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**Lecture – 06**  
**Rules of Modelling**

So, back to the rules of modeling. We were talking about what are the roles that we would like to use for modeling these are like I would say these are rules of thumb ok or more like a practitioners rules in a way.

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**Rules of Modeling**

Rules of thumb:

(i) There are different ways to model any optimization problem.

→ Models of the same optimization problem can pose very different model-solving challenges.

The following rules of thumb should help a modeler (individual) to avoid some model-solving pitfalls.

→ avoid building models that are difficult to solve (computationally).

Rule -1: Keep it Simple – use functions that you recognize from calculus, and in these use the simplest ones.

→ In particular, use linear functions whenever possible because they are the simplest.

$$\frac{x_1 + x_2}{x_3 + x_4} \leq 5$$

↓  
Computationally hard

⇒

Simplify to linear.

$$x_1 + x_2 \leq 5(x_3 + x_4)$$

↓  
linear form - easy to solve computationally.

So, the number 1 thing that you should understand these there are different ways to model a problem or the any optimization problem. There are many ways there are multiple ways to model any optimization problem ok.

So, when you have multiple ways to modelling any optimization problem, it is already going to create an issue for you ok. Models of the same optimization problem; same optimization problem we have models of the same optimization problem can pose very different model solving challenges; very different model solving challenges. So, what we are saying here is that, if you have different ways of modeling the ways of modeling we just do not want to model the problem, can you just do not want to model optimization

problem you actually want to solve it. So, depending upon how you model it, you will end up resulting in having different challenges in solving the problem ok.

So, this is one fact that you need to understand. Then the following rules of thumb these are not really official rules, but these are guidelines thumbs should help a modeler model of who is a modeler? Individual right. The individual who is doing the model is the modeler should help modeler to avoid some model solving pitfalls.

So, there are some issues in the model solving when you come to solving a model you can build a model, but sometimes you will end up building a model that will be very tough to solve. So, what do you want to do is, you want to avoid such scenario where you end up building a model that is difficult to solve. So, the aim is to avoid building models that are difficult to solve and remember to difficult to solve means computationally.

Why? You are using a computer to actually solve the model the modeling is done by hand the solution is done by the computer. So, if you build a model that is difficult to solve computationally, then you want to avoid such kind of modeling exercises. So, these rules of thumbs are designed to avoid building such models. So, the first rule the major rule in that regard is rule 1 first rule is keep it simple ok. I would go and say this is part of the keys principle keep it simple and stupid principle, but its keep it simple ok. Keep it simple means use functions use functions that you recognize from calculus ok. And in those use the simplest ones.

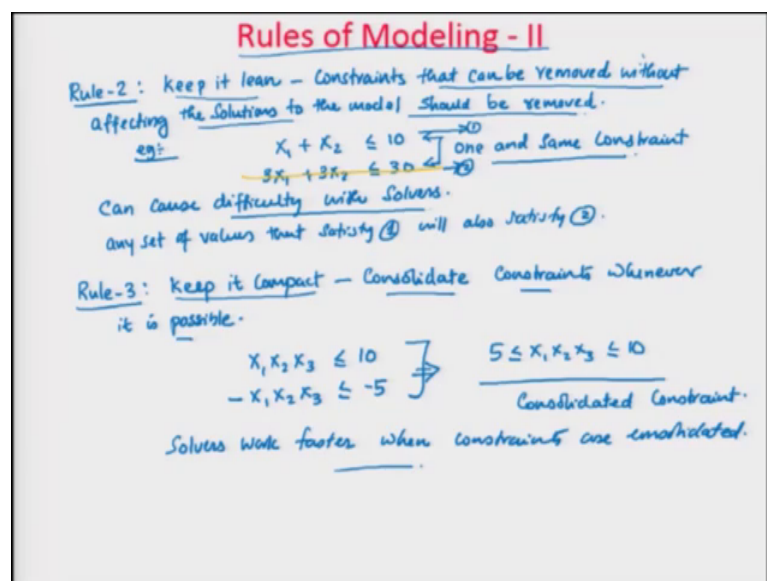
So, as much as possible try to use the functions that functions, that you recognize from calculus that do algebra calculus allows you to do differentiation and integration in a way, that actually helps you to use the gradient method of solving the problem ok. And even in the calculus functions that you use, ensure that you use the simplest ones. So, in particular use linear functions whenever possible; use linear functions whenever possible because they are the simplest ok.

