Data Analysis and Decision Making - 3
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Lecture 10

A good morning everybody, good afternoon, good evening, to all of you wherever you are in this part of this globe and welcome back to DADM-3 which is the Data Analysis and Decision Making-3, a NPTEL MOOC lecture series and as you know this total course durations is for 12 weeks which is 30 contact hours which when broken down into numbers of class would be 60 considering that each lecture is for half an hour and each week we have 5 lectures after (when) that we have an assignment so you have already completed assignment 1 and today is second weeks last class which is the 10th lecture for the whole series.

So after this you will be undertaking the assignment number 2 and in all totality you will have as you can understand there would be 12 assignments and then one final examination. So, if you remember we were discussing about utility and I will come back to concept of the utility later on when we consider the stochastic programing and all this concepts in details. Now, for the safety first principle which we were discussing, so safety first principle had 3 criteria one was basically trying to find so they were two issues.

One was what is RP and RP bar? RP bar is basically the average value and P is basically the overall conglomerate decision which you want to make, how that is going to be formulate? I will come to that later on just a be patient. It would be coming up later on and for the other two ones what basically related to RL, RL is some minimum bound for your decision, not the decision variables for the decision you want to take and it can be change to RF which is the risk for interest rate as per the concept of finance.

So say for example we are trying to solve some problem in Mechanical Engineering, the problem would be definitely trying to maintain the ductility of that material or maintain the strength of the material or the flow of heat whatever it is or insulation capability or the height of a cooling tower in a thermal plant, so all this things can be done, so I am giving you a very simple examples.

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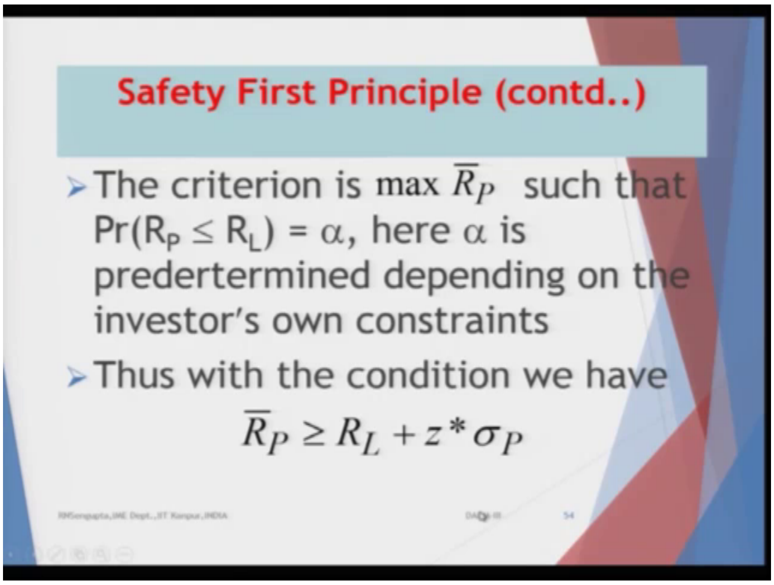

So when you are considering maximization of RP bar what you are trying to do is that you are trying to shift the average value of the distribution more on to the right, so the distribution by itself will move.

Such that what you want to do is that as the average value moves it can be it may be possible that you keep the value or the probability of the overall area for RP. Now RP is the random variable for RP being less then RL is bonded by some alpha value so that is the level of reliability or reliability confidence you want to put. So here alpha is the predetermined depending on the investors own choice and own constraints or the experiment what he or she is doing. Thus, if he consider the very simple case of normal distribution so it can be easily broken up into the case.

So let me make one blank slide and explain it to you. So it would be easier for you to appreciate so, so now this is where we would like to this I will keep it in order to explain that how with this was derived.

