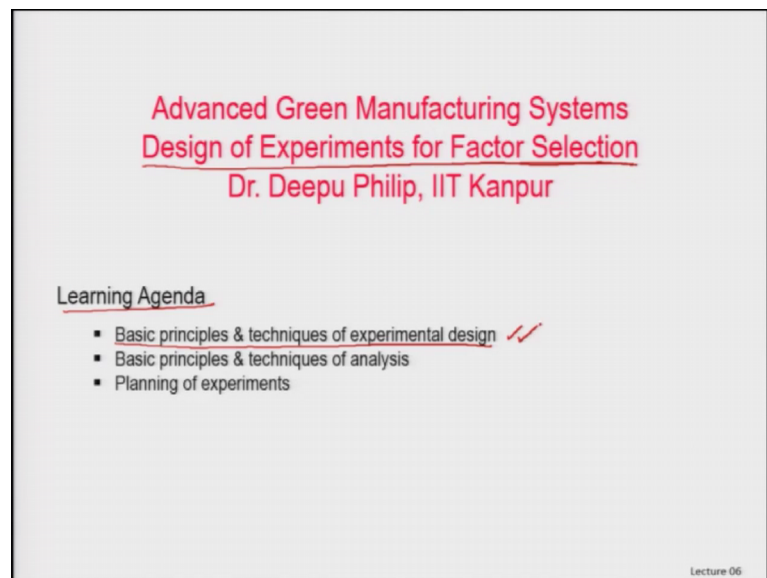


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**Lecture – 23**  
**Design of Experiments for Factor Selection**

Good afternoon students. Welcome to yet another lecture of Advanced Green Manufacturing Systems course.

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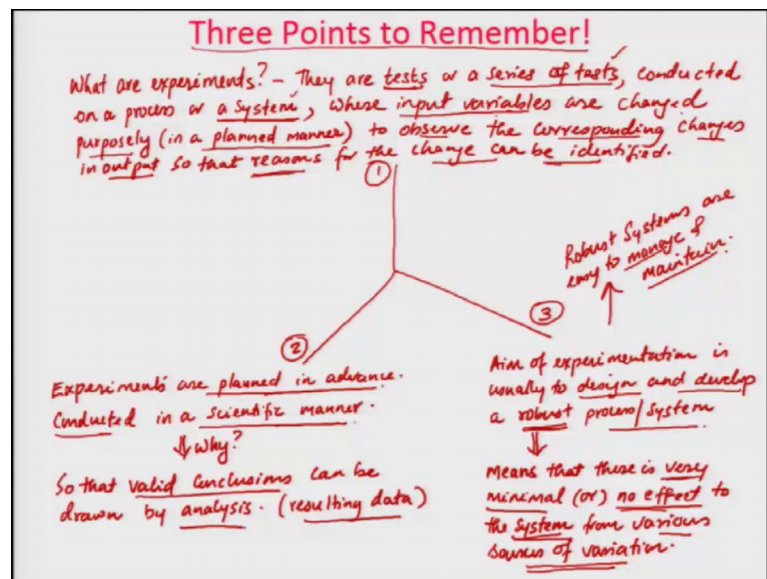


And I am Dr. Deepu Philip from IIT Kanpur, and today we are going to discuss about how design of experiments or the concepts of design of experiments is used in selection of various factors for optimization. We already seen the modeling and optimization aspects of different problems and how can we write the math model for optimization problem and how can we use excel to solve this optimization problems as well. We have formulated and solved multiple examples and found out the optimal settings for those parameters, but some of the things that we are not discussed and that is how do we came up with the coefficients and the constraints of each one of those.

So, how did we selected which factors are more important and which factors we modeled? So, that requires experimental design and today, what we are going to talk

about is how the experimental design concept is going to be used. So, if we look at the today's agenda we have the Design of Experiments for Factor Selection and our focus today is going to be how the factors are going to be selected. And today's learning agenda or what we are going to discuss today is the basic principles and techniques of experimental design and then we also talk about how what are the basic principles and techniques of an analysis and how do we plan for experiments. So, we will try to focus on this concept today.

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So, we will start with the three points, the major three points to remember and I usually like to present it as three major points you know. So, the first point we can think about it in we can write it in multiple fashions. Let us think about it doing this way and three different points.

