Advanced Green Manufacturing Systems

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Lecture – 37
Numerical Analysis in Factorial Experiments - Part 1

Good morning students. Welcome to the yet another lecture of Advanced Green Manufacturing System. And we are today going to talk about the Factorial Experiments, continuing on the previous design of experiments and factorial experiments, and how it is to be used in the course specifically. And we have seen the experimental setup, and how why the factorial design is an important design and all those aspects, but we are not done an example.

And what we are going to do today is do an example, work out a problem which will give you how the problem when you get the numbers, when you are actually go collect the data, get the data in a particular format, then how will you solve the problem. And once you solve the problem, how will you identify what is to be done, and how would you interpret that results that is what we will be covering in today's topic.

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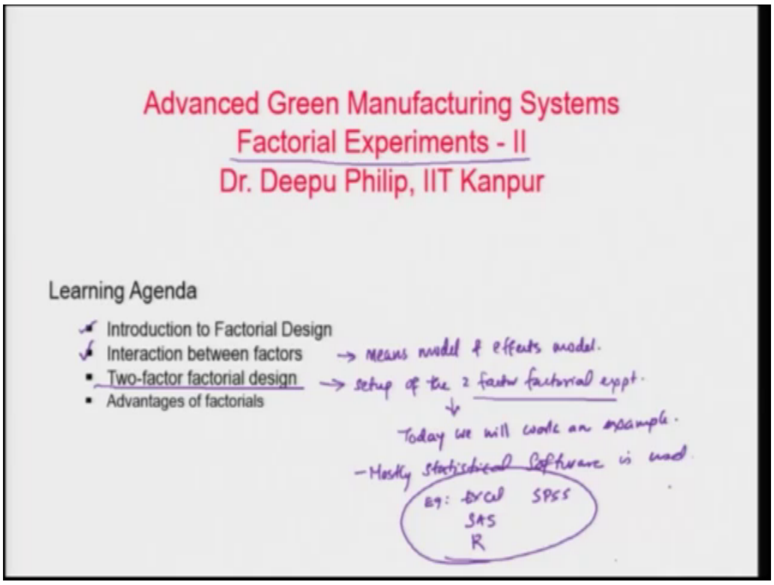

So, if you look at the today's agenda, it is factorial experiments part 2. And you have already gone through the introduction to factorial design, we have seen why what is the factorial, and why is it important thing. We have seen the interaction between factors, why the interaction between factors are important, and we also saw the means and the effects model means model and effects model we have seen that already ok. And then we also talked about what is the two-factorial design, and why is the two-factorial design is important. So, we seen the setup of the two-factorial experiment; 2 factor factorial experiment ok, we already seen that.

And we now going to work on, today we will cover. Today, we will work on example ok. And then mostly statistical software is used ok. It can be examples will be excel ok, some people use SAS, some people use R, some people will use SPSS, those kind of things ok. So, any one of them can be used to study a, to do a two factor experiment and analyze the data. But, today we will do the manual work, so that you can see how it actually works.

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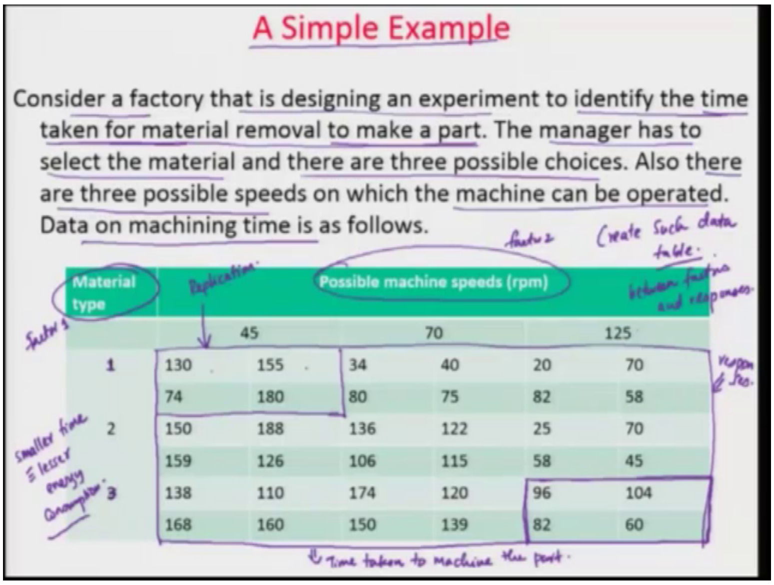

So, the examples that we mentioned earlier in the class ok and this is important that I will refresh the example once again to you ok. A factory is designing an experiment, we have a factory consider a factory that is designing an experiment ok. And what are we trying to do is you want to identify the time taken for material removal to make a part. So, how much time it takes to remove the material to make a specific part is what we are actually going to study.