So, for example, if you write a function  $x^1 + x^2$  divided by  $x^3 + x^4$  is less than or equal to 5 this could be as computationally hard or the because of this question approach, the computer might take longer time or longer it will be are very easy to solve using a software, you can write the same as  $x^1 + x^2$  is less than or equal to 5 times  $x$

3 plus x 4 ok. If you write it like this then this is a linear form easy to solve computationally.

So, the first one is keep it simple; which means you use functions that you recognize from calculus such these kind of functions, and in those functions use a simplest one. If you have a fraction like this or you have a question version like this converted into a simpler linear function. So, this is the process we can call it as simplify to linear. As much as possible try to use linear functions a wherever you can done.

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The second rule of modeling there are 4 rules the second part of it is rule 2. I will call it as keep it lean. The first one is keep it clean second one is keep it lean. So, which means constraints that can be removed without affecting constraints that can be removed without affecting the solution to the model should be removed.

So, for example, an example of this would be, we have a constraint called x 1 plus x 2 is less than or equal to 10 and let us say you have a constraint called 3 x 1 plus 3 x 2 is less than or equal to 30 ok. If you think about that one and the same constraint though these constraints are one and the same writing the constraint 2 times. So, it can cause difficulty with solvers ok. We are using Microsoft excel or something to solve the problem, that solvers always will have difficulty when you see these kind of constraints which are two times. So, so second part of the second rule is keep it lean, which means constraints that

can be removed without affecting the solution. So, the model should be removed; if you remove one of these constraints.

So, for this example if you are saying that I am going to eliminate this constraint as part of this ok. I do not want their constraint because it is a scaled constraint of the other one, it is not going to affect your final solution because that a set of values. So, in this case if I call this as 1 and this as 2 any set of values. So, any set of values that satisfy 1 will also satisfy 2. So, there is no need to put duplicate constraints or such kind of constraints that are duplicated in this regard right.

So, most of us always would do not like you to do these kind of things. Then the third rule 3; rule 3 is keep it compact. So, keep it compact means what you are saying is, consolidate constraints whenever it is possible. Say whenever you are whenever it is possible for you to consolidate the constraints then do that.

So, for example, is that if you have a constraint that says  $x_1 \times x_2 \times x_3$  is less than or equal to 10 and you have another constraint that says  $x_1 \times x_2 \times x_3$  is less than or equal to 5 then you can consolidate this by saying is that,  $x_1 \times x_2 \times x_3$  less than or equal to 10 you can do a consolidation.

So, this is a consolidated constraint. So, when you consolidate a constraint then that becomes easy for the solver to do this. So, solvers work faster when constraints are consolidated. So, this is an advantage that you will have if you try to do this. So, whenever we can do, whenever we say keep it compact; that means, you are consolidating the constraints whenever it is possible for you all right.

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**Rules of Modeling - III**

Rule -4: Keep it continuous - it is better to use continuous variables whenever possible; over those variables that are constrained to have integer values (or) a finite set of values.

$2 \leq X_2 \leq 5$        $X_5 \in \{2, 3, 4, 8, 10\}$

⇒ Why?  
→ optimization solvers (they solve the models) work faster with continuous variables than discrete variables.

↓  
they allow for the usage of calculus techniques.

- (1) Keep it Simple - use simple calculus functions, linear whenever possible
- (2) Keep it lean - eliminate duplicate constraints
- (3) Keep it compact - consolidate
- (4) Keep it continuous - whenever possible use real variables.

Then comes the third rule of modeling rules of modeling 3 ok. The rule 4 the fourth rule we are gone through 3 rules of the fourth rule is keep it continuous.

So, it is better to use continuous variables if possible or stuffy possible whenever possible ok. It is better to use continuous variables whenever possible over those variables that are constrained to have integer values or a finite set of values. So, what we are trying to hear is, it is as much as possible whenever it is doable use continuous variables or the value of the variable.