So, the first point in this regard, let us take the first point. The first point is what are experiments? What are experiments, and there are many answers to this. But for us, for this particular course the important answer for us is they are tests. Ideally speaking they are bunch of tests or a series of tests. They are tests or a series of tests and these tests are conducted comma conducted on a process or a system; on a process or a system. So, these are a tests or a series of test conducted on a process or a system. We are more focusing on the manufacturing system here. So, we will be more looking into the system

aspects of its system where input variables are changed. Input variables are changed purposely. Purposely in the sense, in a planned manner; in a planned manner.

So what we are doing here is, the input variables various input variables we are changing them in a purposeful fashion or in a planned manner. Planned manner for what? To observe the change in output, the corresponding change in output. This corresponding changes in output in output. So that reasons for the change, reasons for the change can be identified ok. So long sentence, but we can break it into pieces. So, experiments are tests or they are a series of tests. Mostly we focus on a series of tests and they are either conducted on a process or a system. We are talking about the manufacturing system in this case, but we can do it on a bunch of unit manufacturing processes as well, not an issue, and what do we do in these experiments? We continuously change; we purposely change the input variables.

And then, we observe the corresponding changes in the output variable. So, that why this changes are happening. The reasons for these changes can be identified. So, that is the first point for us to remember. We are trying to identify the reasons for change in the output when we change the input variables in a planned fashion and such kind of tests are called as experiments. Then second point we need to remember about is experiments are planned in advance.

Experiments are planned in advance, they are planned in advance, conducted in a scientific manner; in a scientific manner ok. So, you plan the experiments in advance; there is planning involved in the experiments you conduct the experiments in a scientific manner. There is a rhyme and a reason. There is a methodology for doing this. Why? Why are we doing this, why do we plan on why do we conduct in the scientific manner? So that valid conclusions, valid conclusions can be drawn by analysis.

So if we plan the experiments and conduct it in a scientific manner, then the resulting data so, the resulting data can be analyzed, the resulting data can be analyzed so, that we can draw valid conclusion when we analyze the resulting data. That is the second most important point that you need to consider. The third point that you need to consider as part of this is aim of experimentation is usually to design; a usually to design and develop a robust process or system.

So, the main aim, why do we do experimentation, we usually like to design and develop a robust process or a system. So, what is a robust system, what is this robustness; means when you say robust what does it mean? Robust means it means that there is very minimal or no effect to the system from various sources of variation ok. When you say the when you trying to develop a robust system, what we are saying that there is minimal; very minimal or no effect. The system is not affected by the different sources of variation or various sources of variation have little or no impact on this system.

So, when you have a robust system then why do we look for a robust system because one thing that you need to think about is, robust systems are easy to manage and maintain ok. So, we can easily manage and maintain a robust system because we know that the system has very minimal or no effect on its behavior due to various sources of variation ok. So, these three points are the most important aspects of experimental design that you guys need to remember as part of this course ok.

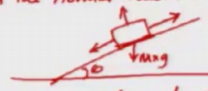
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Types of Systems

Based on the method of study systems can be classified into two broad categories:

- Mechanistic Systems: ~~X~~ where the phenomenon (or behavior) is well understood so that mathematical models can describe the system behavior.

Eg: Frictional force is well defined by  $F = \mu \cdot N$ ; where  $\mu$  is the coefficient of friction and  $N$  is the normal reaction of the body to the surface.


- Empirical Systems: ~~✓~~ They require observation to analyze factors and conduct experimentation to obtain information about results so that experimental models can be developed  $\Rightarrow$  empirical system.

$\Rightarrow$  The phenomenon (or) behavior is not well understood.

Continuing on this, we will talk about the types of system. Let us talk about what are the different types of systems? The reason we need to talk about this is because the experimental design depends upon what type of system that we are going to discuss in the class. The there are various methods of available to study the systems, but based on the method of studying the systems that is available, they are typically classified into two broad categories.

The first one is called a mechanistic system, the other one is called as an empirical system ok. So, what is a mechanistic systems? They are systems where the phenomenon of, where the phenomenon or behavior the phenomenon or behavior is well understood, well understood ok. So that mathematical models, mathematical models can describe the system behavior, describe the system behavior. So, when we are talking about a mechanistic system, the behavior or the phenomenon of the system is very well understood.

