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**Lecture – 28**
**ANOVA – Part 3**

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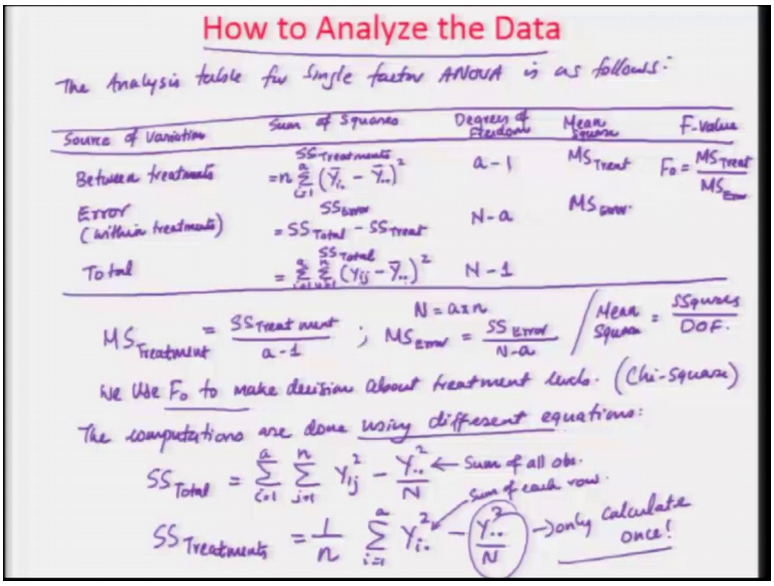


Good afternoon students. Welcome to the course of Advanced Green Manufacturing Systems. We were just going through how the equations can be used and we are now going to see how these things are getting done in practical, but remember that we are interested in going to use these 2 equations for calculating the total values and fill this table ok.

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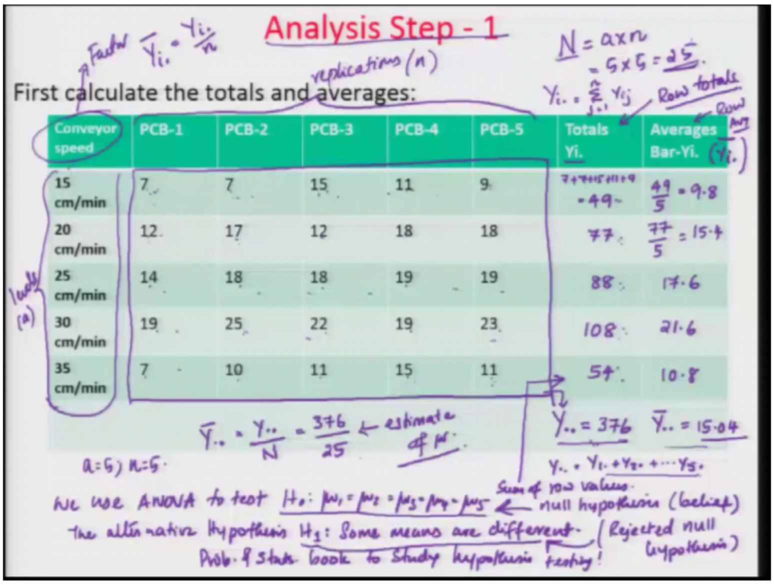


So, this was the data that we looked into it looked into the system. This is your factor as we mentioned earlier ok. These are our levels a levels, these are our replications n right. So, in this case we have what we have what is called as a is equal to 5 little n is equal to 5 right.

So, you can calculate big N ask a times little N which is 5 times 5 25. So, the big N here is 25 in this figure I told earlier is we are supposed to calculate something called Yi dot; Yi dot is the. So, Yi dot is calculated as i dot is equal to sigma i j is equal to 1 to N Yi j. So, for each value of i for each value 1. So, in this case it is this is given by 7 plus 7 plus 15 plus 11 plus 9 ok. That will give you a value of 49 right. Then this the second value you can do 12 plus 17 plus 12 plus 18 plus 18 right.