And the manager has to select, so what the thing is manager has couple of things to do. The first part is to select the material ok, what is the material, and there are three possible choices. So, in this case, you can see the material type. The first part, there are three possible choices; material 1, 2 and 3 right. And there are three more other choices, there are additional three more possible speeds ok, possible machine speeds, this is the second choice ok, on which the machine can be operated. So, these are the 45, 70 and 125 are the RPMs that can be operate that the on which the machine can be operated.

So, what we are collecting here, the data for all of these data, what we are talking about ok, this one implies this is the time taken to machine the part to machine the part. So, by simple way we can say that smaller time equals to lesser energy consumption or equivalent to lesser energy consumption ok. So, you might be interested in having a smaller machining time ideally in this regard, so we will talk about that later.

So, typically in an experimental system, when you are collecting the machining time, this is the way you collect the data, I have mentioned earlier also. So, what does this happen series for the material type 1, and for the speed of 45, there are four values. Four values means, you made these are these four are what we can call it as the replication.

So, you made four different test pieces of material 1, and used it in the machine at 45 RPM, and measured the machining time ok. Then similarly if you think about it in this case, this particular box, you can see that this is another replication ok, where you used material 3 ok, material 3 was used, and they wrote RPM is 25 125, and at that you tested four pieces, under quarter the machining time. So, this kind of a data table is you create.

So, first thing is create such data table right. So, you have creating such data table, to where you capture the data. And so data table where between factors and responses ok. So, these are the factors, so this is your factor 1, and this is your factor 2, and these are your responses ok. So, your the responses are as part of this table fine. So, with these responses, now the next question is how do we get to the analysis or how do we compute the values.

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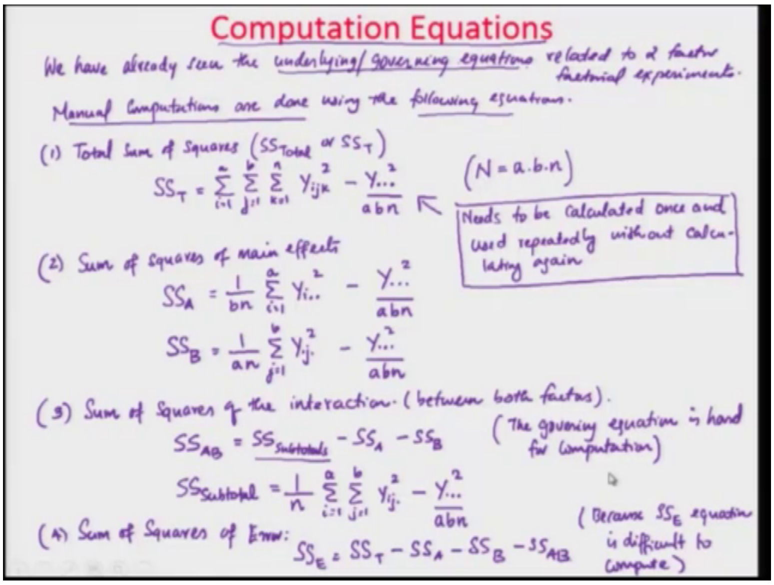

So, the first thing that we need to look into the computation equations so we have already seen, we have already seen the underlying or the governing equations related to 2 factor factorial experiments. We know what the underlying equations or the governing equations are. But, manual computations manual computations are done using the following equations.

So, what we are trying to do is this manual computations the when you want to do it on using a pen and paper, we use this particular equations that is we are going to see, what are those equations. So, the first equation, the part number 1 is the total sum of squares total sum of squares ok, it is also known as it is written by sum of squares of total or SS T ok. This is a usual notation that we follow. And to calculate the total sum of squares, we use this equation SS T is equal to ok, it is a summation it is a three step summation sigma i is equal to 1 to a ok.