So that mathematical models are available to describe the behavioral of the system. So, let us take an example; example ok, the frictional force. Frictional force is well defined is well defined by  $F$  is equal to  $\mu$  multiplied by  $N$  and where  $\mu$  is the coefficient of friction coefficient of friction and  $N$  is the normal reaction,  $N$  is the normal reaction of the body to the surface to the surface. So, we all have studied friction where we studied an inclined plane, a body kept there, its mass times gravitational weight is acting here, there is a normal reaction toward, there is a force that is coming here, all these kind of things we have studied and that there is a coefficient of friction between these kind of forces.

We studied this. Remember if you and that is a. So, I am not going to explain too much of this, but these kind of systems are called as mechanistic system. This is not what we are interested in our course this is not our major focus point. We are mostly focused on empirical systems ok. So, empirical systems are they require observations. They require observations to analyze. Observation to analyze, to analyze factors, factors and conduct experimentation conduct experimentation conduct experimentation to obtain information to obtain information to obtain information about results information about results about results so that, experimental models can be created; can be developed.

Such systems are called as empirical systems. So, what happens is the phenomenon of the system is not very well understood. So, you have to observe the system so that you can analyze various factors and you then you do it mandates you to do experimentation so that we can find observations about the results. We can observe how the result changes and using that whole thing we can build what we call as experimental models or empirical models so that, the system can be studied. Such systems are called as empirical systems. This is what we will be focusing in this class ok. The reason for this is the phenomenon, the phenomenon or behavior is not well understood ok.

We do not really have a complete understanding of the behavior of the system. That is one of the reasons why we build empirical models of such kind of systems. So in this class, we are focusing on empirical systems or in this particular course, empirical systems is of importance to us right?

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**Variables in the Experimentation Process**

In any experimentation process, there are three major variables:

- 1. Response Variable:** They capture the output of the system (or) capture the behavior of the system.
  - Usually denoted by ' $y$ '  $\Rightarrow$  response variable.
  - $\rightarrow$  "Controlled variables."
- 2. Process Variable:** - They are varied in a controlled (planned) fashion during the experimentation, which results in changes of the value of ' $y$ '.
  - Usually denoted by ' $x_i$ '  $\rightarrow$  where ' $i$ ' the specific index of the variable.
  - They are also known as controlled variables.
- 3. External Variables:** - These are usually outside the controllable space of the experiment  $\Rightarrow$  They cannot be controlled.
  - Usually denoted by ' $z_j$ '  $\rightarrow$  where ' $j$ ' is the specific index.
  - Also known as "uncontrolled variables".

So going back to the experimentation process, we are now going to talk about the variables in the experimentation process and if you remember in the earlier discussion we had, we talked about variables of the optimization model or optimization or building the optimal optimization models. Now, we are going to talk about variables in the experimentation process ok. So, there are many different variables people talk about it, but for the purpose of this course in any experimentation process that we discussed in this class, there three major variables that we are going to focus on and all the three variables are listed here, they are called response variable, a process variable and an external variable ok.

So, what is a response variable? They capture the output of the system. Output of the system, or capture the behavior of the system ok. So, what we are talking here is, the response variable is the output when we change the factors. What is the output the system is giving and we going to talk about this as a how the; this output is in a way it captures the behavior of the system ok usually denoted by usually denoted by  $y$ , alphabet  $y$ . So most of the time, people will talk about when they say the  $y$  is ok. This is why

some sometimes also called as the response variable  $y$  ok. So, this is the response variable. The most commonly used notation for this is  $y$ . But what it does is, it is capturing the behavior of the system or in a way what it contains is the output of the system.

Then we talk about something called as process variable ok. So, what's a process variable? They are varied in a controlled fashion, in a controlled or planned fashion. They are varied in a controlled fashion during the experimentation, during the experimentation ok. So we vary these factors or these variables in a controlled or a planned fashion during the experimentation which results in results in changes of the value of  $Y$ . So, what it does is when we vary these variable, process variables in a planned or a controlled fashion during the experimentation, it will result in changing or varying the values of the response variable  $y$  ok, alright.

So usually denoted by usually denoted by  $X$ .  $X$  is typically used to denote that with a subscript  $i$ .  $X$  of  $i$  where  $i$  is the index,  $i$  is the specific index of the variable. So, if we have five factors then  $X_1$  means the first factor,  $X_2$  means the second factor,  $X_3$  means the third factor and so on. Right and because this variables are controlled they are also known as they are also known as controlled variables. Why are they called a controlled variables? The reason they are controlled variables is because they are varied in a controlled fashion. You are controlling the variation or the changes in the variable. So hence it is also called as a controlled variable.

