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Lecture – 20
Solving optimization problems using MS Excel

Good afternoon students. Welcome back to the course advanced green manufacturing systems. Myself Prabal Pratap Singh, I am the course TA and today we are going to learn how to solve the optimization problem using Microsoft Excel.

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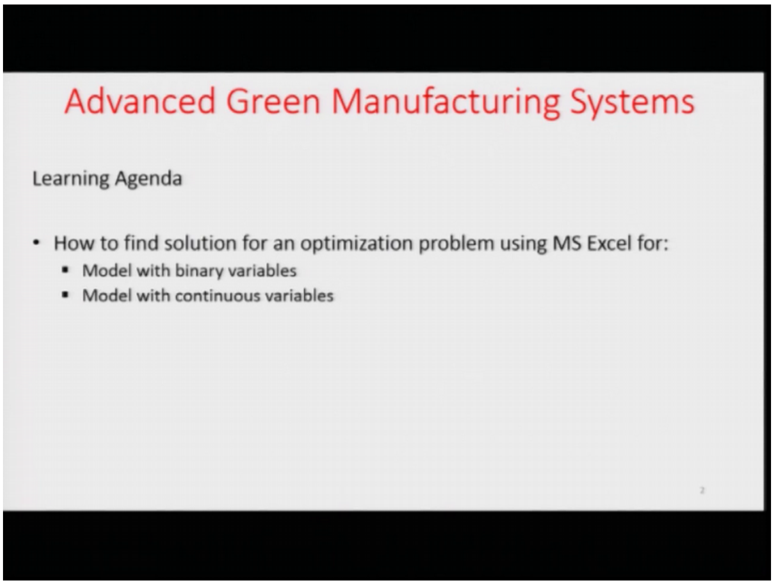

So, our learning agenda for today is how to find the solution for an optimization problem using Microsoft excel for two different problems. The first one is model with the binary variables and the second problem will be the model with continuous variables. So, these two problems have already been discussed by the course instructor Dr. Deepu Philip in the previous lectures and this lecture is the exact continuation of lecture 5 and lecture 7 in which these two problems have been discussed by creating the model manually.

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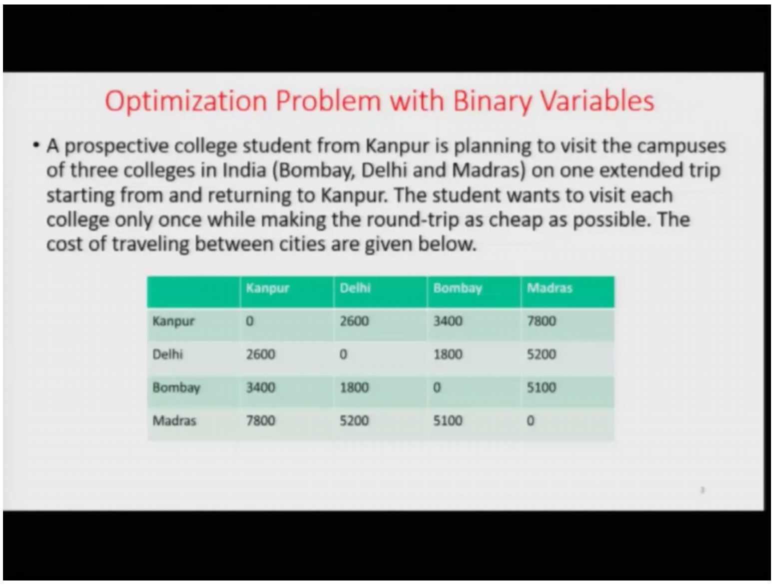

So, as we know that the first problem which was discussed is optimization problem with binary variable. In this problem prospective college students wants to travel from Kanpur and he is planning to visit the campuses of three colleges in India; in the city Bombay, Delhi and Madras on one extended trip starting from and returning to Kanpur. The student wants to visit each college only once while making the round trip as cheap as possible. The cost of travelling between the cities are given in the matrix below and now we are going to first recap the exact mathematical model which was discussed in the class and then we will try to solve the same problem by coding it into Microsoft excel.

There are various different options to get the solution for an optimization problems since this is a linear programming problem with binary variables. So, first we are going to learn how to do it with Microsoft excel the other alternatives are Python for in if you talk about open source programming languages or like the commercial packages like Lingo also provide these facilities for extensive optimization problems.

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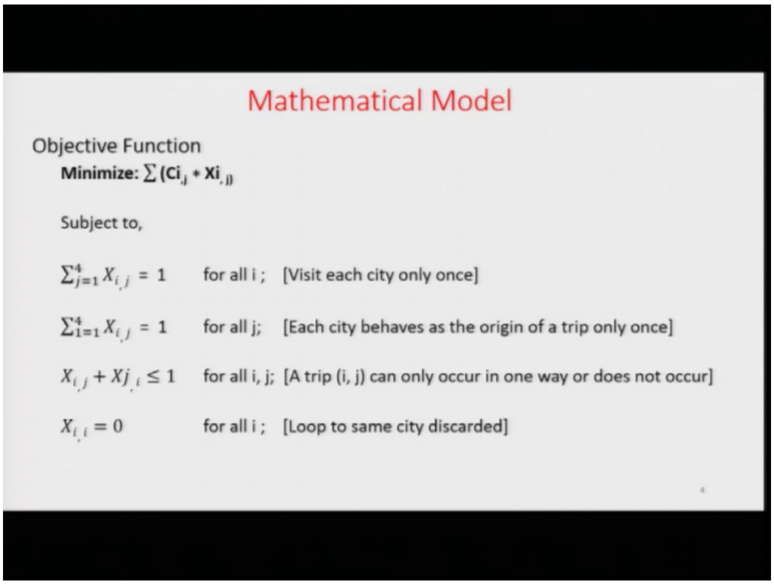

So, this problem has the objective function as given here in which we have to minimize the total cost of the trip and the variables are Ci j into Xi j. So, we are here reducing the cost to the minimum and this objective function is subjected to these different constraints in which the first constraint will talk about that each city should be visited only once, the next constraint will talk about each city behaves as the origin of a trip only once and the next trip next constraint will talk about that a trip can only occur in one way or does not occur. There is also constraint in which we are saying that all the X 11 X 22 X 33 or X 44 since we are only talking about four cities will be equal to 0 because we do not want to loop to same city and these values should be discarded in the optimization model.

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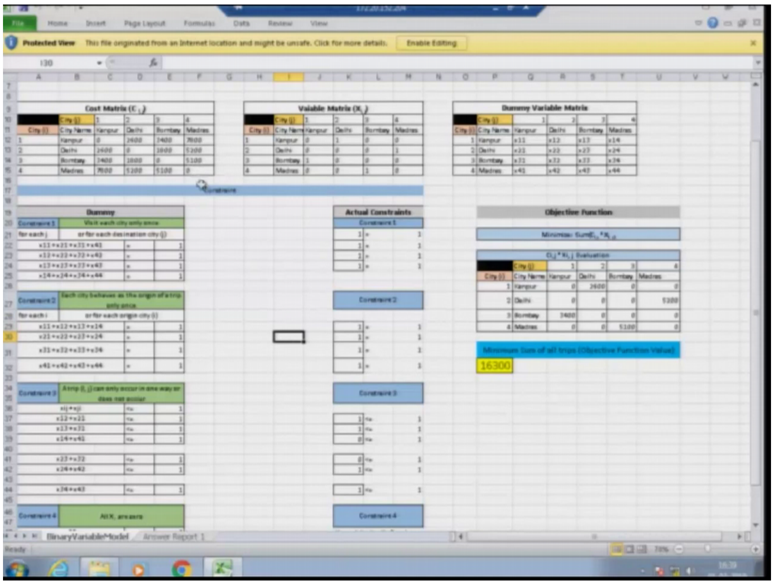

So, let us start with the Microsoft excel. So, we have already coded this into an excel file and it is shown here. This is the complete model and what this is showing is that this yellow colored cell is showing the minimum sum of all trips. Now this is a kind of decision making which can be done by using Microsoft excel and we are going to learn to make this type of a spreadsheet from scratch.