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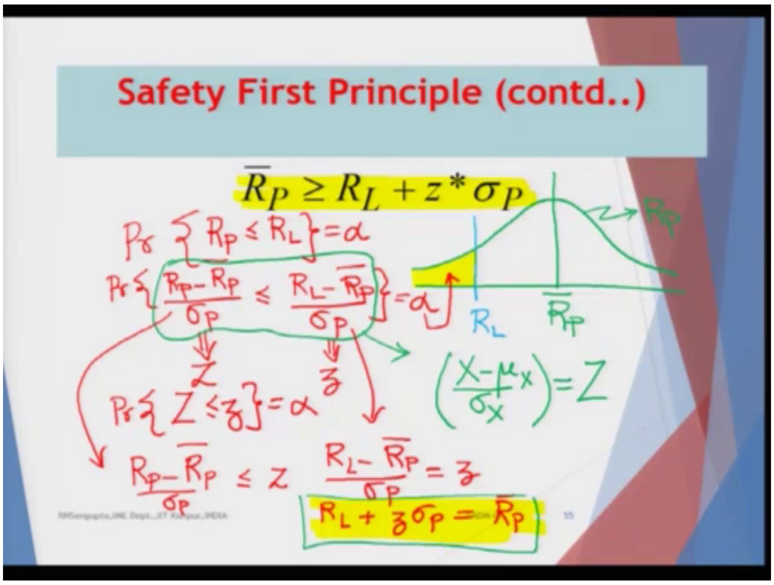

Ok, now what we have is if you considering the probability fact, it will be probability of RP bar let me check (so) because there can be two ways how you, so this is probability of RP is less than RL alpha value ok. So probability RP less than equal to RL is equal to alpha. So let me draw the diagram also, so consider this as a normal distribution, this is the means value, this is RP bar and this is the distribution of RP.

Now coming back to here, so considering the normal distribution so this probability RP minus RP bar by sigma P less than equal to RL minus RP bar by sigma P is equal to alpha, so consider this value of RL is here, so let me put RL I am using different colors in order to differentiate and the area which we want to so this is the area alpha value. So technically so this is the alpha value which I am talking about. Now what it leads to? So this is Z, capital Z and this is small z, so you have probability of capital Z less than equal to small z is equal to alpha.

So this fundamental concept which we are utilizing, this one we are utilizing the concept again using different colors is basically x minus Mu by sigma x Mu suffix x these are suffix x is basically the standard normal normality of xZ, you did just two simple replacement of the variables. So once, so now see here in Z case Z is known as RL is known, RP bar is known, sigma is known you find it out capital Z find other value of alpha also given for that you find out capital Z and in the capital Z you find out the value. Such that you can find out the distributions and its mean and whatever is not given you can find out from this 1 equation.

Now if I expand it, so obviously this became small z, so if this became small z so you utilize this formula, so I am going to use the color same so that is RP I am just utilizing the inside part RP minus RP bar by sigma P is less then equal to small z. So this will be RL sorry this is RL ok this value, so this is equal to capital Z and the right hand side other part so this is and this part RL minus RP bar by sigma P is equal to so obviously this is greater than so you will definitely have the concept I will put the equal sign first, so small z so it would be RL plus because if you take RP bar on to the right hand side so it will be this.

Now see here, so if this minimum value has to be alpha and obviously the less than sign greater than will come accordingly. So the equations based on which I will have this is what coming out from here, I will just highlight it because rather its getting a little bit cluttered so I will highlight it using one color so this is the equation we were talking about, this greater than sign would basically come from the fact that the value of alpha is this one is less than or greater than capital Z were the capital Z is less than equal to small z or greater than equals to small z. So that will depend on which type of the distribution you are looking on.

Now this is important to note, we have considered that I have explain that in DADM 1, so this left hand side, right hand side would depend on two things, so if you have done hypothesis testing it would mean that whether you are taking the left tail or the right tail considering the hypothesis testing problem how you are going to formulate, so in that case it can be done for the case of normal distribution when you using the optimization case it can be done as greater than or less than.

Now if you further extend it, so consider RL is fixed, so you want to basically (ex) keep increasing the value, so in the case the blue frontier root with the efficient frontier in general case for the portfolio optimization problem and if you keep RL fix and keep increasing the value so what you have in the other problem is.

