Data Analysis and Decision Making-3

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Lecture 11

A warm welcome, a very good morning, good afternoon, good evening to all of you wherever you are undergoing this course, taking this course and as you know this is a DADM which is Data Analysis and Decision Making-3 course under NPTL mock series and this course we have already completed 10 lectures which is 2 weeks is already complete. Now this total course duration is for 12 weeks, contact hours is 30 which when broken down to lectures is 16 number and each week we have, so that means each class is for half an hour.

So they are on roughly on and they tune it about 29 to 30 minutes and each week we have 5 lectures, so with the completion on the 10th lecture, 10 lectures in total. So you have already would be in position to take two assignments, so after each week as I said there are assignments. So hence after the end of the course you will be doing or completing 12 assignments plus one final examination. And my good name is Raghu Nandan Sengupta from the IME Department at IIT, Kanpur. So if you remember we were discussing a very simple problem of two machines, two jobs and each machine has 8 hours in each day considering one shift.

And the number of hours required for to process product one or product two in machine 1 and machine 2 were given. And there we were basic a formatting a problem with an idea that you want to optimize the total output considering the revenues are given such that you can understand that revenues give us the information about trying to about the profit, so you want to basic a maximize the profit.

Now if you remember, I also mentioned that the (())(02:07) of the last class, 10th class it was basically that each of the constraint are such that they would be made in totality or some of their overflow would be there, overflow in the sense some unutilized number of hours would be there for machine 1 and machine 2. Considering the fact that the number of units to be produced at each end need not be an integer but technically that we need integers because you are producing products so that they cannot be 2.5, they cannot be 1.32, so they has to be, have to be (interior) integer values and these are greater than 0 because you cannot produce negative values of any product.

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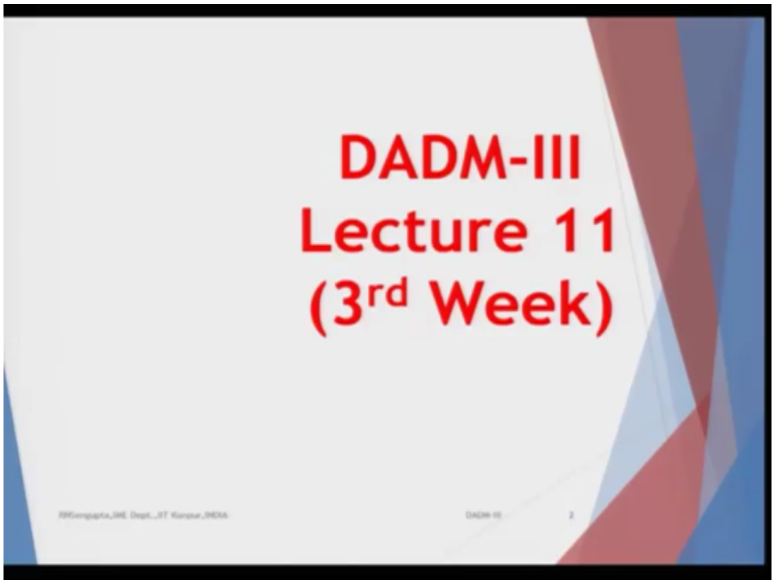

So this is as I said this is the 3rd week starting and we are in the 11th lecture.

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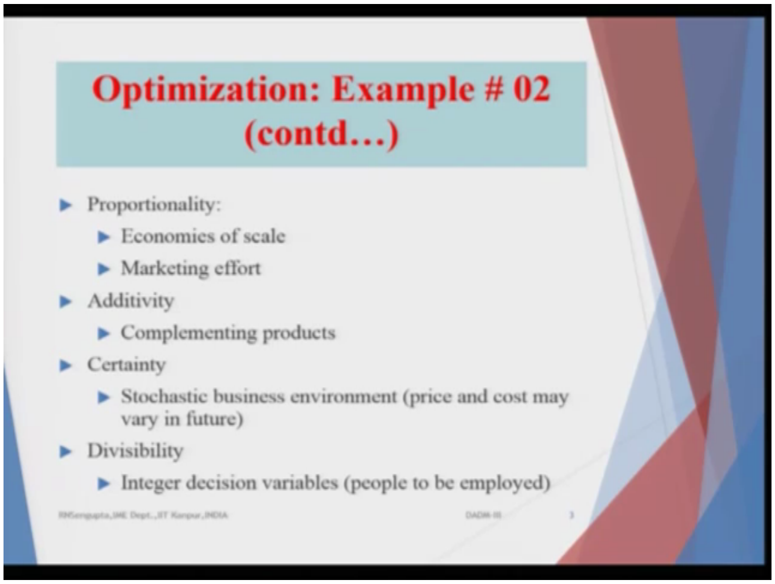