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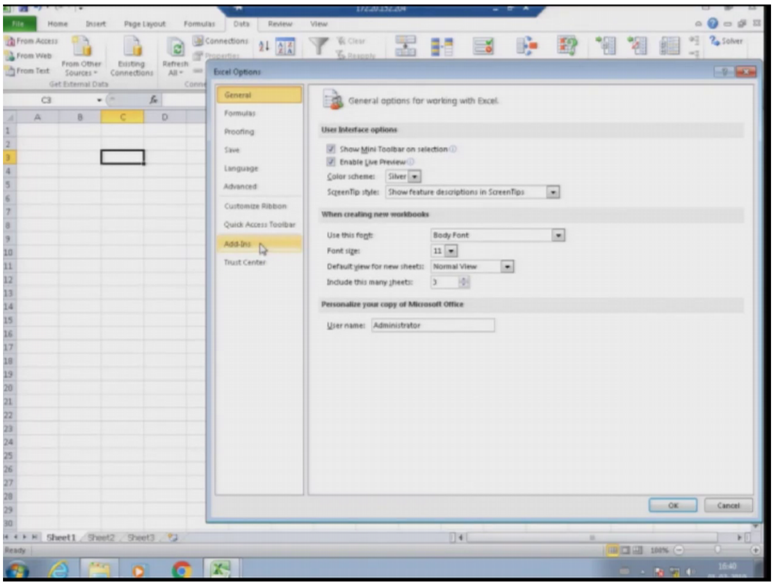

So, let us start a new spreadsheet now the basic installation of a Microsoft office will give you different applications one of which is Microsoft excel, but the basic installation will not provide you with the solver capabilities.

So, to get the solver capabilities, we first need to inspect whether we are getting the option of solver here since we have already coded this and we have already activated this option. So, it is been shown here as the solver for analyses, but may be in your personal computers it is not available. So, to get this available, you first need to go to file menu and choose the options. There you will get an option of selecting Add-Ins.

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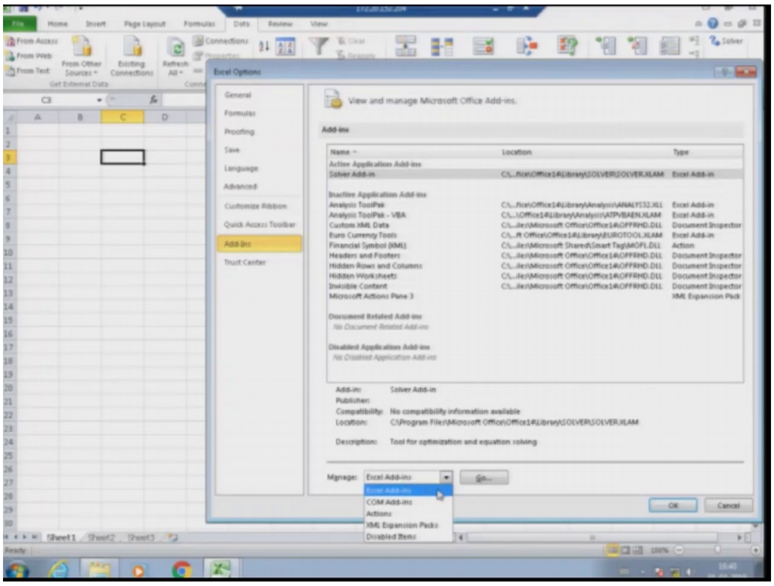

There are various difference Add-Ins which are provided by Microsoft and we need to select Excel Add-ins and then click on Go.

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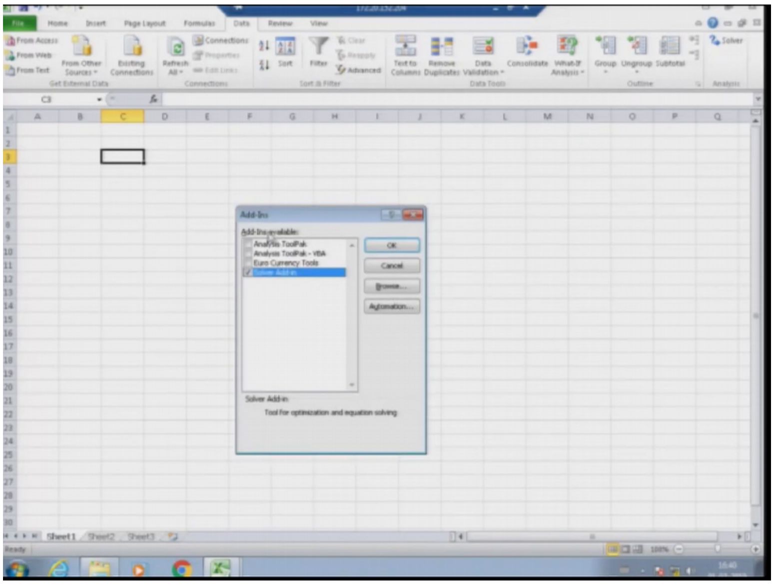

There are four different variables as here we are as here we have, but we only need Solver Add-in for this purpose. So, select this and click ok. In your computers, it will first install and then maybe it will ask you to restart your Microsoft excel application. Then once restarted, you can use the solver dialog for optimization problems. So, let us first copy the question which we need to optimize. So, this is the question which we are talking about and the data given here is the cost matrix. So, there is nothing new in this so, we can directly copy this.

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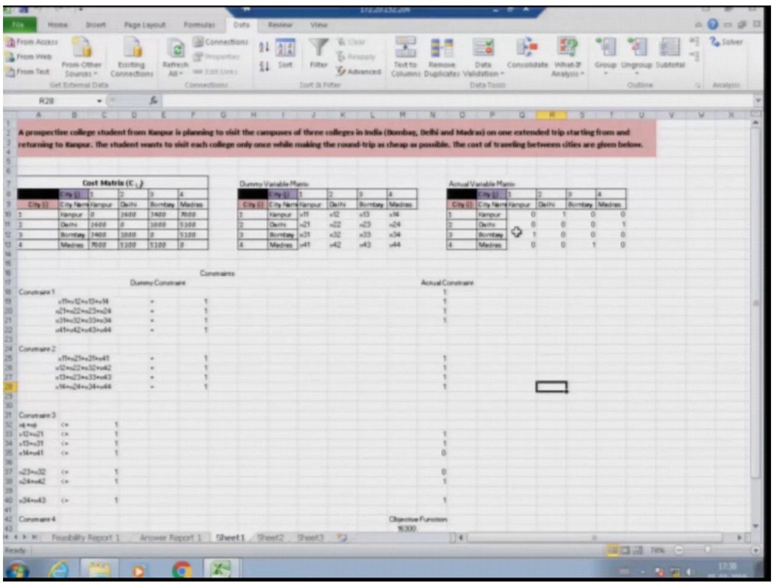

Now we have the question and what the question is saying that we have few cities and these cities have some cost related to them for travelling from one city to another.