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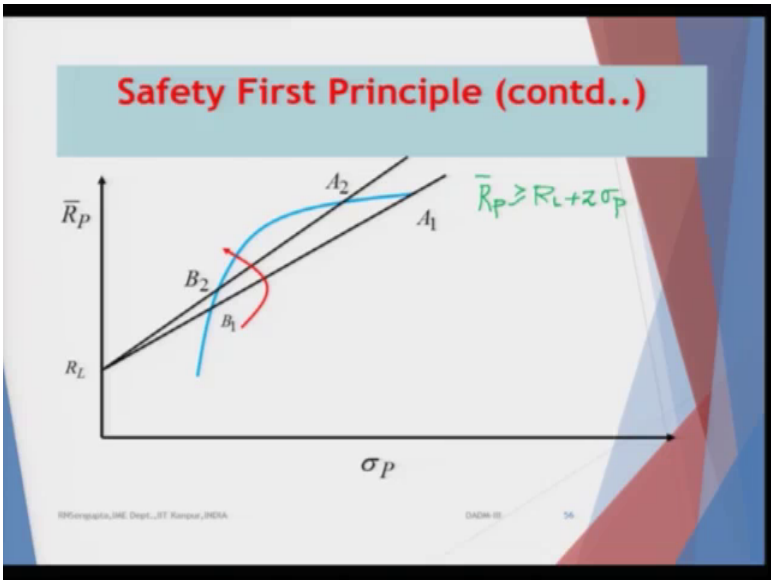

This, so this means RP bar is greater than equal to RL plus z sigma P, so for keep increasing the value of small z that will depend on the value of RL and the alpha value also.

So obviously as it keeps increasing it will be a basically moving counter clockwise till it is tangent to this point where it exactly lives I am not going to draw it here because I will start showing it later on. So this leaves the surface say for example, when I am hovering my on the this electronic pointer. Now as it leaves it that point would basically give you the best combination of the decision variables based on which you are going to find out the optimum portfolio depending on what the overall utility you have. I will come to the optimum best decisions within few minutes.

So the moment it leaves outside obviously there would be non-feasible regions, so the concept on non-feasible, feasible region I will come as I said within few minutes, Now consider your projects A,B,C,D there are 4 projects and you are investing and the average value of RP and sigma for all this 4 projects are given.

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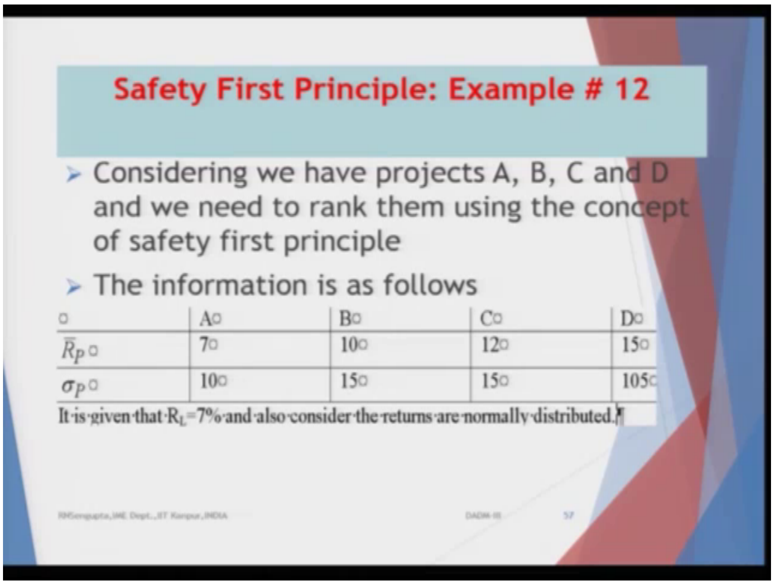

So considering your 4 projects A, B, C, D and we need to rank them using the concept of safety first principle, so the RP bar value of A,B,C,D are respectively 7, 10, 12 and 15 while the values of the standard divisions are 10, 15, 15 and 105. So this are the standard division obviously will square them to find out the variances. So if RL is given as 7 percent and also consider the returns are normal distributed you would rank the decisions based on the concepts of safety first principle. So obviously you can use it (utilize) optimization problem also. So I will just give the basic framework for the normal distribution and remember one thing, the problem can also be solved for other distributions also.