So, that will give you that is 12 plus this is 40 77 right. So, you get the value as 77 in this regard then the this Yi dot is 14 plus 18 plus 18 plus 19 plus 19 ok. That should give you 88 ok. And then you have the third one fourth one as one 19 plus 25 plus 22 plus 19 plus 23 should give you 108 ok. The last one is 7 plus 10 plus 11 plus 15 plus 11 should give you 54 right. So, these are the individual sums. As I said earlier again these are the sum of the rows ok.

So, remember this is the sum of row values ok. Each individual row values are summed ok. Then I can do something called as Y dot dot; Y dot dot. And the Y dot dot is equal to Y 1 dot plus Y 2 dot plus all the way to Y 5 dot in this case. So, I sum this out which is

49 plus 77 plus 88 plus 1 naught 8 plus 54 that should give me 376 ok. So, this is the total sum ok. This is equivalent to summing all these values sum should give you this one right.

Now, what the next one we are going to do here is Yi bar I wrote it as bar, so, Yi dot bar ok. So, that is the average of these. So, this is calculated as 49 divided by 5. So, there is 1 2 3 4 5 5 observations right. So, the Yi dot bar is equal to Yi dot divided by n ok. So, that is the way you do you divided by the n, n is 5 here right this will give you the value of 9.8 ok. This is 77 by 5 ok, which is 15 point 4 in this regard right. The third number is what we calculate as 17.6 which is 88 divided by 5 then 100 and 8 divided by 5 is 21.6 and 54 divided by 5 is 10.8 ok.

So, these are your averages right then I can do something called as Y dot dot bar ok. So, I can find the entire averages of it which is Y dot dot bar is equal to Y dot dot divided by N I can do that. So, this is in this case it is 376 divided by 25 ok. I can get it the value it comes close to 15.04 right. So, this is an estimate of mu an average scanned of an estimate of an average ok. So, going through once again quickly. First we do the totals these totals are the row totals ok. And what you do is you some each individual values of the row sum them together that will give you the row totals you do the 5 row totals. Then you do the average this is the row averages.

And what you do is you take the sum total divided by the number of observations in the how many replications are there you get this value. This is your total grand total this is given by summing up of all the values. So, I told you how do we calculate Y dot dot. You can do this one or you can calculate all the values you get it to the 376. And this is Y dot bar then you calculate this how do we do that here is equation that I told you ok. So, this is the first step of analysis step one right.

Then the thing is we use ANOVA analysis of variance to test H 0 as mu 1 equal to mu 2 equal to mu 3 equal to mu 4 equal to mu 5 ok. We are initiate testing the that. So, this is your what you called as null hypothesis ok. You believe you believing that all the means the treatment levels of no belief on the no impact on the soldering defects ok. And the alternative hypothesis the alternative hypothesis is H 1 what are we going to do there alternative hypothesis some means are different.

So, we are saying that not all of them are the same. Some of them have a different impact on that. So, if you reject the null hypothesis, so, if rejected null hypothesis will allow us to believe this ok. So, it is like the same way as please take a textbook basic probability statistics book to study hypothesis testing. But in this case the way I am assuming that you guys have already gone through it, but they always the null hypothesis the belief is that the treatment levels of no impact on the means and if that belief is not right. Then we believe that some of the means are different that is alternate that we are going to take and we have already seen how this numbers are actually calculated right.

