Advanced Green Manufacturing Systems

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Lecture – 38
Numerical Analysis in Factorial Experiments - Part 2

Good morning students, welcome to the yet another lecture of Advanced Green Manufacturing System.

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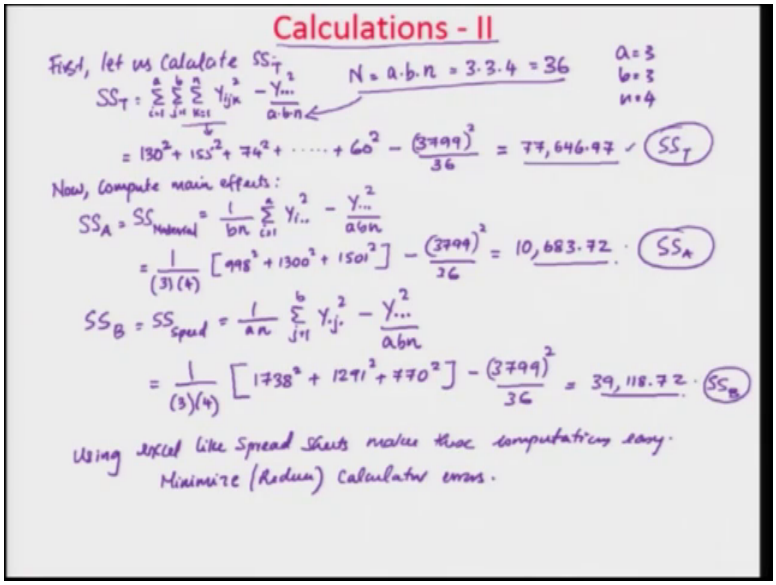

Now, what we need to do is, we need to do how the calculations are done in this specific case. So, now, what I am going to do is, I am going to show you how the calculations are done with these values for that particular thing ok.

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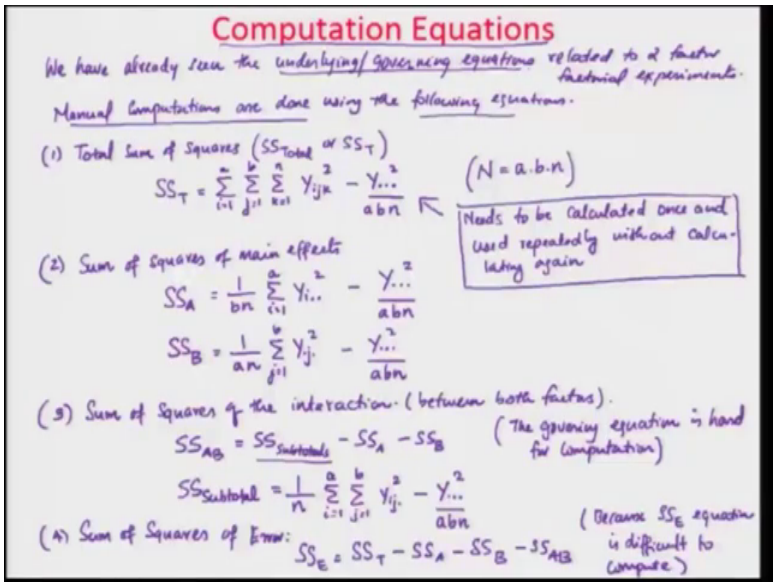

So, we have seen previously what are the values of SS T ok. So, first let us calculate SS T sum of squares of total ok. So, SS T as I said earlier is sigma i equal to 1 to a sigma j equal to 1 to b sigma k is equal to 1 to n ok, so three values, Y i j k square minus Y dot dot dot square divided by a times b times n right, this is what we were talking about. So, if you expand this ok, this is equal to, so let me use big N, big N is equal to a times b times n which is 3 times 3 times 4 so that is equal to 36 in this particular case ok. So, that is what we will be using as the denominator right here. So, this will be equal to when we compute this, it is 130 square.