But the issue is that for other distribution trying to formulate using the standard normal derivative would not be possible, you have to do some mathematical modeling or try to use simulations in order to basically generate the number of data or find out an exact solution.

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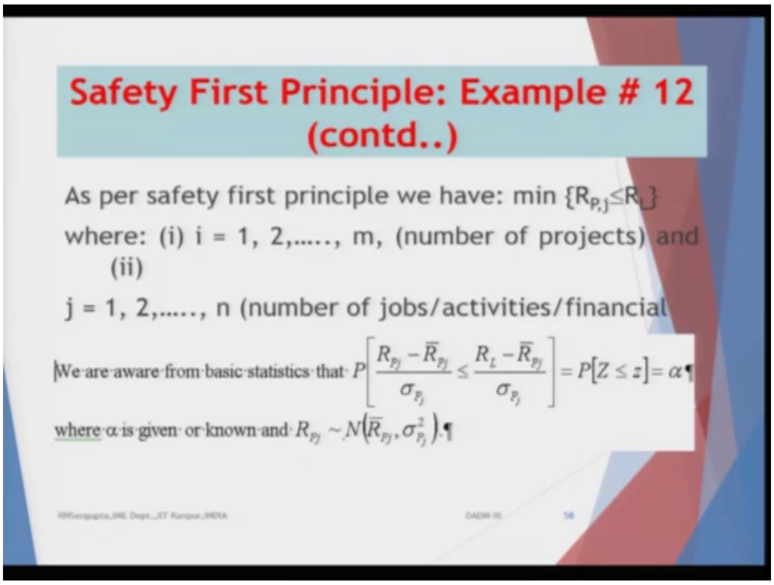

As per safety first principle we have to basically minimize the concept of RP j, j is basically the project is less than equal to RL.

So were 1, I is equal to 1, 2, 3, 4 and there is the numbers of projects and j is the numbers of job of activities of financial decision which you have. So each would basically have different decision variable I want to basically optimize them. So obviously it would be if I use the safety first principle, so this RP j minus RP bar j so for each project you find how you have the (mini) average values, standard divisions are given so which get converted into standard normal deviate based on the fact that you considering the normal distribution. Again I am mentioning that normal distribution need not be true always.

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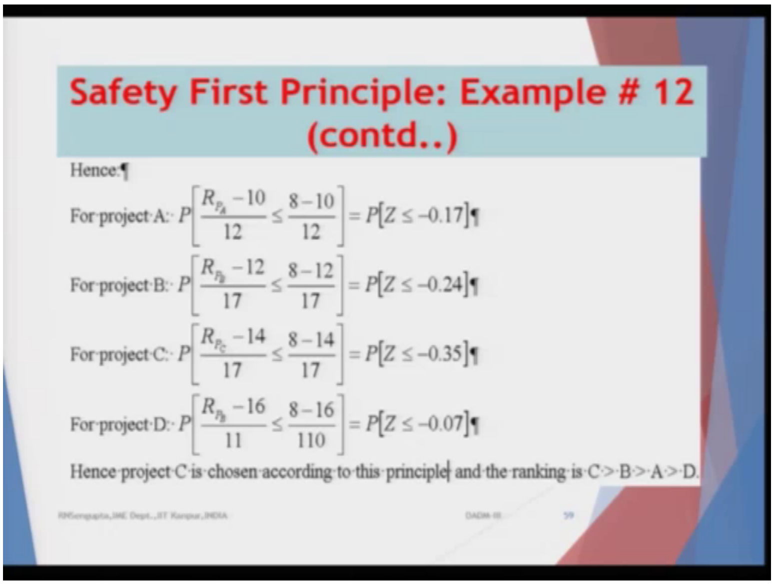

So once I have the this, so for project A the value of standard deviate z is less should be less than equal to minus 0.17, for B it is should be less than equal to minus 0.24, for C it is less then equal to minus 0.35 similarly for finally for D it is less than equal to 0.07. So if I find out the values it would technically means the if you are trying to basically find out the probabilities, so hence you will rank them considering that how far the values would be onto the right. So obviously you will rank then as C is better than B is better than A which is better than D.