Now whenever we are trying to consider this problem so I will again repeat few other assumptions which we have will be encountering time and again. So they would be concept of proportionality, so when economy of scale, so the more you produce more you gain. So obviously in that case we will consider the proportionality having to be there in such a way that it is linear in nature, it is not increasing at an increasing rate not increasing rate at a decreasing rate. That means if you have produce more, more and more so in that case the profit would not be increasing in more than exponential manner,

In the sense that if you are produced 2 units, I get a profit of 4, if I produce 3 units I get a profit of 6, if I produce 4 units I get a profit of 8. So but in case if it is increasing at an increasing rate which means that more I produce so if I go to 5 rather than 10 it will be, say for example something more than 10, 11. When I produce 6 rather than 12 it will be say for example more than what it should have been at 12.5. When I go to 7, it is not 14, it is say for example 15 and another case whenever I consider then it is decreasing in that sense the decrease would be such rather than 6 producing output of 12 it will less than 12 rather than 7 producing an output, profit output of 14 it will be less than 14 but the proportionality will be decreasing.

We will also consider that the marketing efforts would be such that the overall average return which will get from the production would be positive, positive in the sense it is increasing proportionally it will positive such that any utilization or non-utilization would be coming out from the constraints and they would give us the concept of shadow price and the slacks.

Additivity would mean that if any complementary product is added or if you remove one product and add another one certain that one product can replace the other. So in that case the constraints (of the) would be changed but considering the fact that the proportionality and the overall feasibility of the, feasibility means of the solution, so obviously the solution overall area would change, solution set would change but the feasibility would remain such that we are able to find the optimum solution as required.

We will also consider again I am repeating certainty, on the values no stochasticity, no uncertainty that means once given the data we will proceed considering that it is a true value based on which we will do the calculation. So obviously it would not mean that as the production increases the number of hours which is available keeps fluctuating over and below 8 or say for example if the price fluctuation, if it is 1 or 2 on the number of hours required for machine for one product in machine 1 or machine 2 is 2 or 3 whatever it is. Those values of 2 and 3 would not be changing randomly with respect to the some distributions, so once finalized they will be fixed.

Similarly the cost structure, if you have sell one product and the overall profit which I am going to get from product one is 2 rupees or 2 dollars per unit sold it would continue to remain as 2 rupees or 2 dollars as we increase or decrease the number of products which are being sold for decision variable one or decision variable two as the case may be. So certainty would be in the stochastic business environment, price and cost may vary in future but we would not consider that, so as I said the cost structure would be the input cost, the raw material cost, the labor cost, electricity cost so we will consider them to be fixed, we are not considering them they varying.

In the same way price, selling price based on which we are selling in the market hence the revenues would be the total profit would be selling price minus the cost price. Those would not be changing randomly, we will consider them be fix and then continue solving the problem accordingly. This concept of divisibility would be the integer decision variables so number of peoples to be employed or say for example number of machines to be utilized or number of trucks to be employed to transport goods or the number of products to be made.

So in the initial problem I mentioned that the number of products which are to be (bought) to be produced are all integers but in the problem solving when you solve you will find out they come to be non-integer values. So technically this is also an very big assumptions will be considering to solve the problem and later on we will see when we solve the integer programming problem how this problem can be basically compensated in order to find out the best solution which was give us integer value without compromising on the constraints.

So see in this case we see that it is integer values but say for example on the other hand we are considering the problem of trying to mix paints to produce some liters of paints. So consider there are two paints and they are the decision variables X1 and X2 but in this case paint amount of paint sold or bought (or) which is a decision variable can be in decimal also. It can be say for example 2.32 liters, it can be 3.996 liters. So in that case we will consider continuity to be there such that the values can be consider as continuous, so any (var) variable would be possible as X greater than equal to 0.

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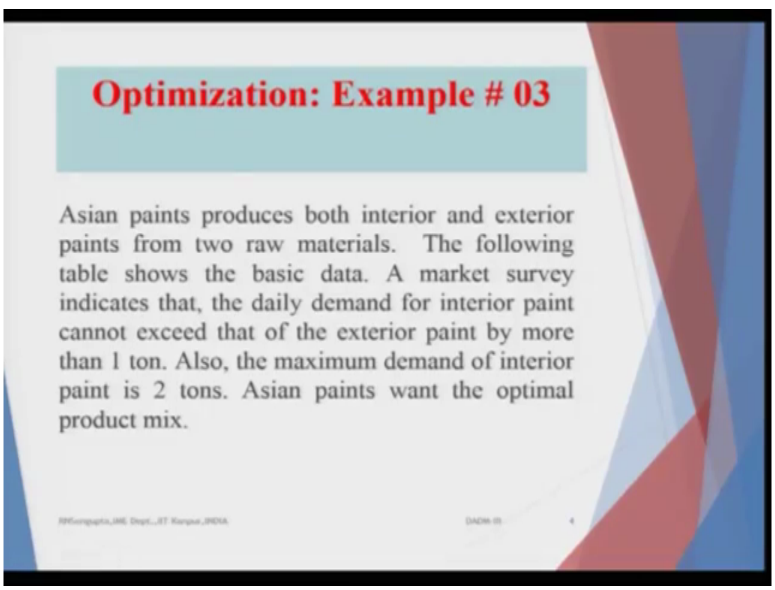

Now consider so I will come to the, so that was the general feel of our 2 dimensional problem so I will (consider) again consider a second problem and an try to basically give you a pictorial diagram idea that how the problem is to be tackle or what is the conceptual framework of the problem. So Asian paint produces both interior and exterior paints from two raw materials, so raw materials can whatever the raw materials can be, they can be paint by itself, they can be some oil which is to be used some chemicals used.