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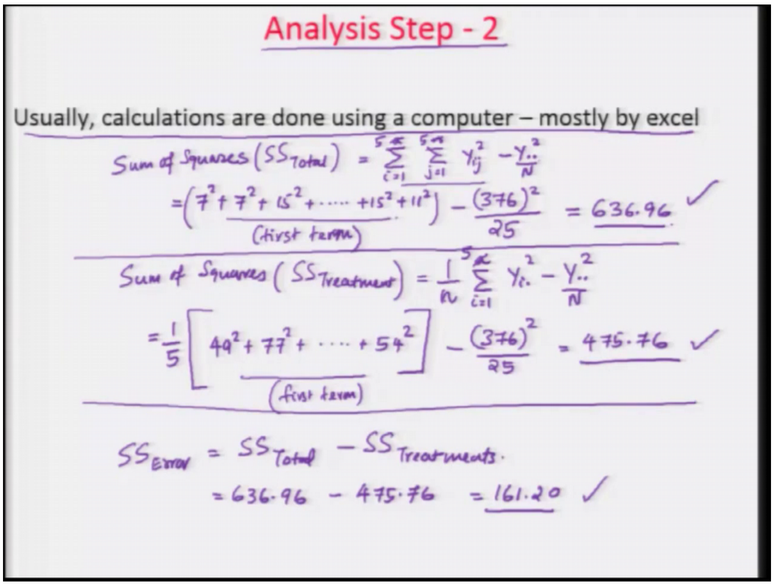


Now, second step of the analysis is ok, analysis step 2, we do the computations, but remember most of this kind of time you have large number of daytime in when you are doing in a factory you have many number of a machines and lot of experiments a lot of data that you are collected. So, doing this manually is not really an easy thing, but this is a small example problem. So, I can do this kind of things manually to show you how the problem computation works.

But in real life you would use a software program like sometime say SPSS or (Refer Time: 09:08) or something like that or things like excel to do this, but here we are actually doing this by hand just to show you the mechanics of the system alright. So, first thing we need to do is we need to calculate the sum of squares SS total. So, let us first calculate sum of squares of total. Remember in the previous equation that I told you we calculate the SS total here. And here is a example of the SS total then you calculate SS treatment from there you calculate everything else right.

So, going back to this let us calculate the SS total, this SS total is given by the equation sigma i is equal to 1 to a sigma j is equal to 1 to n little n ok. So, in this case the a is this is 5, this is also 5 ok, a and n are 5 5 each ok. Yi j square minus Y dot dot square divided by N this is the equation that we actually we do not need a parenthesis I am sorry ok. So, this you square the sums and then you from this you subtract it. Which if you elaborate it is written as if we go back these are the values. So, 7 square plus 7 square plus 15 square plus 11 square plus 9 square plus 12 square plus all the way up to this last 11 square ok.

So, we do that here 7 square plus 7 square plus 15 square plus all the way to 15 square plus 11 square ok. So, the first term this part of the term is this ok. It is this one this is the first term t r m term ok. How did we calculate this is 7 plus 7 plus 15 plus all these values we square them square them square; them square; them and this is the last 15 and 11 and then you sum them up right. Minus this is the first part of the process this is the first part first term minus Y dot dot;Y dot dot you calculate it as 376 is right here 376 ok.

So, we say it as 376 square divided by n, n was calculated again previously here as little n was calculated as 25 not little N big N was calculated as 25. So, you divided by 25 ok. So, again the first term is you square each individual values; each individual values are squared and summed it together right. And that gives you the first term and then the second part is the 3 this total sum square divided by 25 and when you do this number I have calculated this already.

So, I will give you the number 636.96 right this is the value you will get right. So, please remember this how this calculation was done right. Then the next one we do is sum of squares SS treatment, sum of squares of treatment is calculated by the equation. So, we already told you what the equation is previous case 1 over and divided by multiplied by this equation right.

So, we are going to do that using here. 1 over N multiplied by sigma i equal to 1 to a i is equal to 1 to a Yi dot square minus Y dot dot square over N right. So, this is the equation that we did here. Which we are exactly going to do here value of a is 5, so, we will be doing this 5 times ok. So, which is equal to 1 over 5 little N is 5 1 over 5 multiplied by the Yi dot square is the Yi dots are exactly here. So, it is 49 square plus 77 square plus 88 square plus 1 naught 8 square plus 54 square right.

So, what we are going to do here is it is 49 square plus the next one is 77 square plus all the way to 54 square ok. This is the first term ok. Where you are having each one of the sums you square them ok. And that and square them and sum them up. So, this is what you how was your first term right ok. Then minus your Y dot dot Y dot dot as I said earlier is 376 right here sum total of is that is 376 whole square divided by big N is 25 ok. So, you have the both the terms and I get this as 475.76 right. So, this is second number I have already calculated.