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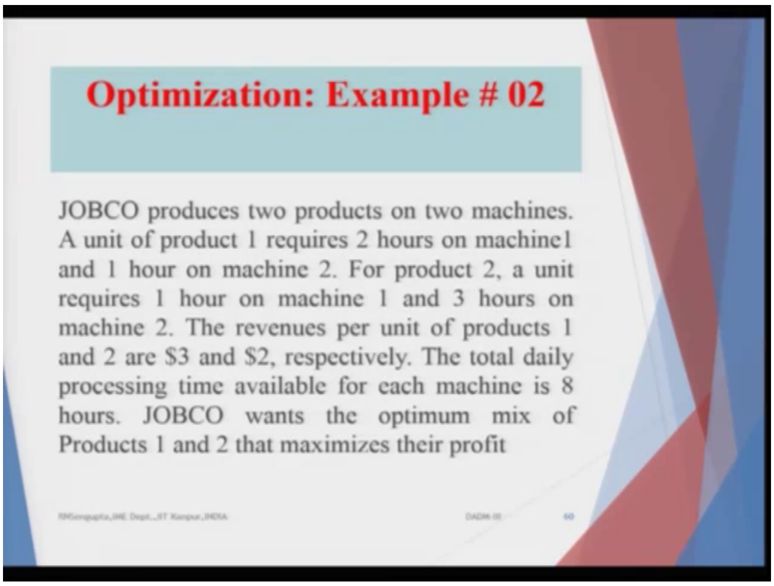

Now we will slowly go into the area of optimization and this class would be at least this part for 2 or 3 lectures would be (own) only of conceptualization on how the model has been framed. So consider an example I will start with the example and then come to the general concept, so consider the company JOBCO produces 2 products in on 2 machines. So now mark the word there are 2 machines, if we extended it what it will became? It will became clearer as we proceed.

A unit of production 1, require so product on 1 from machine 1 requires 2 hours on machine 1 and 1 hour on machine 2. Similarly for product 2 which is the other product, 1 unit requires 1 hour in machine 1 and 3 hours in machine 2, so obviously the combinations are there. The revenues I am just very simply considering from product 1 and product 2 is given as 3 and 2 some units.

So the total daily processing time available for each machine, now remember this is important is given by 8 hours per day that means machine 1 also has 8 hours considering as a shift, so obviously if the 3 shifts were, we are continuing so it would have been 8 into 3, 24. If there are 2 shift so it 8 into 2, 16. So considering anyone shift you have basically maximum 8 hours per machine per day and the company wants to optimize what is the word optimization I want to come to that later, if you want to optimize such that is able to optimize the product combination in order to meet its criteria.

Now let us pause here for 1 minute, so when I am saying the word optimization from the point of view of the company given the information which you have you will consider very simply the revenues are given and you want to increase the revenues per unit of production which you are getting for product 1 and product 2. But on the other hand if say for example, the cost what given, only the cost in that case the combination would have been such that you would definitely like to decrease the overall the cost that means minimize the loss in such a way that you are able to meet the criteria.

Now remember one thing, whenever the problem in formulization is done, whether you formulate as a maximization one on the minimization one before that one should be aware that what is the decision variables and how you going to optimize it point 1. Point number 2 the constraint should be formulate in such a way that they would give you some practicality in the situation what you are trying basically formulate so I will come to that the practicality with few examples later on.

So here as the avenues are given obviously avenue means some positive thing which is coming to the companies pocket so obviously it will try to increase in that sense the optimization problem would be maximization one.