The following table which was below shows the basic data. A market survey indicates that the daily demand for interior paint cannot exceed that of the exterior paint by more than 1 ton that means the demand between them is such that the demand on the interior cannot exceed that of the exterior by more than 1 ton so it has to be limited by that. Also the maximum demand on interior paint is 2 tons in a particular unit time. So Asian paint once the optimum products mix in order to basically find out what is the profit considering the profit is the fact.

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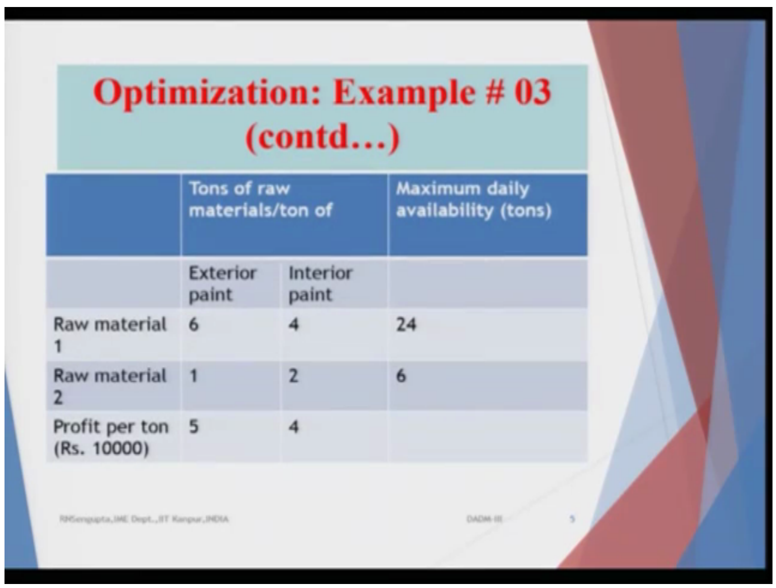

So let us consider the overall picture, so let us go one by one. So if I consider raw material 1 which for our case I am telling you before and is basically decision problem X1, so (number) tons of raw materials required per ton of exterior paint is 6, so this is I would not highlight and just try to basically make a note of that to all the students who are watching this video. So raw materials required for which is of type-1 would be 6 tons to produce 1 ton of exterior paint. Similarly raw materials required per ton of interior paint to be manufacture is 4 and the maximum daily availability in tons for raw materials 1 which is, which again I am saying will be may be decision problem one depending on how the problem has been formulated is 24.

That means you cannot utilize more than 24 tons of raw materials one. It can be 23.9 whatever it is but obvious in that case they would be left some leftover, so what is the concept leftover I am going to come to that within few minutes. Now for raw material 2 similarly the tons of raw material 2 required to produce exterior paint 1 ton of exterior paint is 1 ton that means 1 ton is required for raw material 2. Similarly for raw material 2 which is require to produce 1 ton of interior paint so that quantum is 2 tons.

Again the maximum availability of raw material 2 given per day or per unit time is given as 6, so these are basically the so-called the constraints which you have corresponding to the fact that the availability in tons is given, some value is given in not in rupees but in kg's or tons or liters whatever in this case it is tons.

Now in this problem remember, in the initial problem we had done it was basically producing products that is why I had kept mentioning it that the numbers have to be integers even though the formulation was not in that line in the sense that the answer could have been say for example 2.3 number of product 1 or 3.29 of product 2 but I also did mention that we will be considering that in the later part when we solve the integer programming problem. But in this case it is as I said it is paints, hence the number of tons or number of liters, number of kg's can definitely will be non-integer values and they can be in decimal places also hence trying to solve this type of problems using (integer problem) using linear programming would make much better sense.

Not that we cannot solve the problem in the initial case using linear programming but obviously they would be some caveat, some important points to be noted which I mentioned accordingly that the integer values would give us some excess amount of either or time left for machine 1 or machine 2 and the case may be. Now coming to the cost factor or the profit factor or the revenue factor in the initial problems it was the revenues, so you wanted to maximize it in the same case that it is given for this problem on the Asian paints it is profit per ton and it is given as 10,000 so for selling exterior paints it comes out to be 5 selling interior paints it comes to be 4.

That means if you sell 1 ton of interior paint you get a profit of 4, I am only talking about the units I am not multiplying with 10 (to the power) 1000 or whatever it is and selling exterior paints would basically give you profit of 5 per ton.