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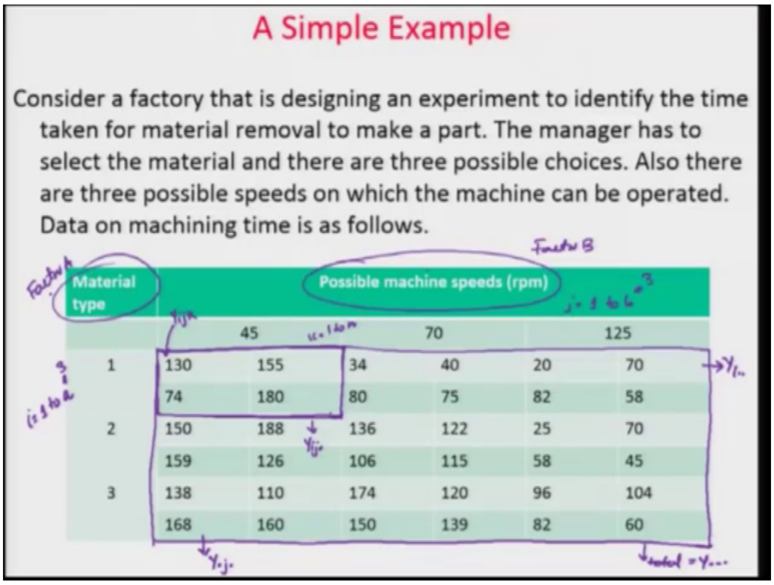

So, i if you think about is this is your factor A factor A, and your i is indexing from in this case is 1 to a, and a is 3 here ok, so you have three levels for this. Then sigma i j is equal to 1 to b that is a second set of factors, which will be this particular case factor B ok. And the index is from j is equal to 1 to b, and b in this case is also 3 right, so that is j is equal to 1 to b, and sigma k is equal to 1 to n, k is the replications right.

So, if you think about it, we have the case right here, four replication for each level ok, k is equal to 1 to n ok. So, these three things if you do that, then we say Y i j k square, you square each individual observation, so the Y i j k is this is your Y i j k ok. And then from that you subtract, what we call as Y dot dot dot square divided by a times b times n ok. So, the Y dot dot is you would take all of these values all the 36 values as part of this ok, total sum is Y dot dot dot ok.

We sum all of these values that is your Y dot dot ok. So, then another factor in this case is the a b n, sometimes people also denote this with capital N, which is N is the a times, b times n ok. So, this is the total number of observations that we talk about in this regard right. The so this is how you calculate the total sum of squares.

Then number 2 ok. So, next one, what we calculate is the sum of squares of main effects squares of main effects ok, this is a second value that we need to calculate. So, in this case, there are two factors; the first one is factor A, and second one is factor B. So, this is factor A, this is factor B. So, we want to calculate their main effects, so that is calculated by SS of A sum of squares of factor A is equal to 1 over b n b for the factor the j the column, and n is for the replicates, sigma i is equal to 1 to a Y i dot dot square minus Y dot dot dot square over a times b times n.

So, this value you can see that needs to be calculated once and used repeatedly without calculating again ok. So, this once you calculate, you keep on using it in all other equations pretty much. So, this Y i dot dot is the column thing sorry the row thing, you have your Y i dot dot, so which means for each row, you sum all the other all the values that are available for all the columns ok. And that is the sum of squares of A.

And now sum of squares of B, the second factor is 1 over a n, a is a number of levels for the factor a, a n sigma i is equal to 1 to b ok. And we say Y dot j dot square ok, so that is you are indexing on sorry the index is on j minus Y dot dot dot square over a b n. So, this Y dot j dot, before this index is pretty much if you look into this, this is your Y dot j dot ok. So, for each column for each individual values of i u sum the columns ok, the i and n are indices indexed. So, these two will tell you the sum of squares of the main effects.

Then the 3rd one that we are more interested in doing or also is important for us is the sum of squares of the interaction; sum of squares of the interaction, interaction is the interaction between a and b between both factors ok. So, with that what we now need to think about is how do we do, how do we calculate that? We have already seen the equation earlier, but you can think about thus sum of squares of AB ok, which means the interaction is sum of squares of subtotals minus sum of squares of A minus sum of squares of B ok.