Now, the first thing which we need to do in solving an optimization problem is to first select the variables. So, here we have already discussed that the cities are the variables and the i denotes the origin city and j denotes the destination city. So, we have created this matrix and now create dummy variable matrix. Now if you see that how these values have been filled up. The curiosity will be like how can we code all the mathematical constraints which we have created in our mathematical model and then get the solution from this solver excel. So, as we are doing it manually analogously, we will first create the variables. So, actually if you can see, we have created r X i j variables here. So, just to demonstrate that the variables are created, these cells are actually the variables which the follower will use.

So, let us say this is x 11, then this is x 12 this is x 13, this is x 14; same way all the values can be filled in and these values will be used to optimize the objective function in the optimization problem using the solver. Now this is just for the demonstration purpose that is why I have written this is the dummy variable matrix, but to actually use this matrix for the purpose of optimization we will create this matrix again. But we will keep these values these cells unfilled so, that the solver can change the values in these cells to get the optimized results.

So, let us create the actual variable matrix and just reset all these cells in the variable matrix to null so, that the solver can use it. Now we have initialized all are variables. Now let us start with defining the constraint for the problem.

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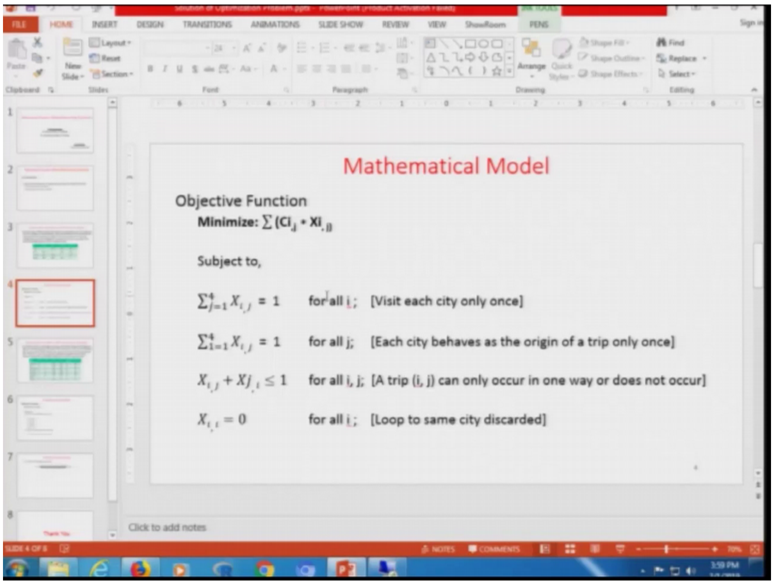

So, let us see what is the first constraint the first constraint says that the any X i j for all is should be equal to 1; that means, each city should only be visited once. So, let us encode this into our excel. So, you have already seen that if we are giving the basic input in any cell, then we can directly write into it and this is the function were we can see what are the value of the cell.

So, you can see this is the basic string, but when you are going to encode anything which is having an expression, we need to first start with equal to sign. So, let us start putting the expressions for this mathematical model by using the equal to sign with the start. Now we need to give this constraint as x 11 plus x 12 plus x 13 plus x 14 equal to 1. So, let us write the dummy constraints first so that we can stay clear what we are going to encode in the mathematical model. The first instance of the first constraint will be x 11 plus x 12 plus x 13 plus x 14.

The next will be x 21 plus x 22 plus x 23 plus x 24. The next will be x 31 plus x 32 plus x 33 plus x 34. The last constraint of this first constraint will be x 14 plus x 42 plus x 43 plus x 44. So, these all instances of the first constraint will be equal to 1. This will constraint the value to make any one of the origin city to be equal to 1, otherwise there will be loops in the solution. Similarly let us first see what is the second constraint. So, the second constraint talks about that each city behaves as the origin of the trip only once. Therefore, we can see that we are changing the value of i here from 1 to 4 for all the j's means first the j will remain constant for first instance. Then in the second instance, the j will change.

So, similarly let us define constraint 2. So, the first constraint would be x 11 plus x 21 plus x 31 plus x 41. The next will be x 12 plus x 22 plus x 32 plus x 42. The third constraint would be x 13 plus x 23 plus x 33 plus x 43. The last would be x 14 plus x 24 plus x 34 plus x 44. These will also be constrained with the same equal to 1.

Now, the next constraint is that we should not get the loops in the trips. So, this was defined in the lecture like x i j plus x j i should be less than equal to 1. So, we will code this now. So, this constraint is important because otherwise the loops will keep the objective function minimum, but that minimum value will not be accurate. So, this x ij plus x ji less than equal to 1 should be for all i comma j. So, we write all the instances because we are going to encode this. So, x 12 plus x 21 less than equal to 1; x 13 plus x 31 x 14 plus x 41 x 23 plus x 32 x 24 plus x 42 x 34 plus x 43.

So, these are all the instances of constraint 3 and we can verify this. Yes, now this is the last constraint and this is required because there are no such thing as city i to city i means we are not going to travel from the city in which we are in.

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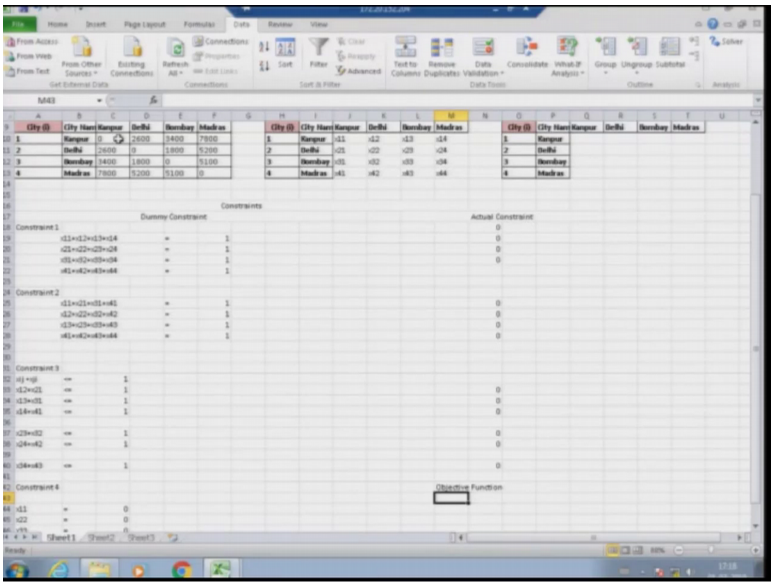

So, this is constraint 4; x 11 x 22 x 33 x 44. Now these are all the dummy constraints. Here we are going to actually encode these constraints because now we have clear picture about what do we need to do next. So, as I have told you that we have actual variable matrix here and each blank cell is variable. So, to select x 11, we are going to use this cell to select x 12; we are going to use this cell.

So, let us start writing the left hand side of this equation first. So, start with equal to and using the control button on your keyboard. Start picking up the blank cells of the variable which we have mentioned here in the dummy constraints. So, x 11 is this so, click on this now write plus because we are going to add all these constraints. So, end by pressing the control button, we are going to select the next variable and the next variable here is x 12. So, this is how we are going to select multiple variables. Now select the third variable x 13 and x 14. So, you have encoded the left hand side of the first instance of the first constraint.