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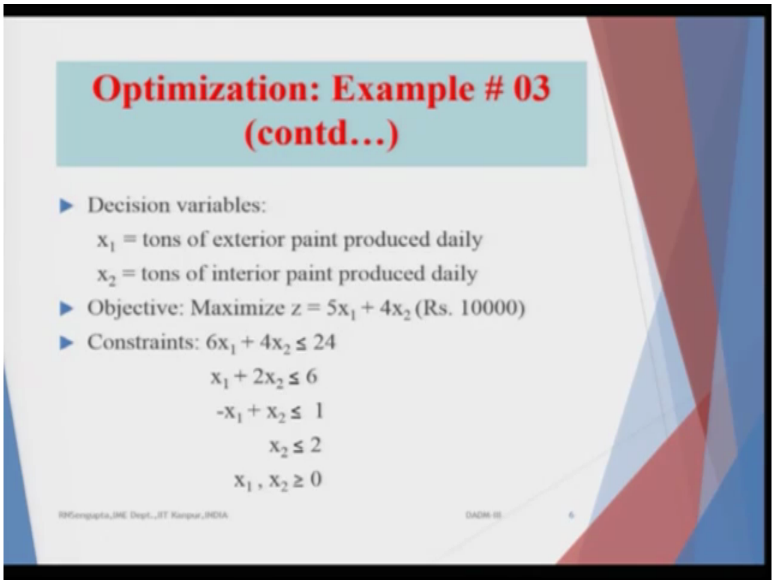

Now consider the values to be decision variables to be to be considered. Now remember one thing the choice of decision variables for this problems have to be thought out very carefully because the answer is that what, they can be two ways of tracking the answer, (1) number 1 is that, the amount to be produced is amount of paints to be produced would depend both on the exterior paint and interior paint plus on raw material 1 and raw material 2.

So we will consider here X1 as the tons of exterior paint produced daily and similarly on the other lines we will consider X2 to be the tons of interior paints produced daily on a unit time scale. So in that case if you are considering the profit remember because you have to basically do the profit in that case the profit comes out to be 5 into X1 plus 4 into X2 considering 5 and 4 are the respective values of addition of profit for 1 ton of exterior paints and 1 ton of interior paints respectively.

Now coming back to the formulation which we need for the case of the constraints, now watch here very carefully we assumed.

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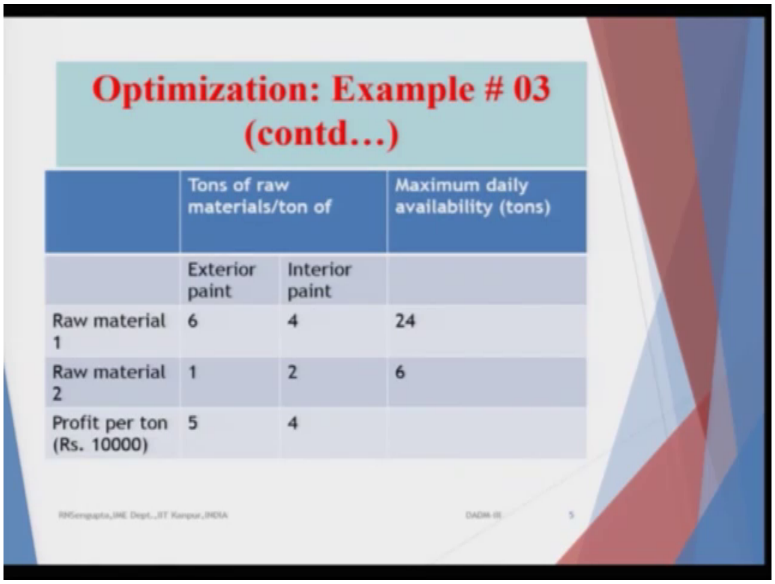

And let me go back to the last slide. So this will give a good picture. As per the problem formulation it is said that you will need 6 tons of say for example tons of raw materials required for ton of exterior paint it was 6 for raw material 1 and 4 for raw material 1 for interior paint. Similarly the values corresponding to raw material 2 were 1 and 2. And the maximum value available for raw material 1 and raw material were 24 and 6.

Now (you have) we have considered the decision variables as exterior and interior paint, so if that is the case the constraints would be formulated and shown here and if it is not that if the decision variable was something else then the constraints would have been formulated a little bit separately which I will come to that within few minutes.