So, what we are saying here is that if we can calculate the subtotals in this regard, then from there we already calculated SS of A, and SS of B, you subtract them out of it. You should be able to calculate the sum of squares of AB ok. The governing equation is hard to compute, equation is hard for computation. So, we do the sub-parts of it. So, the SS sum of squares of total SS of subtotal ok, the subtotal is given by the equation 1 over n ok.

Now, A and B are gone ok, the subtotal is 1 over n sigma i is equal to 1 to a summation of j is equal to 1 to b ok, what you are doing is Y i j dot square ok. So, you are doing this new term Y i j dot, we have already seen that what is this in the previous case, and Y dot dot dot square over a, b, and n right. So, this right term, right hand side remains the same, but this particular term Y i j dot square. So, what we are talking about this, this is the Y i j dot ok, this whole thing for this entire four this is the Y i j dot fine, so then this is done. So, now you have the sum of squares of A B can also. Once you calculate this using this, then use the subtraction you will be able to find out.

Then the last one we need to do is sum of squares of error ok. And again error equation is also obtained through you know computation, so SS E is equal to SS T sum of squares of totals minus sum of squares of A minus sum of squares of B minus sum of squares of AB ok. So, from total if you subtract the effects of A, the B, and AB together that, left out is the sum of squares of error. So, this is also done, because SS E equation is difficult is difficult to compute manually ok. So, what we do is we use this subtraction method to do the same right ok.

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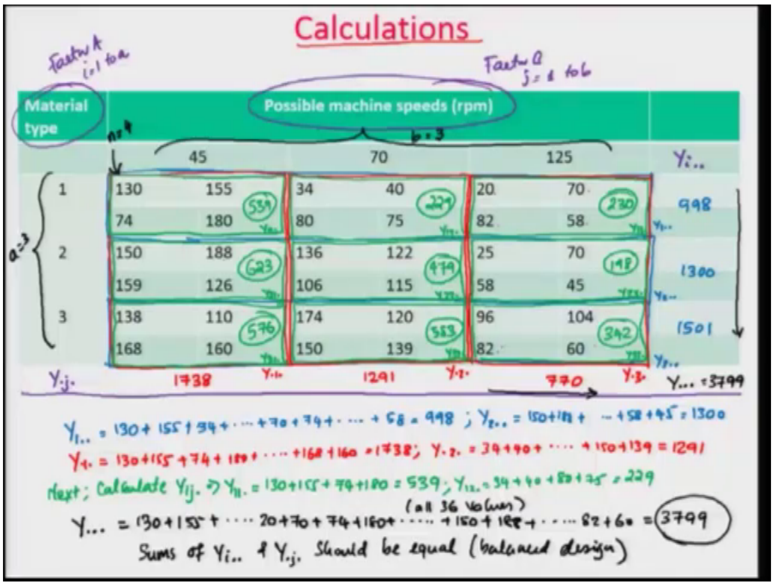

So, we will move to, what we call as the calculations aspect of it. How do we do the calculations? So, the same example is given to you guys the calculations, and how do we do this. So, the material type is your factor A, factor A varies from i is equal to 1 to little a. This is your factor B j is equal to 1 to little b ok, and then there is the n in between right. So, the first thing is our calculations in this regard is, so the this side the column side ok, what we are the raw side, actually not the column side, what we are going to do is this is your Y i dot dot ok, this is what is going on there in this regard.

Then for you the next thing that you need also need to calculate in this particular case is what we call as the, we will add one more column, below this or one more row below this, we call it as Y dot j dot ok. So, the Y dot j dot will be for this particular thing, so we will see how these values are calculated ok. So, for the value of one in our case, this much ideally speaking will give you your ok, first value of Y i dot dot ok. So, this Y i dot dot is given by if you think about is, so Y 1 dot dot, this is for the factor 1 is equal to 130 plus 155 plus no 34 plus all the way to plus 70 plus 74 plus all the way to 58 ok. This whole thing, you sum it up and you get the value of this to be 998 ok. So, this is the first set of value that you calculate.