So, we have calculate the sum of squares of total and sum of square squares of treatments we are done both right. Then how do you calculate sum squares of errors? Is calculated by SS total minus SS treatments. So, where did I get this. I got this in the matrix remember this the error sum of squares were calculated as sum of squares of total divided minus sum of squares of treatment.

So, using that what I am going to do it is equal to 636.97 this is what I calculated right there minus 475.76. This is the sum of squares of treatments both of them put together I get it as 161.20 ok. So, I calculated my sum of squares of total sum of squares of treatment and there I calculate the sum of squares of error. So, what I have done is, I have calculated all the sum of squares from the for, so, that I can start filling the table in this regard right.

So these calculations as I told you earlier please and remember that these calculations are always done with the help of the computer software.

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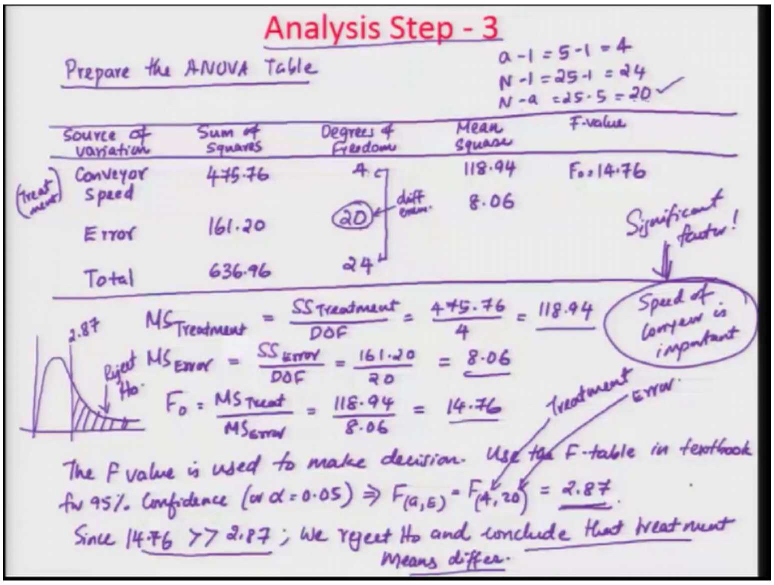


Now, we get into the last step of analysis the third step of analysis what the first thing we need to do is we prepare the ANOVA table ok. So, you prepare the ANOVA table. Where did we see the ANOVA table. This was your ANOVA table right this is what we call as our ANOVA table. So, let us go back and try to prepare the ANOVA table.

So, I just draw this ok. So, first thing is we have source of variation ok. And we have was the source of variation it was remember this was between treatments. So, the main source of variation for us is the conveyor speed this is your treatment that is your source variation. Then the next one is you had is error and so, if you think about it. It is treatments error and total right. So, we go with the same fashion total right.

This is our setup and the second one is sum of squares ok. So, where did we get the sum of squares? You have already calculated in 636.97 475.76 161.20. So, we go back and we fill those values ok. So, if the first one is a totally 636.97. So, 636.97, I always write it in this fashion. So, that I do not make any mistakes. Then the treatment is 475.76. So, it is 475.76 this is your treatment by the way please remember that.

And that the error is calculated as the difference of the 161.20. So, it is 161.20 right we write the sum of squares and this regard then we calculate what we call as the degrees of freedom ok. So, the degrees of freedom as I said earlier in the ANOVA table ok. Degrees of freedom is a minus 1 N minus a N minus 1 right. So, let us do a minus 1. So, the value of the degrees of freedom here is it is a minus 1 is 5 minus 1 equals 4 this value will be 4 ok.