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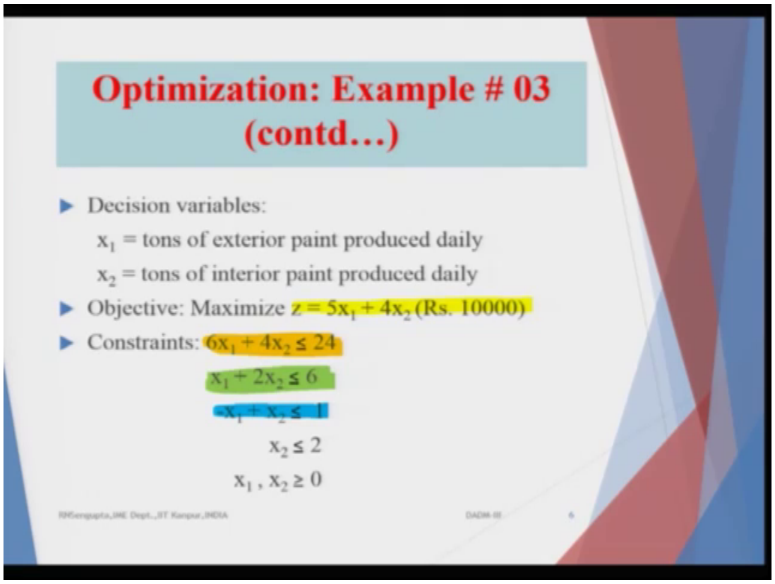

Consider constraints, so we have already considered the maximization objective functions so we will highlight it. So this 10,000 is because per unit 10,000 was the addition so it was 10 into 10 to the power 4 and this was 4 into 10 to the power 4 that is why this value of 10,000 is coming. Now let me highlight constraint 1. Now constraint 1 means we already know 24 tons is the limitation. Now if I go into the utilization of material 1 we know that per ton of exterior and interior paint which we need 6 and 4 quantum of material 1 only.

So that means the total amount of material 1 being utilized to produce X1 amount of exterior paint is 6 into X1 which is the first value. so that is the total and the total quantum of exterior paint needed to produce the raw material 1, sorry raw material 1 needed to produce the interior paint is 4 into X2 because X2 is the value of interior paint which we are going to produce. So if I consider the total utilization of raw material 1 it will be 6 into X1 coming from exterior paint and 4 into X2 which is coming from interior paint.

So the summation of that obviously we know has to be bounded by 24 hence the first constraint basically make sense that we are utilizing the decision variable as this interior paint and exterior paint but we are trying to basically formulate the constraint based on the fact that the raw material utilization of 1 is important and given to us, so this is the important fact. In case, if we had taken X1 and X2 as the raw material 1 and raw material 2, in that case the problem formulation would have different because in that case the objective function would also change, similarly the constraint would also change.

So in that case the constraint utilization would have been based on the fact that if we are using X1 amount of say for example raw material 1 for exterior and interior then the proportionality would be divided accordingly such that X1 the amount which you are utilizing would be less than 24 and similarly if we using X2 as for raw material 2 would be less than equal to 6. But again going back to the initial idea of trying to consider that what is the decision variable? Decision variable is exterior paint 1, exterior paint 2 which is X1 and X2.

Let us come to the constraint 2 based on the fact that it is important for raw material 2. For exterior paint raw material 1 needed (is) 2 needed is 1 ton per 1 ton production, so it will be X1 amount needed for raw material 2. Similarly for raw material 2, the amount needed for the interior paint is 2 tons per 1 ton of interior paint produce, so if the total amount of interior paint produce is X2, so hence the total amount needed for raw material 2 would be 2 into X2.

So the total, total now utilization of raw material 1 would be X1 coming from exterior and 2 X2 coming from interior, if you add them up, obviously the total sum cannot exceed 6, so that basically satisfies the constraint number 2. So let me highlight it. The 3rd and 4th constraints which have been mentioned when the problem was being formulated, so there were some concept related to what is the total utilization of exterior paint 1? What should be the total combination of interior paint 1 and exterior paint 2? Or the raw materials of exterior paint 2, so based on that let us check what are the 3rd and the 4th constraints, I will highlight it blue color.

Now remember here, if I take X1 on to the right-hand side it means that the X1 is for the exterior 1, so the summation of X1 plus 1 additional ton which we produce should always be greater than equal to X2 considering X2 is basically being utilized for the interior paint being produced by Asian paints. So what does it mean?

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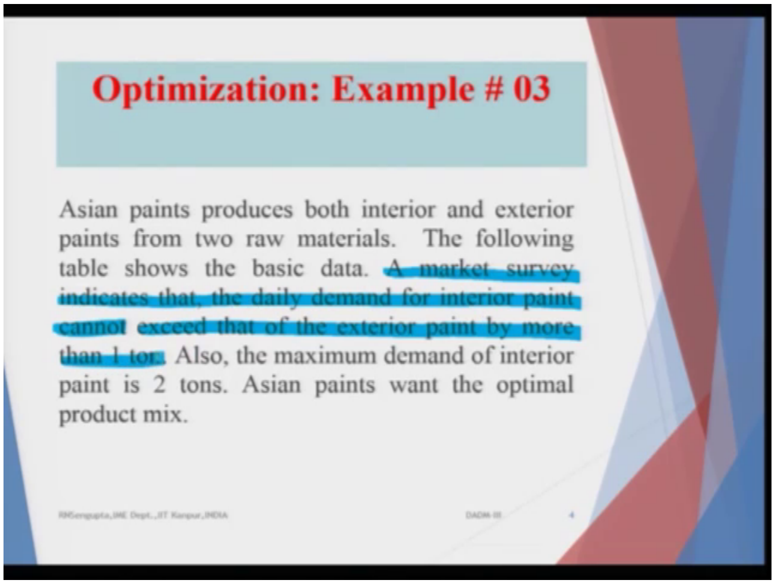

If I go back to the initial formulation it mentions very clearly, I will use the same color to highlight, so a marketing survey indicates that the daily demand for interior paints cannot exceed that of exterior paint by more than 1 ton.