So, if you say that a value variable  $x_2$  if you say 2 less than or equal to  $x_2$  less than or equal to 5 ok; that means, all values between 2 and 5 and there could be a scenario where we can say an  $x_5$ , where the values can be of 2 3 4 8 10 something like this, then it is always better to use a scenario of continuous variable than a discrete or a set of variables that are integer set of values for a finite set of values and why? The question is why do we use this rule? The answer is optimizations solvers, the computer programs which they solve the models.

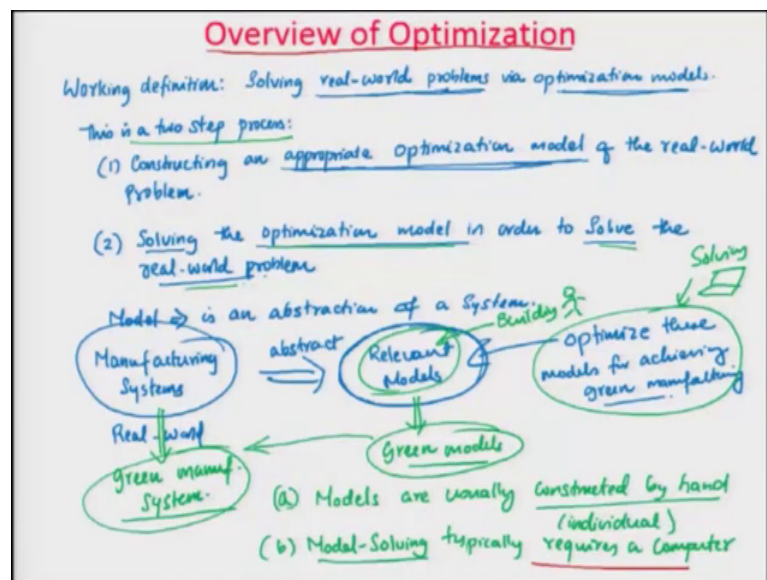
The optimization solvers which will care a computer work faster work faster much faster with continuous variables ok. They work much faster with continuous variables than discrete variables ok. Why? Continuous variables they allow for the usage of calculus techniques. So, since the continuous variables allow you to use calculate techniques or techniques of gradient or such kind of things, the optimization solvers will work much

faster in the in the realm of continuous variables than the discrete variables. So, we have seen there are 4 rules to this.

So, if you think about the rules, first one is rule 1 is keep it simple keep it simple means as much as possible keep the function simple whichever we can be used. So, it is like use simple calculus functions linear whenever possible whenever possible. Number 2 is keep it lean what we said is eliminate duplicate constraints or constraints that can be reduced or lubricate constraints in this case I am actually referring to is eliminate the duplicate constraints means, constraints that can be produced. Then the third one we talked about it was keep it compact. So, in keep it compact what we did is, consolidate constraints whenever is possible and then the fourth rule what we talked about is keep it continues.

So, whenever possible use continuous values for use continuous variables or use real variable. So, let us put it as real variables or what discrete variables. So, if you think about it this all thing what we are covered in the class so, far today is the main 3 aspects of optimization.

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### Components of Optimization

Three main components in an optimization model:

- (1) Variables: These represent model components that can be changed to create (or emulate) different optima (possibilities or alternatives)
- (2) Constraints: These represent limitations (restrictions) on the variable.  
eg: Variable = Fuel efficiency  $\Rightarrow > 0$ .
- (3) Objective function: This assigns a value to each of the different possibility and it gets its name from the fact that the objective is to optimize this function.  
 $\Rightarrow$  The optimized objective function will provide the optimal values of the variables.

In this course the "models" that are used are mathematical models. These mathematical models are developed to represent sustainability aspects of real-world manufacturing systems.

And we talked about the components of optimizations as the variables, the constraints and the objective function.

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### Simple Example

Familiarize with variables, constraints, and objective function.