This is N minus 1 right. So, if you remember this; this is run as N minus 1 as the degrees of freedom. So, N minus 1 is equal to 25 minus 1 is 24 right. And this degrees of freedom error degrees of freedom was written as N minus a right. So, if you think about it thus N minus a that is 25 minus 5 equals 20 ok, this value is 20. One other way to check whether you got the degrees of freedom correctly is the if you look at these ok. These two difference should be this difference remember that.

That is another way to think about you calculate this if you take the difference of the total and that the treatment degrees of freedom then the error should also come to that and you can independently calculate by this particular equation as well right. Then the next one we calculate is what we called as the mean squares ok. So, the mean square is calculated in this case is mean square treatment ok. Is given by sum of squares of treatment divided by degrees of freedom which is equal to 475.76 divided by 4 ok. When you do that you actually get a value of 118.94 ok.

So, the value goes here 118.94 right. Then the next mean square is the mean square of error, is given by sum of squares of error divided by degrees of freedom which is 161.20 divided by 20, which should give you 8.06 this value goes right here right. So, this is your mean squares. I told you again the mean square is the sum of squares divided by the degrees of freedom sum of squares divided by the degrees of freedom. Then what we do is we calculate the F value ok. F value is calculated as I told earlier also is that the F ratio is the mean squares of treatment divided by the means squares of error right.

So, we use that the F 0 is equal to MS treat and divided by MS error which is 118.94 divided by 8.06, you get the value of 14.76. So, this value F 0 14.76 goes right here right. Now that being said now you have calculated all the major values if you think about this table we have filled almost every values right here; now the thing is you have to make a decision out of this right.

So, the calculated value; so, the F value is used to make decisions. Use the F table in textbook and for 95 percent confidence or alpha equal to 0.05 whatever it is you look at the F value. So, you are looking at F of the F is given by usually calculated as you have the F of a and error degrees of freedom ok. So, that is F of 4 and 20 ok. These two error degrees of freedom goes here, for 95 percent confidence interval is 2.87 right. This is the value if you look into the table I would recommend you to look into it ok.

So, this degrees of freedom goes by the degrees of freedom of treatment and the error ok. So, this is the treatment this is the error ok. You calculate both and then you get the value as 2.87 ok. So, what happens here is, if you think about it is like a probability distribution something like this and you are saying the critical values somewhere is 2.87. So, anything about this is rejection reject H 0 you are rejecting the null hypothesis.

So, since 14.76 is larger much larger than 2.87 ok. We reject H 0 we reject the null hypothesis. And conclude that; and conclude that treatment means differ ok. So, you are saying that yes as of now because this value is so large we are saying that the means are different. Means are different in the sense that these conveyer speeds. Whatever the speed you select the speeds have a important. So, the conclusion is that the speed, speed of conveyor is important ok.

So, you have you are now identified a one way to figure out a factor that is significant; means which is a significant factor ok. So, for this soldering machine the conveyor speed is an important factor that is the conclusion that you are able to now take out of this. Same way you do this for multiple factors. And some of the factors will come out to become to be significant some of them will come out not to be significant. So, whichever comes out to be significant those are the factors that you need to take it outside for optimization.

So, I hope now you understand how we selected the factors that are to be taken into an optimization model right. So, we do we select all the set of initial setup parameters and then from there we do what we call as a analysis especially collect data do experiments. And then we test the hypothesis that different treatment levels of this factor has importance or not importance. And when you get do an ANOVA it will tell you yes this factor is significant for you the answer is yes then you take it forward ok.

So, with this what we do is we have come to a conclusion of this analysis of variance single factor analysis of variance. Please work out the example and also do the homework problem, because it is important that, if you do not do this practice it then you will forget this and you will not be able to do a proper job of understanding this. So, please do the practice problem, please do the homework. Please understand this is an important technique for you to learn and please do the need for.

In the next class what we will try to do is? We will try to do the factorial experiments ah. So, that just you have an idea of which level is more important. We are now say that this factor is important. Now the next step will also tells you which of the levels are more important for you. And the ones we finish that then we will go back and look into the optimization algorithms and stuff like that. Thank you very much for your patience hearing and wish you all the best.

Thank you.