Then the second one you calculate similarly is Y 2 dot dot. So, the Y 2 dot dot will be these many values ok. So, this is your Y 1 dot dot Y 2 dot dot will be calculated will be same thing as 150 plus 188 plus 136 plus all the way to 70 plus 159 plus 126, you sum all of this, and you get it as 1300 is 150 plus 188 plus all the way to 58 plus 45 that is 1300. Same way the last one for the third value that we are going to compute is this one or we call as Y 3 dot dot for the third level, we sum all those values up, and same way 138 plus 110 plus 174 plus 104 plus 168 plus 160 plus 82 plus 60 this much ok. And that value will give you, what we call as what is it is 1501 ok. You can see those computations individually, when you look into this alright, so that solves the first part where we can actually see, what is this Y i dot part of it right.

Then the next one is what we are now going to do is we are now going to calculate compute, what we call as the Y j dot Y dot j dot. And how do we calculate the Y dot j dot, so this portion for this particular factor will give you the Y dot 1 dot. So, this is the first level of this factor which is 1738 ok. So, the how is that calculated, so Y dot 1 dot is equal to 130 plus 155 plus 74 plus 180 plus all the way to 168 plus 160 that value is 1738, so that is how you calculate the first set of the column sums ok, this is the first set.

And the next one what we are going to talk about is this particular case ok, the second one which we call as the Y dot 2 dot. So, for this case the Y dot 2 dot is equal to this for the second value or for the 70 value is how do you calculate that that is calculated by 34 plus 40 plus all the way up to 150 plus 139 right. And that sum if you do, it is comes to 1291 ok. So, 1291 is the next sum are coming out of it.

Then comes the third one, third value that we need to calculate, which is the Y dot 3 dot which is given by this particular set that will be Y dot 3 dot, and that will be 20 plus 70 plus 82 plus 58 plus all the way to 82 plus 60. If you sum all this one is down, you will get it as 770 ok. So, this 770 is your all the Y dot values right fine, so this sort gives you both the rows and the columns sums.

The third one that we need to calculate is for the subtotals, so now the thing is we need to calculate is the next, calculate Y i j dot ok, so Y i j dot is for each replications. So, the Y i j dot is calculated in this particular case as the first one is there will be many values ok, which is Y 1 1 dot ok, so this is your 1 1 dot ok. This value Y 1 1 dot that value is 130 plus 150 plus 74 plus one 180, so that is equal to 130 plus 155 plus 74 plus 180, and that gives you 539 ok. So, this 539 is this particular value in this case right.

Then the next value, we can calculate out of this is you can take it as the Y, so when you move to this one, it is 12 dot which is these values, where we calculate these ok. For first one first so is Y 12 dot 34 plus 40 34 plus 40 plus 80 plus 75 ok, you get that values as calculated to be 229. So, this is your second value.

Then the third one, then we are going to calculate here is this ok, this is Y 13 dot ok. And that value will come to 230 in this case ok, you get this is the sum of these four values. Then comes the next column or the next cell, we call this as Y 21 dot the next value, and that is the sum of these columns all put together gives you 623 ok. Then calculates this one which is the Y 22 dot ok; the sum of these four values gives you 479 ok.

Then the third which is of the second row, again Y 23 dot sum of these four values will give you 198 ok. Then the last what you go is this cell ok, which is a third row first cell Y 31 dot, this gives you the sum of these four values as 576 ok, then comes this cell you calculate them Y 32 dot you sum of all these 583 you get this, then the last one is Y 33 dot. For this cell which is called as sorry my bad Y 33 dot, and we get these values to be sum to us 342 ok.

So, now we know how to calculate this many values ok. So, now once we have calculated all those kind of things, then the next major thing that we need to calculate is something called as so now the one thing that we now need to do is Y dot dot dot is what we need to calculate. So, Y dot dot dot is equal to 130 plus 155 plus all the way to 20 plus 70 plus 74 plus 180 plus all the way to what we call as 150 plus 188 plus all the way to last towards 82 plus 60 ok, so you sum all these 36 value ok. So, we can say it as all 36 values are summed at together, and that Y dot dot comes to 379 3799 ok.

So, one other way to think about it is if you sum these guys up ok, and also sum these guys also up, the value should tally right here that is one of the reasons, why it is called the sums of Y i dot dot dot and sorry Y i only two dots, and Y dot j dot should be equal balanced design ok. So, these sums should sum to both of them should sum to 3799 in this particular case. So, the material type, we had the a is equal to 3, and here you had what you call as b is equal to 3 ok. And using this and here we had little n n is equal to 4 ok, so this is what the values that we have ended up using in this particular case ok.

Thank you very much for your patient listening.