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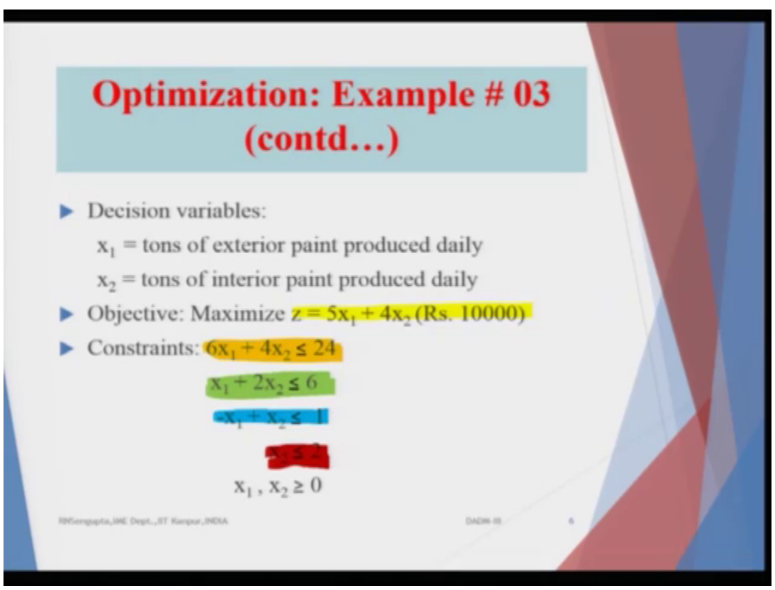

So when I basically bring it in the formulation this would mean that minus X1, I am just taking it to the left hand side, minus X1 plus X2 is less than equal to 1. So this is the 3rd constraint, so the constraint will basically give you the idea that what is the overall feasible set. I will use say for example red, now coming to the 4th constraint. So it gives you what? X2 is less than 2, so X2 is what? X2 is the interior paint less than 2 that means we cannot produce more than 2. So does it come out from the problem formulation? Let us check, I will use the same highlighting red in order to highlight that point which has been given the problem.

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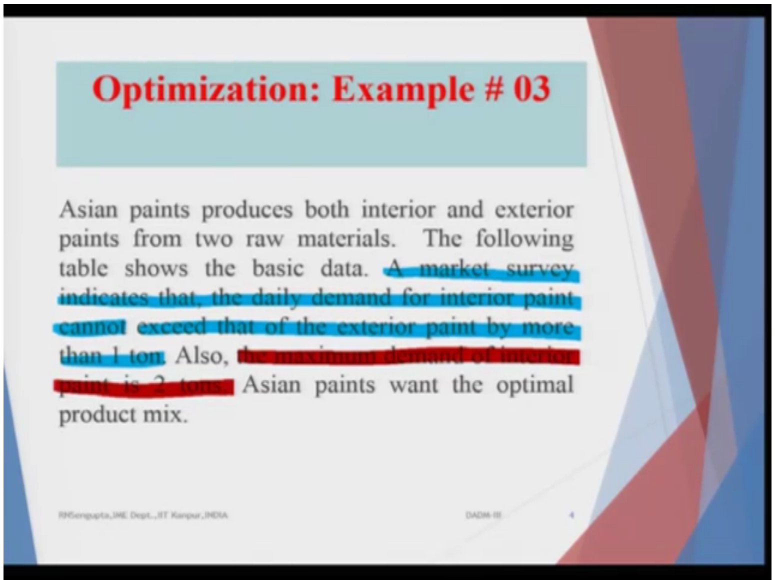

Here it mentions the maximum demand of interior paint is 2 ton that means the total amount of being produced for interior paint cannot exceed 2.