Similarly, we are going to do it for all the instances of the first constraint. So, we have encoded the left hand side and you can see that these are showing the value 0s because we have written an expression here and the variables in the expression are all null. So, it is calculating it to 0 currently, but when we run our solver; it will give some values to these constraints and only one of the variable will get the value one as per the constraint.

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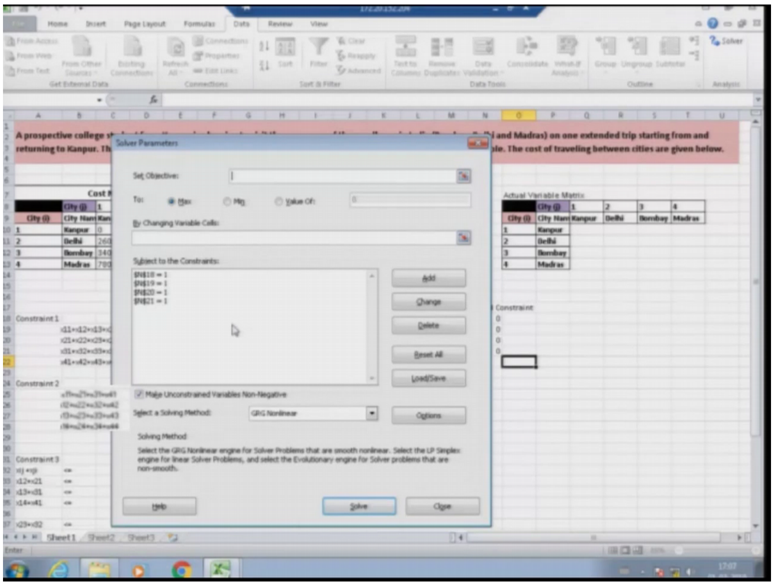

Now, let us encode this into solver. So, go to your data tab and choose solver button. Here I would like to tell you that the set objective is the value which contains the cell having the objective as being encoded. So, currently we have only encoded the first instance of our first constraint. So, we have not yet encoded the objective value. So, let us skip this for now and by changing variables ask for all the variables which needs to be changed by the solver.

So, solver parameters include all the variables and the objective function will try to maximize or minimize as per the given value and then the final value will get generated into a report and the subject to constraint tabs include all the constraints which we are going to encode. So, by using the add button we can add a constraint and we can also add it the constraint which we already added we can delete the constraint, we can also reset all the constraint and also we can load or save the constraints. The option tab includes the different options for the solver.

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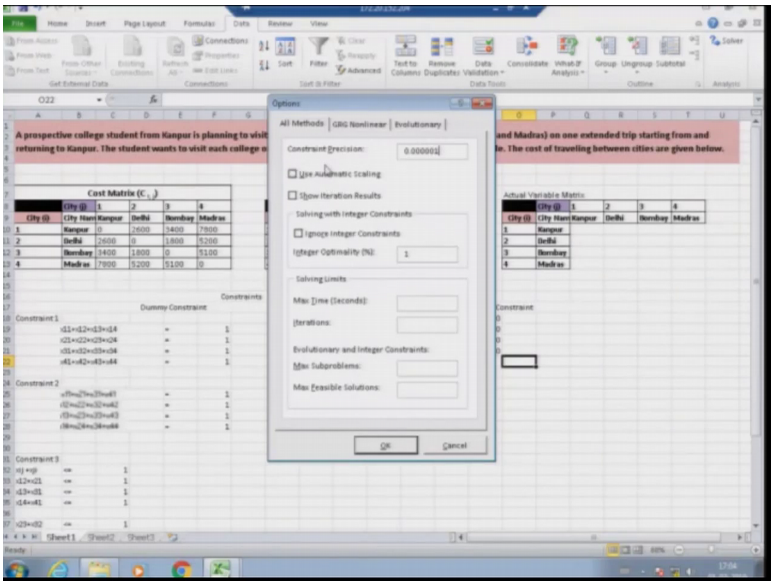

So, there are many different types of solvers and their parameters. We are going to use a very basic simple linear programming solver, but there are various methods which can be used to get to the solution for complex problems. And evolutionary techniques are also one of the solution methodology which can be used by the excel solver, but it has limited capabilities.

So, let us start adding the constraints which we have created.

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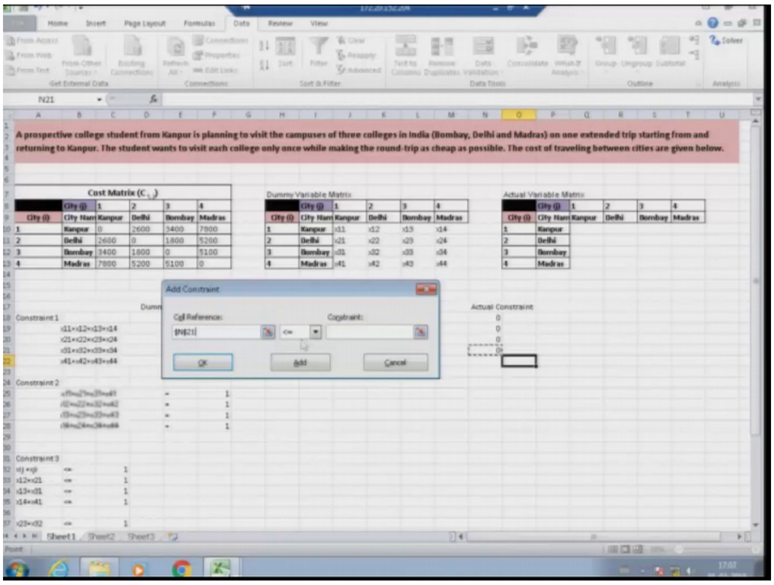

So, after pressing the Add button this new dialog box will appear and the cell reference will ask for the left hand side of the cell and the values in the right hand side can be given here. This button can be used to select the cells or if you want to write the actual address of the cell, then you can see the row or column like if we want to give the value of this cell then we need to write N of 18. So, N18 if you write here, it will select this cell or you can directly click on this cell. So, just click on this and now it is asking for the cell which you want to add here. So, I will click here and then again i will click on this button.

Next is what type of constraint is this. So, as we have already seen this is an equal to type constraint. So, use the drop down menu and select equal to and then write the rhs value which is 14 here. Just click on add; now it is asking for another constraint. So, again select the cell by following the same steps; third one and the last instance. Now we have encoded our first constraint of the mathematical model which has 4 instance and also here you can see that it is showing 18 and 19 and 20 and 21. These are the address of the cell and dollar sign here represents that the value will remain fixed means it will only see this cell in the whole worksheet. Now close this for the time being and let us add another constraint. We will follow the same steps here also and here we will add x 11 plus x 21 plus x 31 plus x 41.

The next instance is x 12 plus x 22 plus x 32 plus x 34. Similarly x 13 plus x 23 plus x 33 plus x 43 and the last one is x 14 plus x 24 plus x 34 plus x 44. Now the last constraint and here we need to add x 12 plus x 21. So, just select x 12 here x 12 plus x 21. Next is x 13 plus x 31 x 13 plus x 31 third one is x 14 plus x 41 x 23 plus x 32 x 24 plus x 42 x 34 plus x 43. Again we will open the dialogue box for solver and add all these constraints.

So, we have encoded the second constraint. Now we encode the third constraint. So, now, we are going to add the constraint 3 and in us model. So, click on the solver tab and start adding the third constraint. So, the first instance of the third constraint is in this cell. So, let us select this and this is the less than equal to constraint and the value of rhs is 1.