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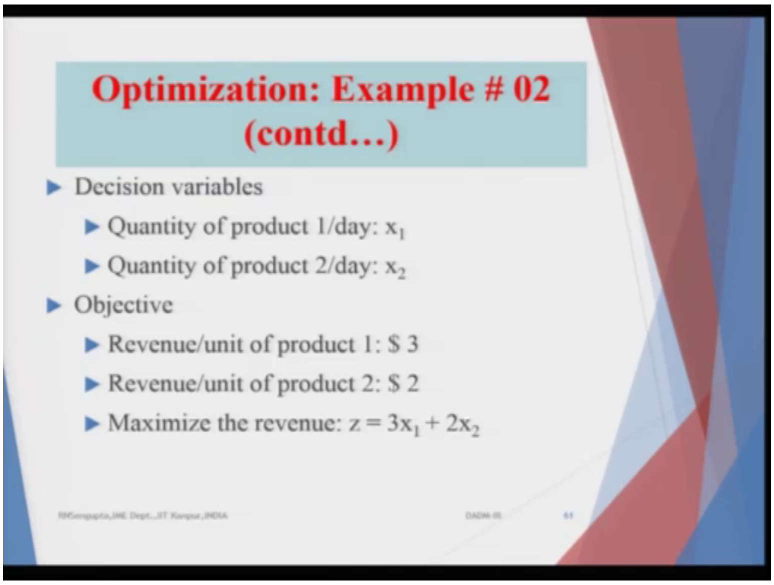

So now we are basically would like to formulate the problem as the decision variables and what is the objective? So the decision variables for the company first is product 1 will denote by either P1 or X1, so here we are taking the (quantity of) product produced per day remember here, the constraint are given per day it is 8 hours it was given per week we would have basically formulate the problem accordingly, so I am going to come to that within in few seconds. So when I mention the quantity of product produce per day is X1 in case if the total duration of time utilization for machine 1 and machine what 2 was given per week then the quantity of product produce per week would have been considered at X1.

In the other sense say for example, if it was given then the total production is given for 6 months the total utilization of the product which you have not the machines say for example total raw materials which you have in that case I will basically formulate the problem as the case were X1 the decision variable or X2 as the decision variable whatever the decision variable is would be considered on the time scale of 6 months.

So hence that formulation concept would be easier for us to attempt. Obviously you can do it for per day and then multiply it by the number of days of working which is there in the 6 month period but we will be basically ignore that for the time range. So the quantity of product produce per day for product 1 is X1 quantity product produce for day 2 is basically X2. And the objective, the revenue of the product which is being produced per product per day is 3 and 2 that means per unit sold, you get 3 rupees or 3 dollars, 3 euros whatever it is and for product 2 you get 2 units or 2 dollars whatever it is. Hence when you want to maximize the revenue would be basically.

So for each product which you sell is 3 units you get in your pocket, so it will 3 into X1 into product 1 and 2 into X2 for product 2 that means you want to maximize the combination of 3 X1 plus 2 X2 such that your overall profit is at the highest level.

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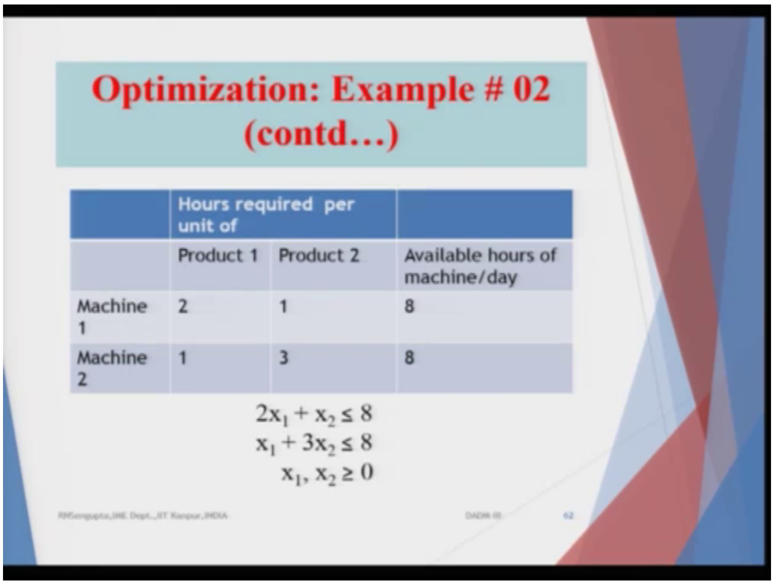