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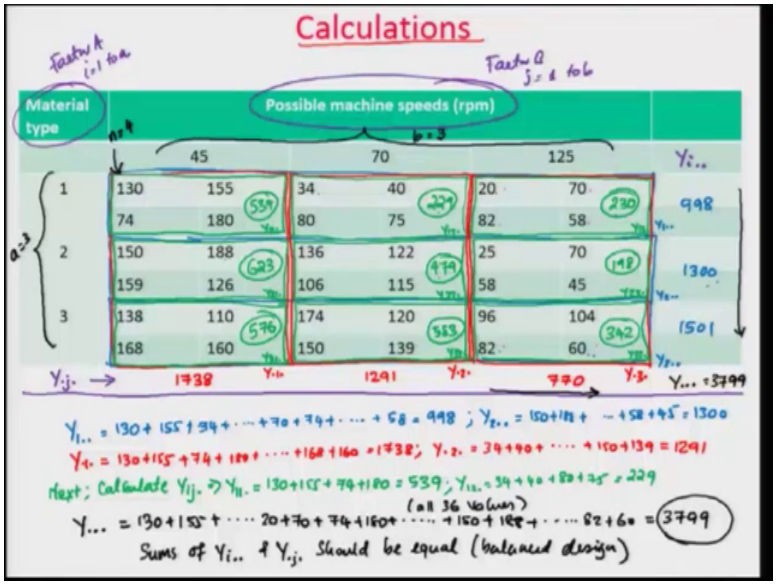

So, where are these values coming from? These individual values you square them right. So, those values will be 130 square plus 155 square plus ok, the third value will be 74 square 180 square, you square all those values right, so plus 74 square plus all the way up to the last value in this regard is 60 this particular value right so plus 60 square ok. You square all those values, this will be your, this whole thing translates to this term minus what is your Y dot dot dot, we know that we calculated it as 3799. So, this becomes 3799 square divided by the bottom value is 36, which is your N ok. So, you square all individual observations sum them up then minus this value.

And when you do this typically I use Microsoft Excel or somebody to set this up in such a way that it actually easy for us to compute these values comes to 77646.97 ok. So, this is the total of what you call as sum of squares of total that we compute in that particular area ok. Then now compute main effects ok. So, sum of squares of A or what we call as in this case A is material ok. So, you can also call it a sum of squares of material also that is also fine or you can call that as A and you sum of squares of A.

That equation remember 1 over b n multiplied then summation of i is equal to 1 to a Y i dot dot dot square minus Y dot dot dot square over a b n this was our equation. So, if you plug in the values, it will come to. So, the Y dot dots are 998, 1300, 1501 right. So, what we are doing here is 1 divided by then value of b is as we seen b is 3. So, a is equal to 3, b is equal to 3, n is equal to 4 ok. So, this will be 3 times 4 multiply it by what it is you have the values which is this 998, 1300, 1501. So, 998 square plus 1300 square plus 1501 square this is the first term minus, the same term 3799 whole square divided by 36 ok.

So, when you do this computation and calculate, you will actually get a value of 10683.72 ok. So, this is a sum of squares of A the first factor or the material ok. So, the second so we are done with A, for the thing is machine speed factor b. So, we can call it as SS of B sum of squares of B is equal to sum of squares of speed is equal to 1 over we call it as a n summation of j is equal to 1 to b, now in this case the summation is on j the factor j and we call it as Y dot j dot square minus Y dot dot dot square over a b n ok.

So, what we are going to do in this particular case is, we are going to use the same values a is 3, n is 4 and then it is a summation. So, where do we get the Y dot j dot. So, if you look into this is your Y dot j dot 1738, 1291, 770, these three values are used in this case.

So, then what we do is we put them 1738 square plus 1291 square plus 770 square ok, all these values are squared up minus 3799 whole square divided by 36. The same values are being used in this particular case and you get the values to be 39118.72 ok.

The most of the time is like remember this using excel like spreadsheet sheets make these computations easy ok. So, once you have an excel sheet, then doing all these manual calculations; otherwise you would require a lot of calculator time to get these values calculated, and you can make errors ok. And minimize errors also minimize or reduce calculator errors. So, once you have done with the main effects and the dots sum of squares of total.

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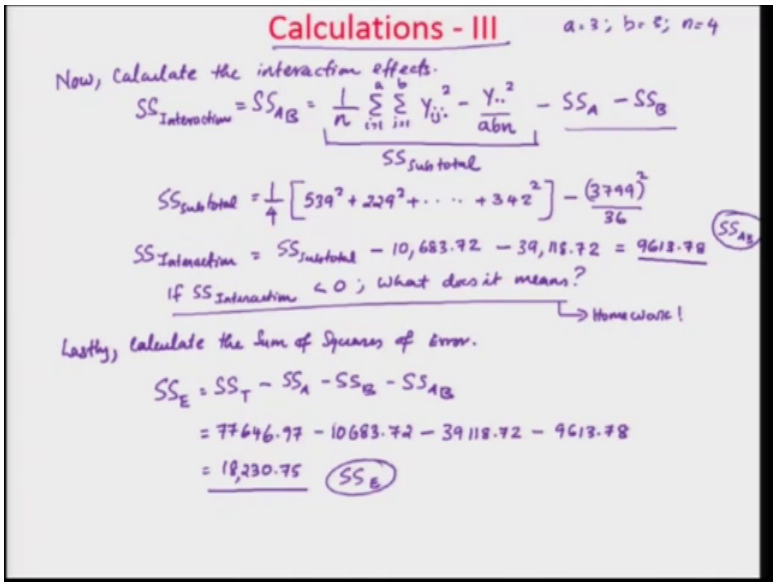