Similarly add the next instance. So, let us check we have given N 18 19 20 21, then from 25 to 28 25 26 27 28. Next is from 32 33 34 35; 32 34 35. So, we have missed one constraint here 35. This is x 14 plus x 41. So, you can also check whether the values which you have encoded is correct or not by double clicking on the cells which you have encoded like we had just encoded x 14 plus x 41. So, this is x 12 plus x 21 x 13 plus x 31 x 14 plus x 41. This is x 23 plus x 32, this is x 24 plus x 42, this is x 34 plus x 43.

Now, the last constraint first encode this x 11 is equal to 0. We can directly encode this into a solver tab and we do not need to encode because there is no expression involved in these constraint. So, we can directly go to solver and add. So, the first constraint is x 11 is equal to 0. So, just click on x 11 u equal to type 0, add x 22 equal to 0, add x 33 equal to 0 add, x 44 equal to 0 add. So, now, we have created all are constraints and just to verify the last ones are t one 3 means t and this is one 3. So, this is the x 44 equal to 0 constraint and we should not miss any constraint here or we should not use different cells because then the solver will get misinterpreted and it will give wrong values or it may not generate the result.

So, the next step is to define the objective function. Now we know that the objective is to minimize the cost of the whole trip and the cost can be calculated by multiplying the cost of each travel from i to j with the variable and since we are talking about a binary variable modeling. So, the variable can only take 1 or 0 as the value. So, if x 2 comma 3 is equal to 1, then only the x 2 comma 3 value which means 1800 will get added into the trip otherwise it will not. So, there will only be 4 values from this matrix which will get added by changing the 1s and 0s in this matrix.

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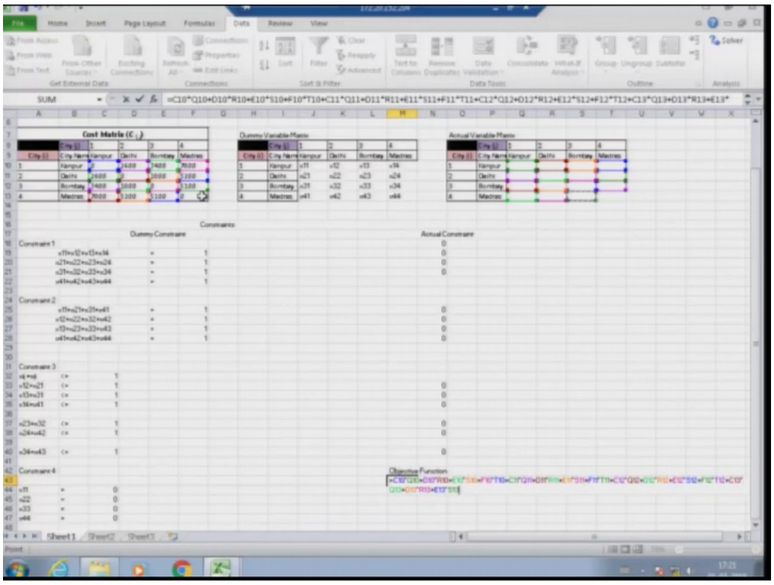

So, let us write this objective function by encoding. So, first select the cost matrix the value from the cost matrix was c ij and then multiply it with the variable matrix x ij and then we will add all these values one by one. So, the first is C 11 in to X 11 plus C 12 into X 12 plus C 13 into X 13 plus C 14 into X 14. Similarly on the second row C 21 into X 21 plus C 22 into X 22 plus C 23 into X 23 plus C 24 into X 24 plus C 31 into X 31 plus C 22 into C 32 into X 32 plus C 33 into X 33 plus C 34 into X 34 plus C 41 into X 41 plus C 42 into X 42 plus C 43 into X 43 and the last C 44 into X 44.

Thus we have created the objective function here and now we need to add this into a solver. So, set objective will ask you for the cell were you encoded your objective function and we can give this and it is also asking for where you have defined your variables. So, we can select all these columns. Now our model is complete we have defined the objective function in a cell we have selected all the cells were we have r variables which are yet undefined with any values and then all the constraints. These constraints will constraint the value of the objective function and now we also need to select whether we are going to minimize the objective function or maximize.

Since here the cost of the travel needs to be minimized, just select minimize. The solving method needs to be chosen also and here we are going to use Simplex l p and just press the solve button, then it will ask you whether to generate a report or not. So, we have created the complete model and we have defined the objective function. So, let us solve this problem by clicking on solve button and here the solver is saying solver found a solution. Now you can chose different things here like you can also do sensitivity analysis which is an advanced part of optimization problem and you can also see the answer. So, just click on and it will create a new spreadsheet answer report one.

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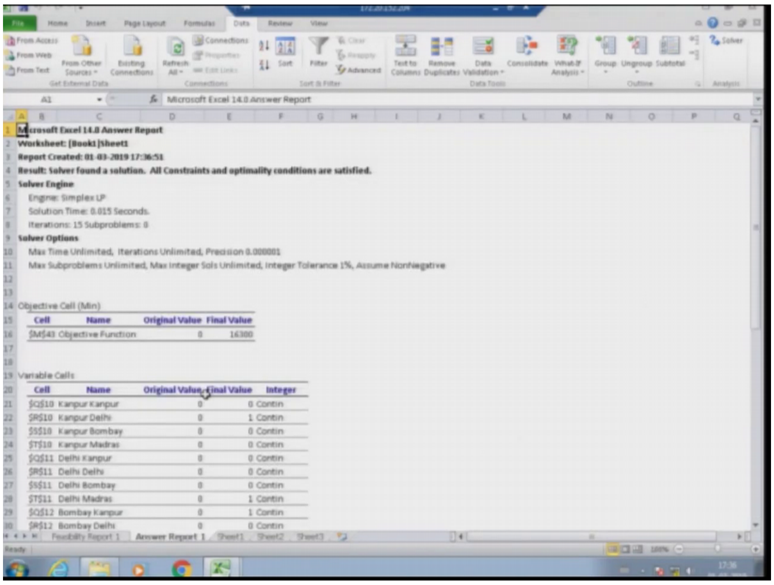

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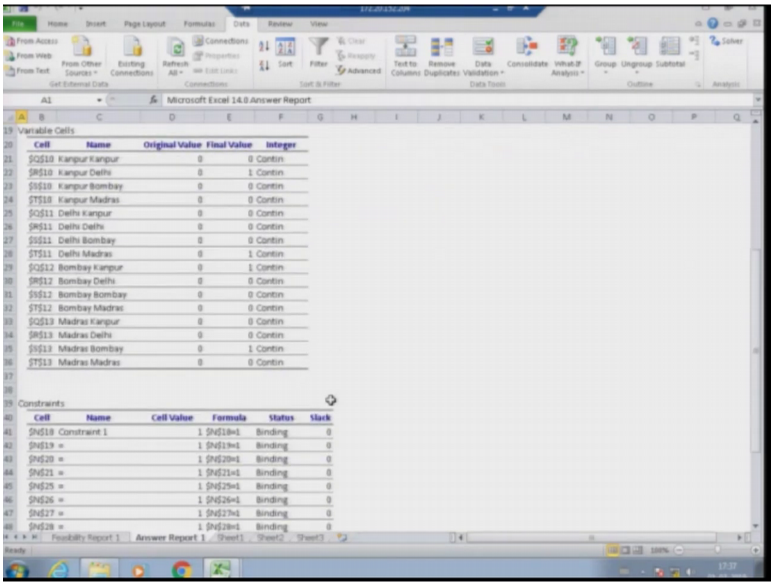

So, it is showing that the objective function is having the value 16300 and it has only given the value of 1 to only sum of the variables in the whole variable matrix like you can see that the final answer is showing that the student will travel from Kanpur to Delhi, then from Delhi to Madras because the i comma j which means 2 comma 4 value is 1.