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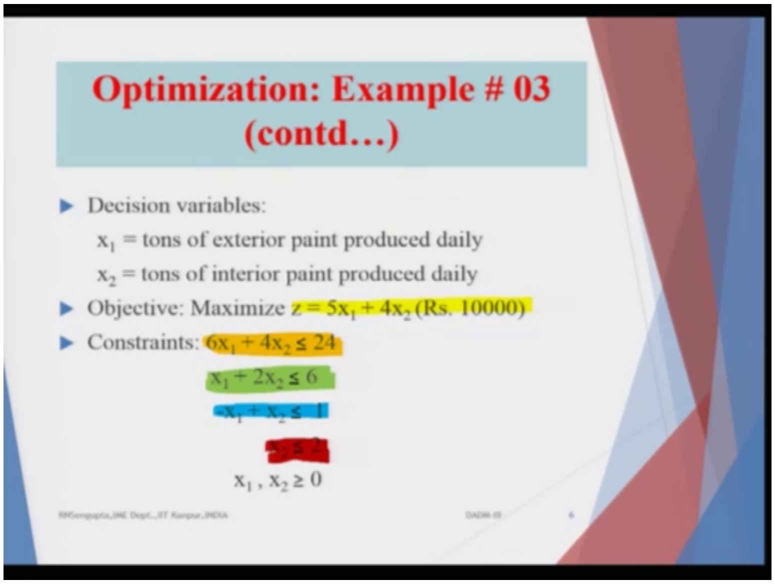

So that point which is being highlighted in red color is easily discernible in the constraint. Now if I see the constraint there are 4 constraints that means they are 4 equations. Now what is the dimension of the problem? That is 2, because we have only exterior paint and interior paint and raw materials 1 and 2. Now the problem of the dimension comes out6 from the decision variables we are producing.

Now (look) let us think it very rationally, in case if the utilization of raw materials was more than 2 but still we had 2 (exterior) 2 pains to be produce exterior and interior by Asian paints then only the number of constraints would increase not the decision variables based on the fact that how we have been able to basically utilize this in the problem. But if you formulate the problem in the sense where you will take decision variables as the raw material used, so in case we have 4 raw materials in that case the decision variable will be 4 and the problem has to be solved using simplest method.

Hence trying to basically draw it in a 4 dimension would not be possible for us to visualize how it problem looks. But other hand, considering the number of raw materials being utilized is 2 but the output which is exterior paint 1 or interior paint 1, if there are now say for example 5 paints paint 1, paint 2, paint 3, paint 4, so in that case the constraints remains two in this case because there are only two raw materials but the decision variables corresponding to the fact that we are considering X1, X2, X3, X4, X5 as the paint we produce would basically change the problem how you going to formulate.

But for the same problem which I just mentioned, if I reversed the idea of the decision variables so rather than taking the decision variables as the paints produce (P1 to) paint 1 to paint 5 we take it as the raw materials which is only two in that case again the problem comes back that we basically have a two-dimensional problem. So how you formulate the problem is basically is to be inculcated as you solve more and more problems, so there is nothing say straight forward technical which will basically able to give it to you that you will basically solve the problem as we proceed.

But the actual algorithm once the problem is formulated, so the main formulation would be important how the problem is formulated, how you solved would be much easier and that we will discuss. So in this way coming back to this problem we had 4 constraints, 2 decision variables.

The first constraint is related to the total amount of utilization of raw material 1 which is the orange one, the second green constraint is basically corresponding to the fact that we are trying to basically utilize the constraint based on raw material 2, the third which is green in color, the blue and red are the respective constraint corresponding to the fact that how the relationship between paint 1 and paint 2 and the maximum utilization of one on the other paint in being given in the problem formulation.

The last one which is very intuitive and very easy to understand is that amount of paint produce of type 1, type 2 or exterior- interior obviously is positive hence X1 and X2 is greater than 0. Now this (X) I am repeating it please bear with me I am sorry for that is X1 and X2 being greater than 0 was in the initial problem where we were basically trying to produce a number of products. But in that case X1 and X2 had to be integers here there is no such bounding conceptually or practically that would not be a problem. So what we will do? We will basically plot the constraints so there are 4 constants, we will plot it and check how they look like.

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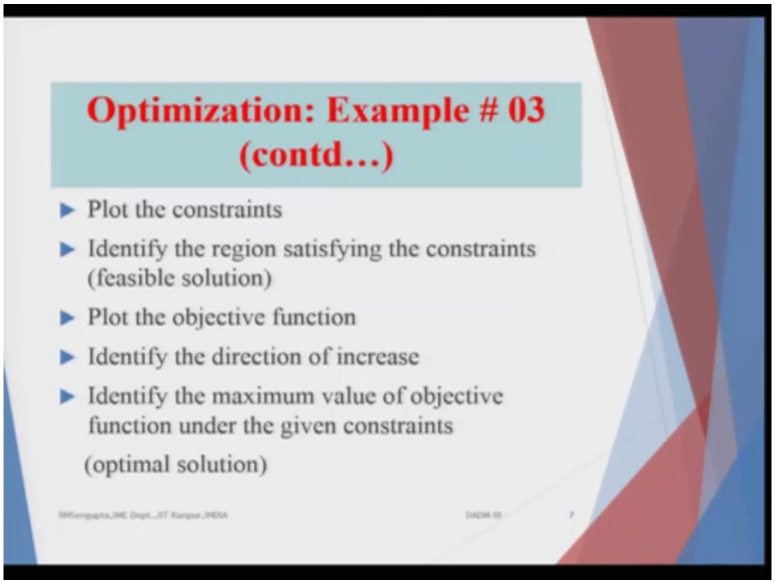

We will identify first the region satisfying the constraints because the (constraints) each of the constraint have to be satisfied in order to solve the problem, so though with the total set of solutions which is applicable to be taken up for by us for the solutions. We will also plot the objective function for the 2-dimension one and check that at what point the objective function leaves the overall and the constraint space of the feasible space in order to give us some idea where is the maximization on the minimization happening.