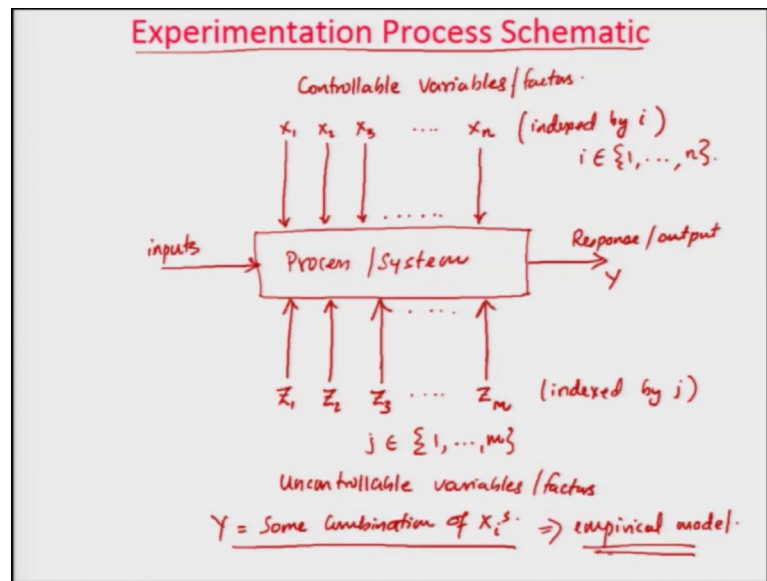
So sometimes instead of the process variable you will hear the name controlled variables or controlled variables ok. Controlled variables can be something else, but let us talk about controlled variables. For this time being, controlled is the word that is used for this alright; then comes the last category of the variable is called as the external variables. These are usually outside these are usually outside. Outside the controlled space of the experiment; outside the controllable space of the experiment.

So what we are talking here is, we they are usually outside the controllable space ok. So ideally speaking, you have no control over them which implies they cannot be controlled. So these variables cannot be controlled that is the most important fact. Usually denoted by, usually denoted by  $Z$ .  $Z_j$  where  $j$  is a specific index, index of the variable. So whatever it is, also known as also known as uncontrollable variables or uncontrolled

variables ok. So, we also call these variables are uncontrolled variables because it is outside the control space of the experiment. So, you cannot control this variable.

A classic example of this would be the humidity or something which you have no control over is controlled by the atmosphere and stuff like that. So, you know now three variables what are response variables, when we mention what is a response variable, the process variables and the external variables or uncontrollable variables right.

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So, just to schematically represent this, let me show you how we can schematically represent this. So say it as process, or let us call about it as system we are studying a process or studying a system ok. You have inputs, various inputs to it and then you have your response or output.

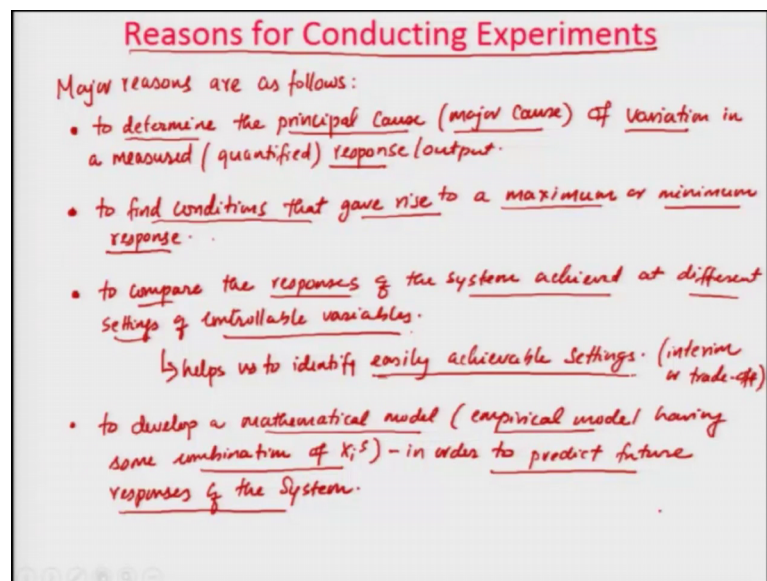
So, remember response variable is your Y then you have variables that you can control ok. So, you have X 1, X 2, X 3 etcetera X n. This is your controllable variables. So these are the values of the variables that you can control. So, the values of them can be controlled and then there is variables from the other side which you have no control over ok. This is Z 1, Z 2, Z 3 etcetera Z m, let's call it as m in this regard. So this way. So, this is your indexed by i. Indexed by j ok. So, j varies from 1 all the way to m and i varies from all the way 1 to n ok. So, these variables are your uncontrollable. Uncontrollable variables or sometimes you can also call them as factors, both of them can be called as



factors. So, your system at the end of the day is Y is some combination of X's. Of X I's ok. That is your, this is your empirical model alright.