Then the student will travel from madras to Bombay and the last trip will be from Bombay to Kanpur. So, he is going to come back to Kanpur by following all the 4 cities in the minimum cost and the cost will be 16300. So, this is the whole process of creating a spreadsheet for evaluating an optimization problem by encoding it using the solver tab to analyze and to make decisions whether which variables need to be given what value. So, I believe this is the problem which you can use to encode any type of different optimization problem which has only binary constraints. So, let us start with the second problem.

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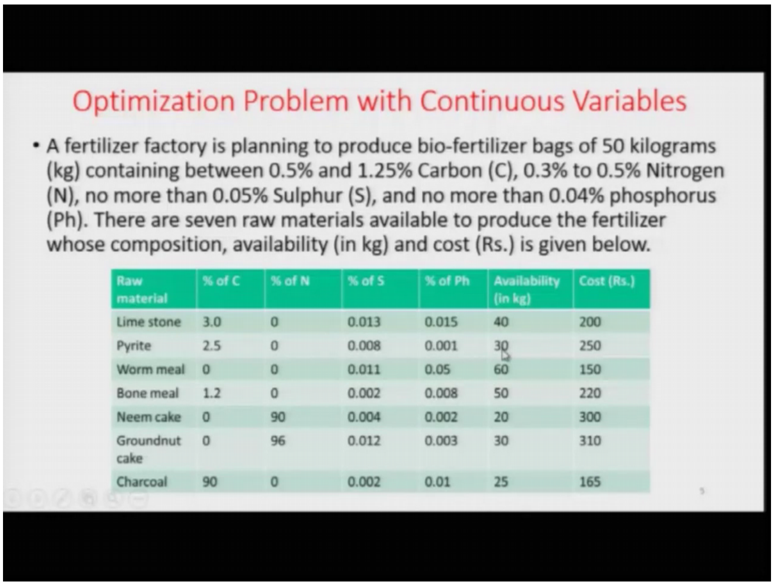

So, this is the second problem which was discussed in lecture 7 by the course instructor and in this problem the difference from the previous problem is that this problem includes the continuous variables which means that the variables can take any real value as its value for maximizing or minimizing the objective function. So, let us first recap what was the problem. So, in this fertilizer factory is planning to produce bio fertilizer bags of 50 kilograms containing between 0.5 percent and 1.25 percent carbon 0.3 percent to 0.5 percent nitrogen no more than 0.05 percent sulphur and no more than 0.04 percent phosphorous. There are 7 raw materials available to produce the fertilizer whose composition availability and cost is given below.

So, this is the matrix which we will be using in our excel for various different operations.

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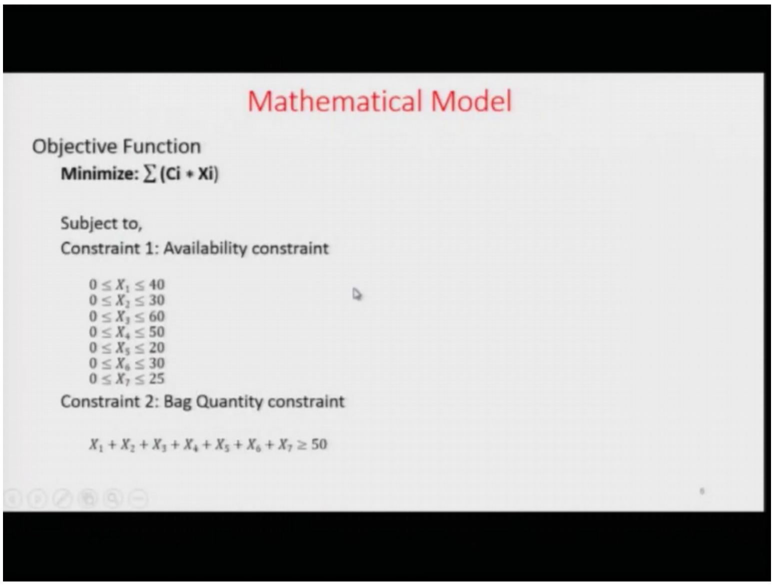

Now let us recap what was the mathematical model. Here also we are going to minimize the cost of the bag and the C i are the cost matrix and X i are the variable matrix. So, we are going to multiply each C i with its own X i and here the X i is not a binary variable rather a continuous variable. And it is subjected to constraints of availability like the variables was X 1 amount of lime stone, X 2 amount of pyrite X 3 amount of worm meal, X 4 amount of bone meal, X 5 amount of neem cake X 6 is groundnut cake and X 7 is charcoal.

So, the first constraints is that the X 1 should be greater than equal to 0 and all X i's are greater than equal to 0 which means we need to have some quantity and the quantity cannot be negative and we have also been given the availability of these materials. So, X 1 is limited to only 40, the next is 30 60 50 20 30 and 25 which was given in the mathematical model as shown here. The next constraint was bag quantity constraint which shows that the each bag should contain this much amount and the bag quantity should be greater than equal to 50 kg.

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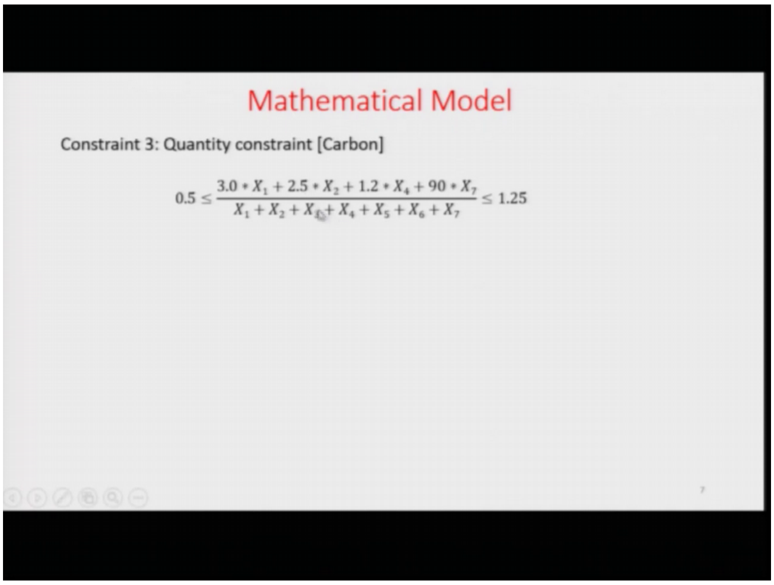

So, the last constraint is the quality constraint and we are showing here the quality constraint for the carbon. So, this is here as defined in the lecture. This ratio is how we are going to constraint the quantity in the final objective and the mathematical model. Now as discussed earlier, we should not have ratios in the mathematical model. Thus when we are going to encode this into our excel file, we are going to rearrange this equation and take the quantity of the bag to the lhs an rhs of this equation.