We will also understand that where the decrease and increase happens, in what rate and in which direction, so why the decrease or why the increase, decrease would be the point when we are considering the point of view from the minimization problem and increase would be from the point of view of the maximization problem. We will identify the maximum value of the objective function under those constraints and also consider the problem that this objective function once maximized would also be applicable to at that point which will be given by the values of the decision variables such that those values of the decision variables are the optimum value of X1 and X2 which will give us this answer

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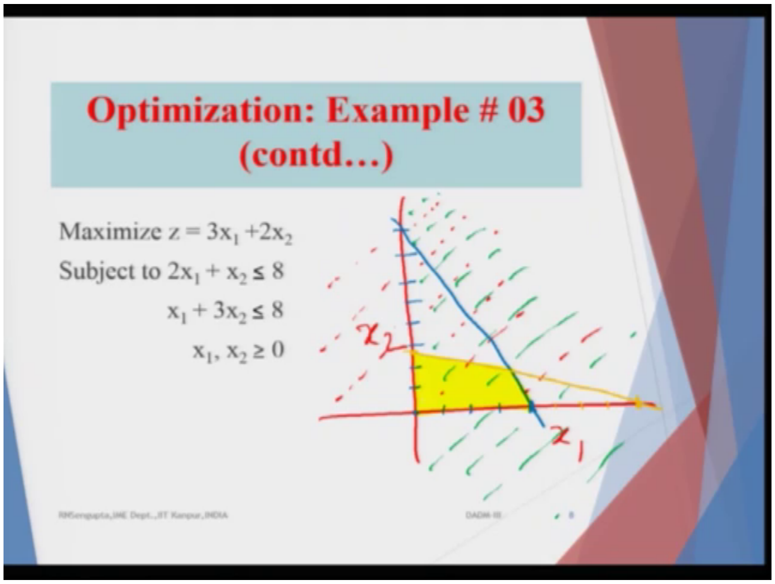

So the problem basically when we consider, so in this case (we basically) now going back to the initial problem the first one, that was the 8 number of hours which was applicable for machine 1 and machine 2, so you will want to maximize 3 X1, I will come to this problem for Asian paints within this class or the next class. So you want to maximize the objective function which was trying to basically produce number of product 1 and product 2 considering 8 number of hours was fixed for machine 1 and machine 2.

Now before I solve the problem look at the constraints, so if I put the equality sign what it is? It is 2 X1 plus X2 is equal to 8 and another constraint is X1 plus 3 X2 is equal to 8. Now less than 8 would be possible when I put 0 on to the left hand side of both the equations, so obviously 0's in both equation 1 and equation 2 is less than 8 which means the origin we are now sure that the origin would be lying on to, if I am considering from my point of view it would be lying onto the left of the equations. Hence we know that it in which direction the feasible set will occur and obviously another constraint would be X1 and X2 is greater than 0.

So very simply if I draw, consider this is the axis I will go into the actual diagram later on, X1 being greater (than) and consider this is X1, this is X2. X1 greater than 0 would be it will become cluttered so I will just mark it varies, so this is for X1 so all these regions obviously it would be in this region also. So technically anything above when I consider X2 is greater than 0, so let me use a different color.

So obviously these values would also come, so first and second coordinate for X1 and first and fourth for X2, so now the common area between them is only the first quadrant. Now plot it this equation, so I only plot one and I continue the other one, so if I consider X2 values another color let me use the blue one, so if I consider X2 as 0, so X1 is basically 4. So let us consider 1, 2, 3, 4 and if I consider X1 as 0, X2 would be 8, 1, 2, 3, 4, 5, 6, 7, 8.

So if I consider the line, so any part on to the left because 0 was important for us, so any part onto the left is an applicable. Finally when I plot the third one, so third one yes I am taking this third one let me use the color say for example orange, so when X2 is 0, X1 is 8, 4, 5, 6, 7, 8 and when X2 is 0, no X1 is 0, X2 is 8 by 3 so will consider it to be say for example 8 by 3 would be 2.2 third, so 2.2 third so it could be somewhere here, so if I consider so again this portion below so technically if I consider I will use another black color in order to make it very simple or else use the highlighter blue color let me use the yellow.

So the overall space comes out to be this I have drawn try to draw it very simplistically obviously in 2-dimension, 3-dimension the conceptual idea would be coming up later on. So I will continue solving this problem in a very simplistic manner and then basically give you a picture that what is the concept of feasible regions, slack in which direction the solution moves in order to give us a good idea that how the LP formation work in a very simplistic manner. Have a nice day and thank you very much.