A prospective college student from Kanpur is planning to visit the campuses of three colleges in India (Bombay, Delhi and Madras) on one extended trip starting from and returning to Kanpur.

The student wants to visit each college only once while making the round-trip as cheap as possible

The cost of traveling between cities are given below.

		City 1	City 2	City 3	City 4 (j)
		Kanpur	Delhi	Bombay	Madras
City 1 (i)	From	0	2600	3400	7800
City 2	To	2600	0	1800	5200
City 3		3400	1800	0	5100
City 4		7800	5200	5100	0

Kanpur - Delhi - Madras - Bombay - Kanpur (start to ending)  
 Madras - Delhi - Bombay

And we took a simple example and the simple example of moving from a college student moving from one city to another and in that process what we did is, we studied the variables constraints and objective function using these examples and how do we choose a variable.

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### Choosing Variable

The most important step in building a model is to decide variables that will facilitate modeling.

⇒ Modeling is an individual exercise (done by hand).

In this example, any round-trip consists of a series of smaller trips between two cities; it makes sense to assign variables to the decision of whether to travel from one city to another city.

(easy to learn modeling using discrete variables)

if there is a variable  $X_{ij}$  which implies student traveling to City 2 from City 1.

City 1 = Kanpur, City 2 = Delhi

if student travels then  $X_{12} = 1$

if student does not travel then  $X_{12} = 0$ .

Note that there is no such thing as trips to the city itself.

$\Rightarrow X_{ij}$

$\uparrow$   
1, 2, 3, 4  
 $\downarrow$   
1, 2, 3, 4

$X_{11} = 0$   
 $X_{22} = 0$   
 $X_{44} = 0$      $X_{33} = 0$

And in this case we choose a binary variable 0 1 and say no because it is the easiest one to understand the modeling aspects you will probably ask me the question.

Now, why you do not use a continuous variable the reason is the, I can always say that I can write it here now easy to learn modeling using discrete variables ok. We will try to work on a continuous variable example later down the road, but it is easy to learn teach this kind of a problem. So, that is why we use this one.

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$X_{ij} \in \{0, 1\}$      $X_{ij} = 0$  for every  $i=j$

The variable  $X_{ij}$  can take any value either 0 (or) 1.

Decision Variable:  $X_{ij} = 0 \Rightarrow$  not visiting city j from city i  $\rightarrow$  no.  
 $X_{ij} = 1 \Rightarrow$  visiting city j from city i  $\rightarrow$  yes

Variable choices can be an iterative process if the problem is complicated.

	1	2	3	4
(i)	Kanpur	Del	Bom	Mad
1	0	1	1	1
2	1	0	0	0
3	0	0	0	0
4	0	0	0	0

↑    ①    ①    ①    ①

From which city you are coming back to Kanpur.

- once the variable is chosen, then the attempt to formulate the problem is made.
- if formulation is difficult (cannot do it)  $\Rightarrow$  Start over and define the variable differently



Then the we talked about what are the different types of constraints and how you iteratively create constraints to reach what you actually want at the end of the day ok.

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**Modeling Constraints**

Now, we deal with the requirement that each city be visited only once.

(j=1)  $X_{12} + X_{22} + X_{32} + X_{42} = 1$  ( $1+0+0+0=1$ )

What we need for our model is a general mathematical expression that encodes this information.

(j=3)  $X_{13} + X_{23} + X_{33} + X_{43} = 1$

↓ Translated as:

$\sum_{i=1}^4 X_{ij} = 1$  for each fixed  $j = 1, 2, 3, 4$ .

(There is exactly one cell in the column of the city that has a non-zero value).

(1)

So, we went through different type of constraints.

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The true round trip can be enforced with a constraint

$X_{ij} + X_{ji} \leq 1$  for all  $i = 1, 2, 3, 4$  and  $j = 1, 2, 3, 4$ .

$X_{23} = 1$ ; then  $X_{32} = 0$ .

**HW.** Why the constraint (8) should not be written as  $X_{ij} + X_{ji} = 1$ ?

This prevents the occurrence of mini-cycles in the solution.