So, empirical model will be your doing some combination of the X I's to get the whatever is your corresponding response and in this case you cannot really do anything with the uncontrollable variables which are your Z Z values Z values and so hence in that process, you have absolutely a your output the Y is dependent on the values of the X's that you talked about. So, that is a empirical model that you are trying to build and in the case the experimentation process that were trying to do for the green manufacturing systems is what we are trying to do is we are trying to build empirical models of your green with the manufacturing system. So that we can study the or we can optimize various parameters to get the appropriate greenness.

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So, why do we conduct experiments? We are told that experimentation different type of variables and those kind of things ok. There are many reasons for conducting experiments you can give billion type of reasons, but the major reasons for me or which are which are of 11 towards us. Major reasons are as follows, which are of 11's to us. In this course I probably say we have about four major reasons we can talk about and we will go by each one of them. The major reasons are so we will say to determine, to determine the principal cause of variation. The principal cause we can also (Refer Time: 24:41) we can say it as the major cause.

Principal cause of variation; principal cause of variation in a measured response in a measured or quantified response is your output ok. Response, response or output; so, the first thing is you are trying to determine, you are trying to find out the principal cause or the major cause of variation. What is creating the major variation in the output the response or the output of the system which is measured is quantifier right. Second one, second reason to find the conditions, to find conditions that gave rise to or gave rise to maximum or minimum response; I told you earlier that optimization problems are minimization or maximization problem mostly.

So now you can see why do we do experimentations. To identify those conditions or those criteria that resulted in the maximum or the minimum response of the system. So that is the another second major aspects, major reason for conducting experimentation is as follows and this is quite relevant to us. The second one that I talked about; the third one is, let us talk about is to compare to compare the responses achieved at different stage settings the responses of the system achieved at different settings, at different settings of controllable variables controllable variables ok. So what we are now trying to do here is we want to compare the responses of the system achieved. What responses did the system show for different values different settings of controllable variables.

This is of importance to us because at some point of time we might know that one particular setting this is helps us to identify easily achievable settings. Some of the settings might be quite hard to achieve, but some settings might be quite easy to achieve and if the systems behavior is reasonably at the easily achievable settings then this could be something that you should look for a; you can think about this in interim or trade off. Might be most of the time a locker setting usually looks at a such a trade off or interim. Your in easily available setting in this regard. The fourth point in all case is to develop a mathematical model. To develop a mathematical model ok, I would clarify this as the empirical model. Empirical model having some combination of X's; combination of X i's ok.

So, the empirical model in this case is or the mathematical model is some combination of X i's in order to, why do we do this? In order to predict future responses future responses of the system. So, if we can build the empirical or the mathematical model which is some combination not linear, can be some combination of X i's then we can use this model to predict the future responses or future output of the system ok. So these are the four major

reasons why we conduct experiments and these are the valid reasons for us for doing experimentation as part of this course.

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**Observational Study vs Experiments**

- Observations can be collected from either observational study or experiments.
- only an experiment allows conclusions to be drawn about cause and effect.

Eg: O/p from a machine in factory is consistently monitored to be of low quality.

What are the reasons?

- Machine is in need of replacement.
- Worker may be bad.
- Factory atmospheric conditions are not compatible.

⇒ The machine output under the current operating conditions (observational study) is of low quality.

⇒ helps in identifying a problem. ⇒ of low quality.

⇒ cannot say what is causing the problem?

⇒ don't know which one is causing it?

Now let us get into the next concept called as the observational study versus the experiments. Lot of the time, people say that I focus on observational study and people say that observational studies of importance and stuff like that but for the purpose of this course, we separate or we differentiate between observational study and experiments. So, let us see what are observation study and experiments. So, the first point that you need to realize is when people talk about observations not observation studies, observations the term. Observations can be collected from either observational study or experiments.