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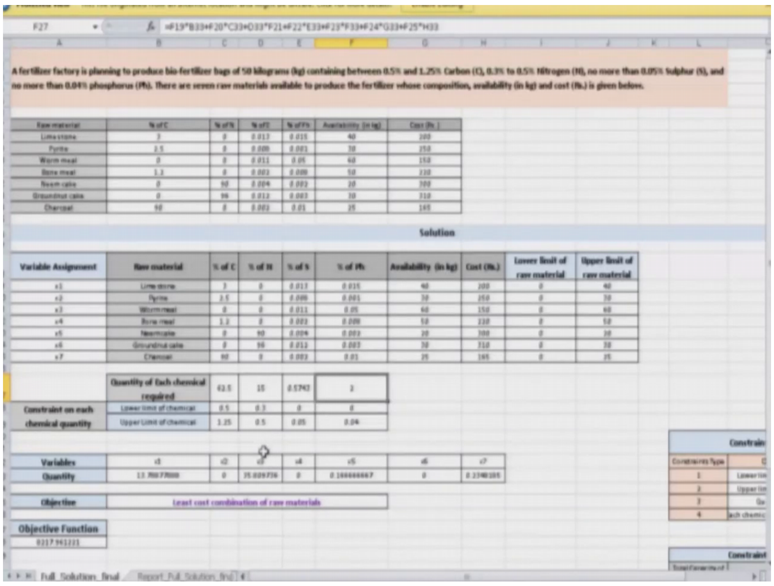

So, let us start with encoding the model. We have already created the complete solution file here and you can see that this is the way which you can create your own spreadsheet for any type of problem. And now let us create the same mathematical model in a spreadsheet from scratch.

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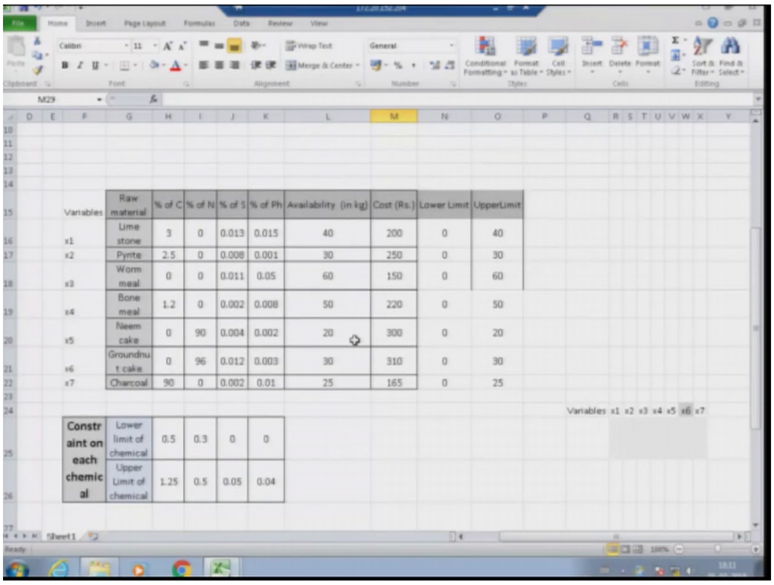

Variables	Raw material	% of C	% of N	% of S	% of Ph	Availability (in kg)	Cost (Rs.)	Lower Limit	UpperLimit
x1	Lime stone	3	0	0.013	0.015	40	200	0	40
x2	Pyrite	2.5	0	0.008	0.001	30	250	0	30
x3	Worm meal	0	0	0.011	0.05	60	150	0	60
x4	Bone meal	1.2	0	0.002	0.008	50	220	0	50
x5	Neem cake	0	90	0.004	0.002	20	300	0	20
x6	Groundnut cake	0	96	0.012	0.003	30	310	0	30
x7	Charcoal	90	0	0.002	0.01	25	165	0	25

Constraint on each chemical					
	Lower limit of chemical	0.5	0.3	0	0
	Upper Limit of chemical	1.25	0.5	0.05	0.04

So, this is the new spreadsheet with only the data from the question and now let us first start by creating the variables. So, let us take this matrix and we know that we have defined x 1 as the quantity of the raw material of lime stone. So, we will write here x 1 x 2 x 3 x 4 x 5 x 6 and the last x 7. These are the dummy variables and we are not going to use these variables in our modeling. But for demonstration purpose we are going to write it so that we do not get confused while we are modeling the equation in the solver tab.

Next is we know the availability in kgs. So, let us define the lower limit and upper limit of this availability. So, the lower limit of all the raw materials is 0 and the upper limit is provided here 40 30 60 50 20 30 and 25. So, as we have discussed in the previous problem that we are going to give empty cells to the variables. So, let us define the variables here x 1 x 2 x 3 x 4 x 5 x 7 and these cells will have the values after we have completed our modeling. So, we can rearrange this then we are going to get the quantity of each material

So, this is the data which was given in the question and we can directly take it from the question and you can rearrange this just below all the materials. So, thus as per the question the lower limit of chemical carbon is 0.5 the upper limit is 1.25, the nitrogen has 0.3 as lower limit 0.5 as the upper limit and so on. Next let us first see the constraints.

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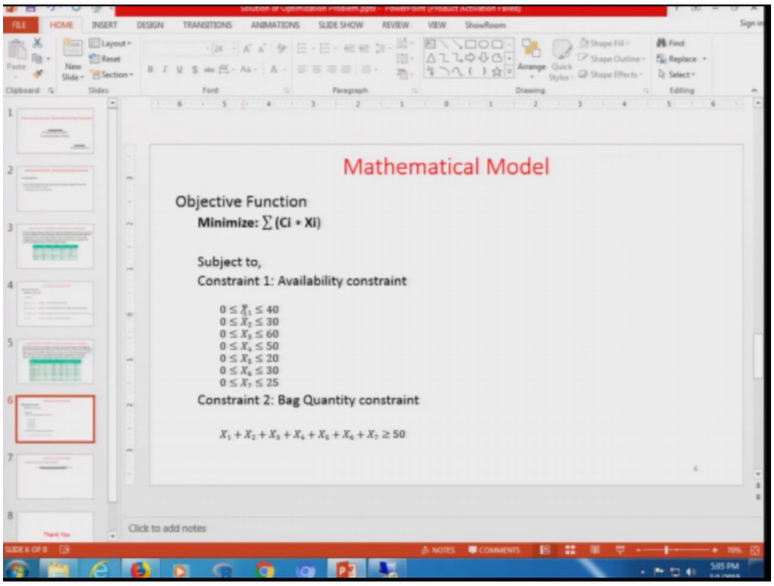

So, the first constraint is the availability constraint and since we know that if there are no expressions in the constraint, we can directly moderate into a solver tab. So, we switch to a solver from the data tab and start encoding the constraint for the availability. So, first encode the lower limit of the availability. This is variable x 1, it should be greater than equal to 0, x 2 should be greater than equal to 0, x 3 greater than equal to 0, x 4 greater than equal to 0, x 5 greater than equal to 0, x 6 greater than equal to 0 and the last x 7 is greater than equal to 0.

Now, we will encode the upper limit of the availability constraint. So, again starting from x 1 this is less than equal to 40. So, again x 1 less than equal to 40, x 2 less than equal to 30, x 3 less than equal to 60 x 4 less than equal to 50 x 5 less than equal to 20 x 6 less than equal to 30 and x 7 is less than equal to 25. So, the first constraint as we encoded, now let us see the second constraint. The second constraint is the bag quantity constraint and since it is an expression, we first need to encode it into a cell. So, close the solver tab and chose a cell of your liking and start encoding the bag quantity constraint.

So, it is the sum of all xi's. So, start from x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6 plus x 7 and now put this into the solver. This should be greater than equal to 50. So, we have encoded the second constraint also. Now is the third constraint and here; what we are saying is that the 3 into X 1, here 3 is the percentage of that particular chemical. So, let us see how we are going to encode this. First we will encode the expression in the numerator and as we have already encoded the expression in the denominator, we will use the same value from the previous cell.