	1	2	3	4
1	K	0	1	0
2	D	*0	0	1
3	B	0	0	0
4	M	1	0	*0

(i)

$X_{12} + X_{21} \leq 1$

Kanpur → Delhi → Bantony → Madras

All the four constraints together results in a round trip with each city visited only once.

And finally, what we said is that the we finally, got a scenario the all the constraints together where the aim of it is all the constraints together or constraints together ourselves in a round trip with the city visited only once.

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### The Objective Function

The series of constraints defined earlier allowed round trips, so now we just need to describe the total cost associated with each round trip.

Since our constraints are in terms of  $X_{ij}$  that can take either "0" or "1" we should describe the total cost in such terms.

How?

→ We can simply multiply each  $X_{ij}$  by the corresponding cost ( $C_{ij}$ ) and then add it up.

	K	D	B	M
K	0	2600	3400	7800
D	2600	0	1800	5200
B	3400	1800	0	5100
M	7800	5200	5100	0

	K	D	B	M
1 K	0	1	0	0
2 D	0	0	1	0
3 B	0	0	0	1
4 M	1	0	0	0

$$\sum_{i=1}^4 \left( \sum_{j=1}^4 C_{ij} \cdot X_{ij} \right) = \text{Total Cost}$$

Minimize  $\left[ \sum_{i=1}^4 \left( \sum_{j=1}^4 C_{ij} \cdot X_{ij} \right) \right]$

↑  
 Minimum Cost Trip

So, then once we talked about how the constraints are created, then we discuss the words and objective function. And an objective function we had a cost one and the decision variable one and the decision variable and the cost gets multiplied together on some day across and since we wanted to minimize the total cost of the trip. So, we use the minimization function. And we saw why it is called as an objective function that aspects we discussed with this.

And then we went into the 4 rules of modeling, which the main aim is to ensure that the model that you are building is not going to be something that is very tough to solve and then the first rule was to.

So, we went to 4 different rules of this. And the 4 different rules came down to the rule number 1 being as keep it simple which means do simple calculus functions and whenever use calculus functions in that also you try to use linear functions whenever possible because linear functions are easy. And you can also if you use calculus functions, then it is very easy because you can use gradient function in that regard which is very fast in calculations.

So, then keep it lean means whenever you have duplicate constraints eliminate them remove such duplicate constraint so, that you are optimization model is lean. And keep it compact which means consolidate constraints. So, whenever you have additional constraints that you think you can write it in one single sentence then consolidate them

accordingly and then keep it continuous. So, whenever you possible instead of using discrete values use real values for the variables and once you use real values, then you can the major advantage is that you can use calculus work faster because the laws for the usage of calculus like techniques, which includes gradient methods and those kind of stuff.

So, I hope that by going through this whole process, this is again not an optimization class this is just a very quick capsule form of optimization hoping to make you guys understand different aspects of optimize optimization. For you to become an expert in optimization, there is no other way other than work on different optimization problems, take real life problems, model them accordingly and keep on modeling the. So, that you understand the language of modeling try to use these 4 rules of modeling the rule of terms of the modeling so, that you come up with really good models.

And once you have these models ready, then try to use various model solution techniques. If it is a linear programming, then you can just help you problem or a relative solver that comes with excel Microsoft excel solver and then for a non-linear are complicated ones there are other things like appeal semi optimization programming language studio which is a product of IBM stuff like that. So, there are multiple options available for you in that regard, but that is outside the realm of this course, the aim of this course is to introduce you to the basics of optimization.

Maybe in the due time process we will try to work one more problem so, that you understand how optimization happens. Maybe we will try to see a problem where different variables are considered at the same time and how do we optimize it as a small example and then from there we will move into other aspects, how some of the more techniques because as of now we only talked about modeling and rules of modeling and some other rules that we should think about optimization. We did not talk about model solving yet all. So, what are some of the model solving techniques available for some of the algorithms that are available to for people who want to do optimization that will be part of the coming lectures. In the meantime continue to read the science textbooks and the process hopefully, you will gain an interest in this and all these techniques that we learned in the class, you will end up using it in your real life to make a manufacturing system sustainable, which is a goal of this course.

Thank you very much.