Now, the next thing for us to do the third part of the calculation we have to do is what we call as the sum of squares of interaction and error that those are the sum of squares we need to calculate now. So, now, what we are going to do is we are going to now do the sum of squares. Now, let us calculate, now calculate the interaction effects ok. So, interaction effects are calculated as I told you earlier SS of interaction or sum of squares of AB both of them is calculated by the equation 1 over n ok, n is the number of replicates remember; a was equal to be 3; b is equal to be 3; n is equal to be 4 that is how we calculated it 1 over n ok.

And then sigma i is equal to 1 to a; sigma j is equal to 1 to b both this 1, and what we had was Y i j dot square minus Y dot square over a b n, this was one of the factors minus sum

of squares of A minus sum of squares of B ok. So, this whole thing is your sum of squares of subtotal ok. So, what you do is you calculate the sum of squares of subtotal in this regard, and then from there you subtract out these two values. So, your S S subtotal is equal to now we calculate it this way.

It is basically 1 over 4 then what are the values that you are going to use? You are going to use these green circled values 539, 229, 230, 623, 479 that kind of values ok. So, we use the mass 539 square plus 229 square ok. So, you can see that 229 square 230 square all the way to 342 square right at the end of the day plus all the way to 342 square ok. This is the first, first part of it then minus Y dot dot square is same thing we already calculated 3799 square divided by 36 ok.

And this value ok, we get it one particular value right. From there what happens is so this is how when you compute this is your subtotal is from this what you do is SS interaction is this particular value SS subtotal minus what we used to do got here previously was the a 4 was 10682 that was 39118. So, what we do is we do it as 10683.72 that is your SS A minus and 39118.72. And when you do this calculation, we get a value close to 9613.78. So, if SS interaction is less than 0, what does it means? This is a question that you need to think about ok.

If this value comes to be negative ok, if this value is non-positive value what is it something important or is it something that you made a calculation mistake, you need to find that this is something called as a homework, you need to find out.

So, now we calculate the SS interaction then the last one is lastly calculate the sum of squares of error ok. So, how do you calculate SS E in this regard. So, remember SS E is calculated as SS T total minus SS A minus SS B minus SS A B ok. So, you are now subtracting all the other error things ok. So, where did you calculate your sum of squares of total, this is your total; from here you subtract this value, this value and the last value.

So, that is equal to if you write that equation, it will come to 77646.97 from that you remove the SS of A is 10683, so that is minus 10683.72. From that again subtract 39118.72, so that is 39118.72 ok, then you subtract this last term that you had which is 9613.78 ok. You did do go through all these subtractions, then what happens is you get the error the remaining error of it which is 18230.75. So, there might be some round of values that you might actually see I just rounded off to two digits and some of this might

also excel might have done some rounding off also. So, now we have what we call as the entire values.

So, if we go back do you have your totals here, this is your SS T or the total value you have your SS of A, you have your SS of B ok, and then you have your SS of AB and you have what you call as your SS of error ok. With all these values put together what we can do is now with these calculations are being done, we can create what we call as our ANOVA table ok.

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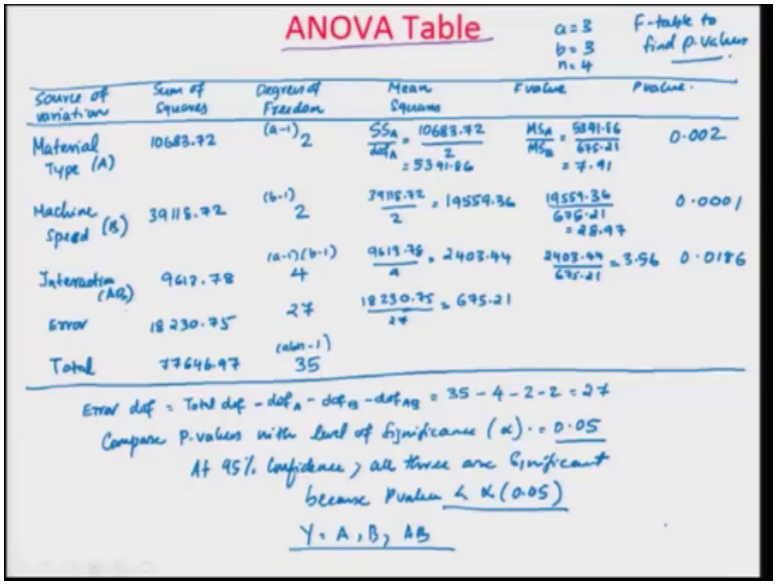