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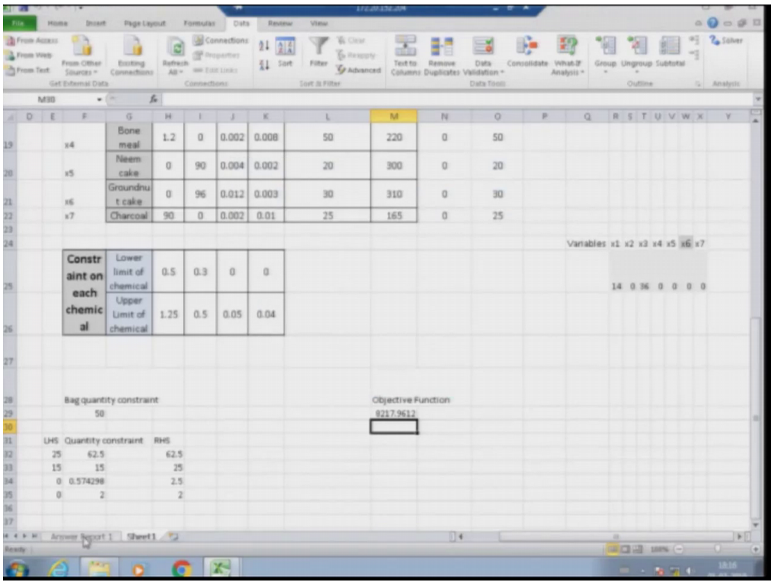

So, these are the values of the content of carbon in each different raw material. So, let us write the quantity constraint here. So, let us start encoding the quantity constraint for the carbon. So, as shown in the constraint it is 3 into x 1 plus 2.5 into x 2 plus 0 into x 3 plus 1.2 into x 4 plus 0 into x 5 plus 0 into x 6 plus 90 into x 7. So, we have encoded the numerator of the first instance of this quantity constraint.

Similarly, we can encode the constraint for nitrogen, sulphur and phosphorous and it will be like the same procedure with different column; 0 into x 1 plus 0 into x 2 plus 0 into x 3 plus 0 into x 4 90 into x 5 plus 96 into x 6 plus 0 into x 7. For sulphur it is 0.013 into x 1 and all the following are with similar procedure. So, this is the last percentage of phosphorous and all the following are with similar procedure and now we need to encode the lower and upper limit as shown in the constraint. So, here we are going to rearrange this equation. Therefore, we need to multiply 0.5 with the total value of the bag.

So, this is the lhs, this is the rhs. So, this is 0.5 into we have encoded the sum of the bag in this cell. So, we will multiply this similarly the lower limit of chemical nitrogen and by following the same procedure we can define the rhs values also. So, we have encoded all the constraints and now we are going to encode the objective function.

So, let us review what was the objective function it is the calculation of cost. So, C i into X i. So, our cost data is in this column for each raw material and our xi's are this. So, let us encode this. Now we need to fill all these new constraints into a solver tab.

So, we have filled the constraints in the solver tab till bag quantity constraint, the next we are going to fill is the quantity constraint of each chemical. So, by clicking on add, let us select this is the cell in which we have encoded the numerator of this constraint. So, select this should be greater than equal to the lhs, add similarly. This should be greater than equal to lhs and for the rhs part the same numerator less than equal to rhs and finally, the objective function. This should be minimized and our variables are in these tabs.

So, after creating the constraints, we should cross check those constraints every time so that they should be no discrepancy. Now we will use simplex lp and solve this and here the solver is saying that it has found a solution. Again as previous, we can use the answer report to analyze and this will be generated in another spreadsheet.

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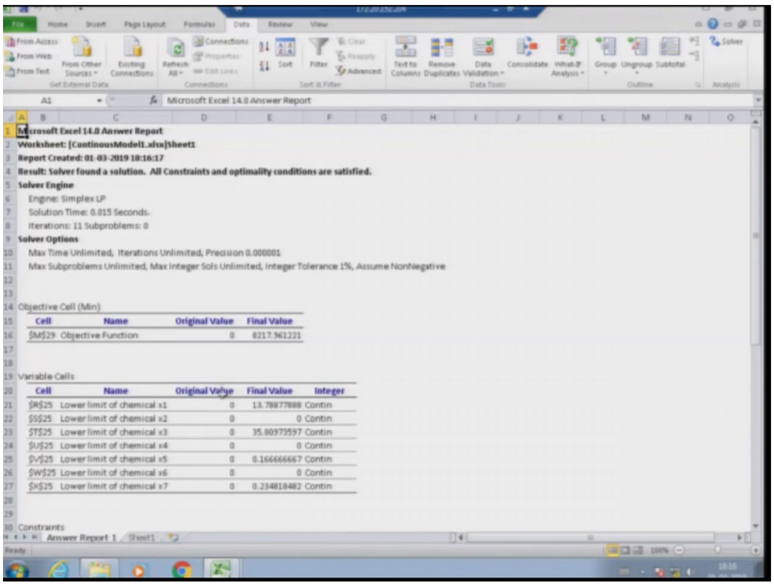

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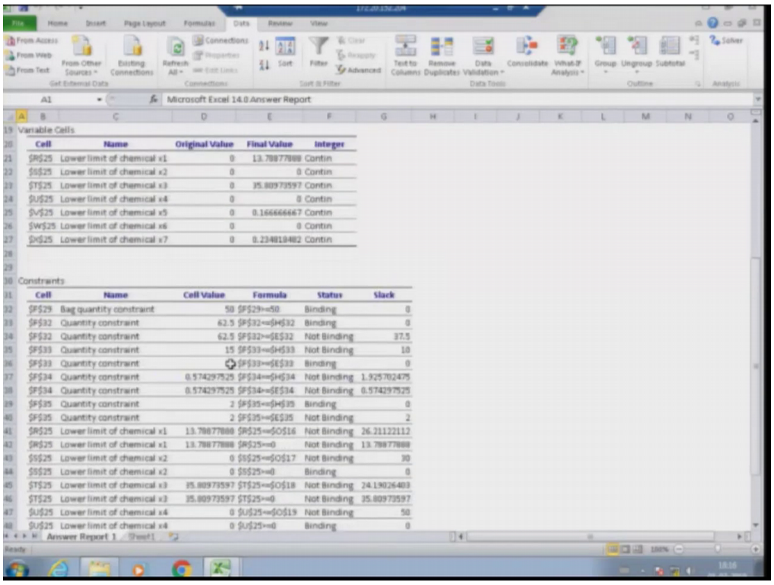

And it is showing that the final value is 8 2 1 7 and all the variables will be given some values. You may be wondering why you are using all these 0 values while encoding in the solver tab because these types of a spreadsheets are future proof. So, if your values of these nitrogen content or any other value like cost or anything will change, you just need to give the different values in these cells and just solve with the button given in the solver and it will evaluate all the different values. You do not need to create these type of spreadsheets from scratch.

You have to create this new spreadsheet when a problem changes, but for the same problem with different data you just need to enter the new data. So, this is how we develop these spreadsheets and we create the models. So, that we can get the optimal solution and this is it for now. Hope you have got the idea of how to encode a spreadsheet from scratch. We will share these spreadsheet and also please be more active in the forums and discuss any problems which you may have in these spreadsheets.

Thank you.