Now let us come what are the constraints? So first let me draw the chart. So this is very simple problem, so drawing the chart is only to facilitate a good understanding other problem. So for machine1 product 1, hours required already we have mention is 2, product 2 is 1 from machine 2 product 1 requires 1, product 2 requires 3 availability on each machine is basically 8 hours per day it can change also, but we are considering the simplistic case of 8 hours.

Now, if I produce X1 amount of product 1 and X2 amount of product 2 then obviously the total combination of 2 X1 plus X2 would basically be less then equal to 8 because you cannot exceed 8.

So say for example say for example, if no X1 is being produced, so in that case the maximum value of X2 would be 8. In case no X2 is being produce and the only X1 is being produce then in the case the maximum value of X1 being produce would be 4 from the first equation. Similarly if I go to the second equation which is corresponding to the fact that I am taking machine number 2, in that case it would be X1 plus 3 X2 is less than equal to 8.

Now in that case, if I produce no of X2 product and only X1 in that case the total production of X1 would be 8 and if I produce no X1 but only X2 then the number of products would be 8 by 3, Now here I should pause, obviously the question would be ask from your side that when we are trying to basically formulate a problem where the units produce are only integers, how is how it is possible the number of units produced for X2 in the second constraint when you are considering the constraint to the machine number 2 is 8 by 3.

We for the timing will ignored that, solve the problem and then basically later on when we go into integer program will check it how it can be solved and obviously it would mean as I have said one on the main assumptions was that the values of X1 and X2 would definitely greater than 0. So this is what is given so X1 obviously you cannot produce negative one but in some formulation later on we will find out when we are trying to give the formulation of inventory related problems it can be done as a, as the variables X1, X2 can be negative depending on whether back ordering or some orders which have been spilled over or they have not been delivered they can be done accordingly. What is back ordering all this things I will come to that later.

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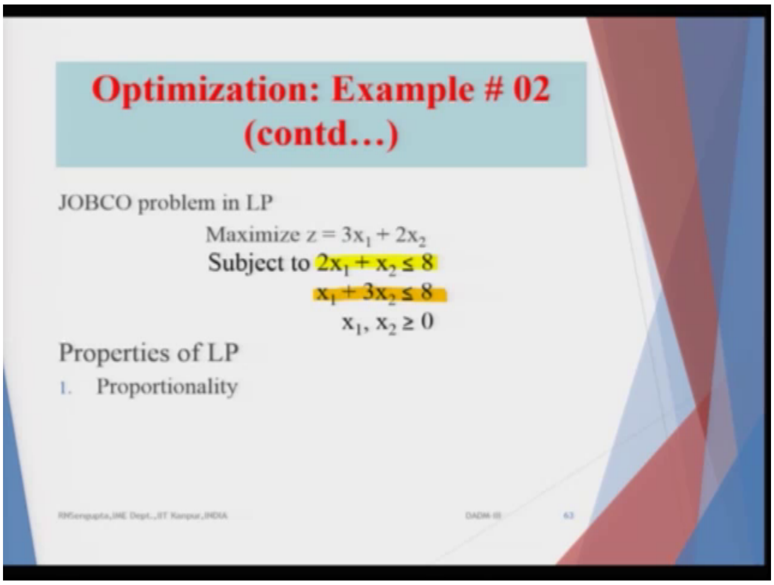

So here the company's linear programing problem is maximize 3 X1 plus 2 X2 subject to the constraints, so now on the constraints are technically 2 but added to the fact that you have the third one also which basically means all the decision variable should be greater than equal to 0. It can be 0 also, but it has to be greater than equal to 0. So maximize 3 X1 plus 2 X2 subject to that constraints the first machine 1 is 2 X1 plus X2 is less then equal to 8 and the second one is X1 plus 3 X2 is less than equal to 8 in both the cases and X1 and X2 greater than 0. So the properties, now let us see the properties of the LP with (())(22:44) formulate here.