So either one of them, both of them will give rise to observations. Either the observations they will give you observations or experiments will also give you observations. Only experiments, only an experiment allows conclusions to be drawn about, conclusions to be drawn about cause and effect ok. So, what we are saying here is in an experiment, whatever the experiment be only in a experiment allows conclusions to be drawn about cause and effect ok. Cause and effect can be what is causing this and what is the effect of it. It can only be concluded from the such conclusions can only be drawn from an experiment cannot be drawn from observations study.

Example ok, let us take an example to differentiate this ok. Output from a machine in a factory floor, machine in factory is consistently monitored, monitored to be of low

quality ok. So there are many machines in a factory and in that factory there is one particular machine it is half machines are monitored but this machine is consistently showing you low quality so; obviously, the question is what are the reasons what are the main reasons? So, one reason can be machine is in need of replacement, is in need of replacement. It is time to replace the machine ok, then worker may be worker may be bad.

That is another reason, may be due to the worker factory atmosphere condition or atmospheric conditions are not compatible. The machine is not compatible to the factory atmospheric conditions so there could be many reasons ok, but so what we are saying here is the machine output, the machine output under the current operating conditions, under the current operating conditions ok. So, the machine output under the current operating conditions this is, the current operating conditions is an observational study. You observe, you observed that the machine output under the current operating conditions is of low quality. So, you know there is a problem that exist it helps in identifying a problem a problem.

So we know there is a problem. This problem is of low quality, cannot say what is causing the problem that we cannot say from this observation. So ideally speaking, you saw that one machine which is doing low quality output the output product coming out of the machine is of low quality and there are multiple reasons you don't know which one of them. So, we said there could be three major reasons and don't know which one is causing it. You don't know which one of them is causing it.

So, the observational study helps you to identify a problem; the problem identified here is the low quality output of the machine, but cannot figure out what is causing it. We cannot say what is the cause of the problem. So such kind of a system or such kind of study is what we call as a observational study where it is able to identify what is a problem, but not able to identify what is the cause of the problem only an experiment. And experimentation study allows you to draw conclusions about the cause and effect or what is the cause? What is the one factor that is driving the low quality output of the system ok?

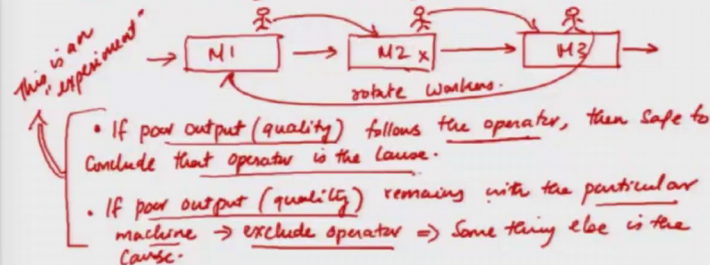
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## Observational Study vs Experiments - II

• It is usually quite simple to determine (or) rule out some of the potential causes.

lets take the reason - Worker is bad. (for eg:)

→ It can be easily answered by moving all operators from machine to machine in the factory for several days and at the same time observing the response of the particular machine.



So continuing on the discussion, now let us see what will happen with the experimentation ok. So, it is usually simple. It is usually quite simple, quite simple to determine or rule out, rule out some of the potential causes. I will tell you why ok. So we had in this case we thought there are three different aspects three major reasons we thought about this there could be more reasons also, but we thought this three one, these three ones ok. One, two and three are the major three reasons we thought why the machine is giving you low cost output right.

So, let us take the reason worker is bad ok. So we, if let us take this one reason for example, as an example we are taking this particular case that the worker is very bad. This is one of the cause that we are looking at right? So, it can be answered, it can be easily answered by rotating operators; by moving all operators will all operators from machine to machine; from machine to machine in the factory, in the factory for several days. And at the same time observing the output, observing the response, response of the machine, the particular machine ok.

