why 1-2 I am going to come to that within 2 minutes.

So this is B suffix 1, this C suffix 1, the corresponding to the change which is happening for machine 1 (and) one only. Now in that case the values which I find out the it is 3.8 and 1.4 the maximum value of the objective function is 14.2. So 14.2 minus 12.8 which is for the first one will give you a change for 1 unit hours is coming out to 1.4. So this is the shadow price of the dual price for one for machine 1 such that the change in 1 hour per unit 1 hour what is the extra profit you will be able to get?

Similarly when I go to machine number 2 without changing machine 1, the line will be it should not be cluttered I will try to draw it, I will use the red color dotted ones to basically mark machine number 2 without machine changing machine number 1. So it will be like this I am not going to mark it total in all the details, so in this case what we will have? So that we now mark the points in green, D remains D there is no change, A goes to A2 corresponding to change in machine 2, B goes to this point B2 and point C remains at C it does not change.

Now if I consider the change in both machine 1 and machine 2 then obviously that point of B now becomes B suffix 1 2 because I am considering changing both machine 1 and machine 2 happening. So similar analysis if I want to find out for the shadow price of machine 2 only it will come out to be 0.2. So 1.2 for machine 1, 1 unit change and 0.2 change in the objectifying value for 1 unit change in machine 2. So obviously you have to find out that shadow price or the

conceptual rate at which you will change one of the machines in order to increase your production would definitely be coming from the highest value which is for the case when you are trying to do the maximization problem.

So it is all green in color the points but again I am repeating A, B, C, D is for the original problem then shadow price corresponding to machine 1, 1 unit change in machine 1 is A then B1 and C1 and D. The next case it is basically A2 only for machine 2 is A2, B2, C remains as it is and D and if both of them changes it is A2, B12, C1 it should be its basically the nomenclature is should be clear, so A2 is basically not changing for both combination of them changing, C1 is also for both changing and B12 is the common point.

So we will continue discussing about the concept of linear programming in details but remember one thing what I mentioning is that it may be simplistic and I am trying to explain it in the pictorial way. But the concept of how you solve the problem is exactly the same what I told, you start at one point traverse the corner points, take the decision in which direction you are going to move depending on the rate of change of the function with 1 unit change in the decision variables taken one at a time, take the decision where the maximum change is happening for the objective function which is a maximization problem.

In the case of the minimization problem you just do the reverse, stop at that point where the rate of change of the function does not start decreasing report that point and basically solve your problem. When you are trying to do with the pictorially, pictorially the concept I have given and then will basically use the concept of simple linear algebra and that linear algebra would be very simple matrix multiplication using the Gauss Jordan concept. I will come to that later but for the timing I will end today have a nice day and thank you very much.