Now let us see proportionality, so the proportionality of the products so if I increase or decrease concerning the conditions are as it is, the profit would be for X2 per unit production would basically be as it increase by 1 in it or decrease it by 1 in it the objective function keep increasing by 2 or decrease by 2.

So they would not be any change in the amount of revenues which you are getting for producing one extra amount of X2 or revenue would not decrease by any value other than 2 for 1 unit decrease in decrease of X2. Now this practicality has to be considered even though in actual situation it is not true to give you an example, say for example you are selling a product and you want to formulate to the problem such that you want to maximize your profit. So it may be possible that you have to basically sell the products at a discount, so hence considering that discounting factors would be consider in some other problems for the time being we will ignore that, ignore that very simply. Number 2, when we consider the constraint so any increase or decrease of the production units for X1 and X2 would always keep the fact that 1 unit decrease in X2 would obviously means say for.

Let us consider this say for example in the first equation let me highlight constraint. Consider X2 is a 0, so in that X1 produced is 4 so there is full utilization in that case. Now considered that X2 has increase by 1 unit, I am not talking about the optimization problem only the constraint which is highlighted, movement X2 increases by 1 unit in that case what you will have is that the equation would became 2 X1 plus 1 is less than equal to 8 which means 2 X1 is less than equal to 7. Now let us pause, in that case X1 would basically be technically be less then equal to 7 by 2 which is 3.5.

Now you will ask yourself and I am (does) just a inculcating the inquisitiveness in you, you will ask yourself that is it possible to produce 3.5 units of X1? The answer is no, in that case what is maximum number of X1 which I can produce would definitely be equal to 3 only so the 0.5 which is left here for X1 would mean that some of the utilization on the time frame for machine 1 is left unutilized. On the other hand consider this, so this was 1 unit increase in X2 from 0 consider X1 was 0 initially so X2 was 8, I am again concentrating on equation 1 only. So in that case if X1 is 0, so that in case X2 is equal to 8, so every the utilization is total so there is no time left in machine 1.

Now consider that X1 increases by 1 unit, so in that case it became 2, 2 into X1 is 2 plus X2 is less than equal to 8 so X2 became less than equal to 6. Now in that case if we produce 6 units you will see that in respect to the earlier example which I gave also related to constraint 1, there is no amount of time left in machine 1 because the utilization is complete. In the first case when X1 is X2 was increasing by 1 unit then the time left for machine 1 as you want to produce 3 units of X1 would be 0.5.

So remember that this combinations of what is the best combinations of X1 and X2 would slowly start turning up and they will give you some concepts of slacks, some consist of shadow prices concept, what are these I will come that within either today or in the next class later on, in the 11th or the 12th class. Now let us go to the second constraint, again let us see, X1 is 0, X2 is total production which is 8 by 3 that means total production for constraint 2 is not possible, so there is some amount left in timing for machine 2.

So any increase in say for example X1 by 1 unit as X2 decreases would also be of the same fact that the utilization of the machine hours may not be totally complete. Similarly would be the case when you exchange these values that means X2 is 0 and X1 is 8 in the second constraint but as you increase start increase X2 the amount of product which we produced in that machine 2 considering the numbers of hours would also be fraction that means they would some unutilized time left in machine 2 also.

Now when you consider both the constraint one which is shown in yellow color and another one which is shown in light orange or some this brownish orange in that case it will be possible that the combination would lead to many of the examples of situations where the number of hours left in machine 1 and machine 2 may not be 0, they would be basically some positive quantity. So how we are trying to basically formulate the problem would also give us an answer that what is an extra amount time left in this case only machine 1 and machine 2 with respect to time it can be for resources also and how they can be reduced and how the prices can be change will consider that in the next class as we proceed.

So with this I will end the 10th lecture and considering that we will proceed of trying to give a pictorial diagram first and then going to solution it will be much easier for me to explain the concept of linier programing later on. Thank you very much and have a nice day.