So, the next step is to create what we call as the analysis of variance table right. So, the analysis of variance table or the ANOVA table as we say seen earlier is built in a particular fashion. So, remember for ANOVA table. So, the table has a very different heading in this regard. We have source of variation ok, then we have what we call as sum of squares remember the columns, then we have the degrees of freedom ok, then we have the mean square ok, F value and P value ok. So, and then the source of variation was factor A. So, in this case what we know is that, if we go back to the table, the factors are material type and machine speeds.

So, we go back and we say it as material type ok. This is your A, then the machine speed this is your B right, then interaction this is AB right. And then what we have is error right, and then we have is last one is total. So, this is our ANOVA table.

And let me write this interaction as AB right here. So, I do not want to get into the sum of squares things. So, the material type A the sum of squares, where do we get those values these values were all calculated. So, we just now plug in these values into the table. So, the sum of squares in this case was 10683.72, for this particular machine speed, it was 39118.72 and interaction we calculated it as if you think about it was 9613, so we put that value 9613.78. And then the total value was put in if you remember it was put in as 77646 ok. So, the total is 77646.97.

And the last value we calculated was error, which was 18000 ok. So, that value goes here 18230.75. So, all the sum of squares goes into this particular column and that the degrees of freedom that need to go here. So, this is like degrees of freedom is a minus 1 and we know a, the value of a is equal to 3, b is equal to 3, n is equal to 4. So, a minus 1 is 3 minus 1, so that value is 2. The degrees of freedom here is b minus 1, which is again 2 right, this value is what is error the interaction degrees of freedom it is a b, minus a minus 1 times b minus 1. So, this value is a minus 1 times b minus 1 as 2 times to that value comes to be 4.

Then what is the total error degrees of freedom this is a b n minus 1, so that is 36 minus 1, so this will be 35. So, how do I calculate the error degrees of freedom? So, error degrees of freedom is equal to error dof is equal to total dof minus dof of A d a minus dof of B minus dof of AB, which is equal to 35 minus 4 minus 2 minus 2 ok. So, if you do that then you get the value to be 27, so that degrees of freedom is 27 right here. There is an equation also A B and minus 1 which you also you can use if you want to, but I would rather do it this particular fashion right.

So, once we have the degrees of freedom, then the next one we need to calculate is what is called as the mean squares. So, mean squares is sum of SS A by dof of A ok, which is equal to what we write here is 10683.72 divided by 2 ok. So, that value gives us what we call as in this particular case it is equal to 5341.86 ok. Second one value that we calculate same way is 39118.72 divided by 2 and that value get it to be 19559.36. This one will be 9613.78 divided by 4 and that value comes out to be 2403.44 ok. And this value comes; the last value comes to be 18230.0.75 divided by 27. And I get that value to be 675.21 right.

Now, the F value in this particular case is calculated by the way to calculate this is the equation here is MS A divided by MS E the error ok. So, what we are doing is 5341.86 divided by 675.21 this value in our case. So, this is, this mean squares divided by the error mean square, you get the value to be 7.91 that is the F value. In this case, it is calculated as 19559.36 divided by 675.21, and that value is calculated to be 28.97. The third one that we calculate is 2403.44 divided by 675.21, which will be equal to 3.56 ok.

So, the three values for the A, B, AB we calculate the mean squares of errors ok. Then the P values are calculated from the F chi square distribution. So, we use the F table, F table to find P values ok. And otherwise you can also use it excel to find the P values. I will give you the P values in this regard it is 0.002. In this case, the P value is equal to 0.0001 and here it is 0.0186 ok. So, this P value is the same as that of ok. So, what we say is that compare P values with level of significance which is alpha ok. So, the level of significance alpha is what we need to compare. And if you say the alpha is 0.05, then all these factors are significantly important for us ok, so that is the way then you compute.

So, in this case we can say that at 95 percent confidence, all three, are significant, because P value is less than alpha, which is 0.05 ok. So, when we say we need to look into all these three. So, our model will have at 95 percent interval our model y will have the A, the B, and AB combined the effects of A, B and AB will be used at 95 percent confidence.

So, I hope that this actually helps you guys to understand how the calculations are made and how they ANOVA is created, and how the decisions are made based on this. So, now what we will do is we will now just quickly look into some of the optimization algorithms quickly, and then finally, we will conclude the lecture towards the end as a group discussion of how do we optimize things.

Thank you very much for your patience listening.