So, what you have done here in this case is you moved all operators from machine to machine in the factory. So, you are switching operators now. Multiple operator so in a way if you schematically think about this, let us say these are the three machines in the factory ok. Machine 1, machine 2, machine 3 and this is the machine in our consideration this is the way the production happening in the factory and there are individual workers, three different workers. So what you do is, you move this worker to here. This worker

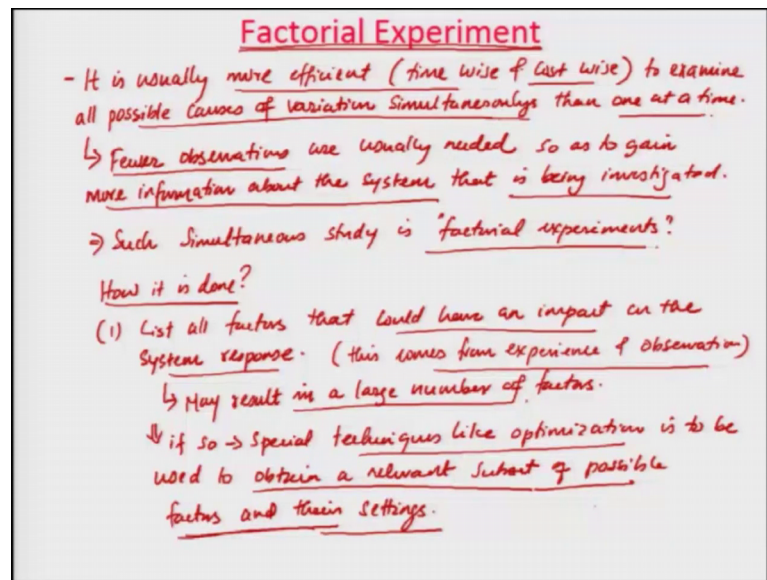
moves here, this worker moves here and then you observe the output of the machines and then you rotate it again ok.

So, you keep on rotating this. So, this is you rotate workers for several days and then you observe this ok. So, then we can find something. If poor output this is the quality follows the operator you can see that the poor output is following the operator then safe to conclude, conclude that operator is the cause is the cause right. So if the output the poor quality follows the operator then you say the operator is the cause ok. If poor output quality the poor output of the quality remains with the machine, with the particular machine then exclude operator. It is not the problem of operator exclude operator something else is the reason. Something else is the cause, not the operator.

So you can exclude the operator out of this, if the poor quality the output of the machine remains with the particular machine even after switching the operator, then you can exclude the operator as the main cause ok. So what do you did here this two ok. The thing that you did systematic fashion, this is an experiment when we say the (Refer Time: 40:14) experiment what we just did here, switching the operator from one to another and observe the quality of the machine and in this case from the output of this experiment, one conclusion you can make that if the output poor output, poor quality output remain with this particular machine, then you can say that the operator is not the cause.

You cannot make this conclusion from an observational study. You have to conduct this experiment and observe to identify the cause and effect of the system; right, alright.

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So, this would ideally bring us to the one particular concept called factorial experiment and we will talk about factorial experiment in detail because this is very relevant to us in this particular course. And let us talk about what is a factorial experiment. A factorial experiment in the previous discussion that we had of rotating the workers in the machine and those kind of the things, it is usually more efficient in time wise and cost wise and cost wise. It is usually more efficient both time wise and cost wise to examine to examine all possible causes of variations all possible causes of variation simultaneously; simultaneously than one at a time ok.

So it is usually more efficient. Time both time wise and cost wise; for example, all possible way things simultaneously and than one at a time. If you do one at a time it is too much time consuming this is because fewer observations are necessary. Fewer observations are usually needed are usually needed. So as to gain, so as to gain more information about the system, more information about the system about the system that is being investigated that is being investigated ok. So you can, if you simultaneously study all these factors or all the different factors together then you can actually require fewer observations to gain more information about the system that you are going to study ok. Such simultaneous study such simultaneous study is known as is factorial experiments.

When we say factorial experiments such simultaneous studies what we called as factorial experiments ok. How it is done? How this factorial study is done, how it is done? Step one list all factors that could have an impact that could have an impact on the system response ok. So, you list all the factors that can result that could result in an impact on the response ok. This comes from experience and observation. So, we observe and you list all those factors that you think that is of relevant right. Send may result may result in a large number of factors. It may result in a large number of factors ok; then if so if so special techniques like optimization.

Optimization is to be used is to be used to obtain a relevant subset of possible factors possible factors and their settings their settings. So, when you do the from list these factors from experience and observation you could probably come up with a large number of factors and these large number of factors, if such a case then studying this will become unique require to collect a lot more data then you can use specific techniques like optimization that we just study before so that we can shrink this big large set of factors into a small possible set of factors. So that there the factors which are those small set of factors that are important to us and what are the settings that aspects can be studied because of this kind of a factorial experiment ok.

Thank you very much.