Advanced Green Manufacturing Systems
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Lecture - 19
Solving Optimization Problems

Good evening. Today I welcome all of you to yet another lecture on the topic on the course Advanced Green Manufacturing Systems and you have already been seen through different type of topics and courses and other things and previously I left you guys with the basics of modeling and how do we do use mathematical modeling approach to model optimization problems, so that you can model green manufacturing systems and mathematically and that just parameters according to that.

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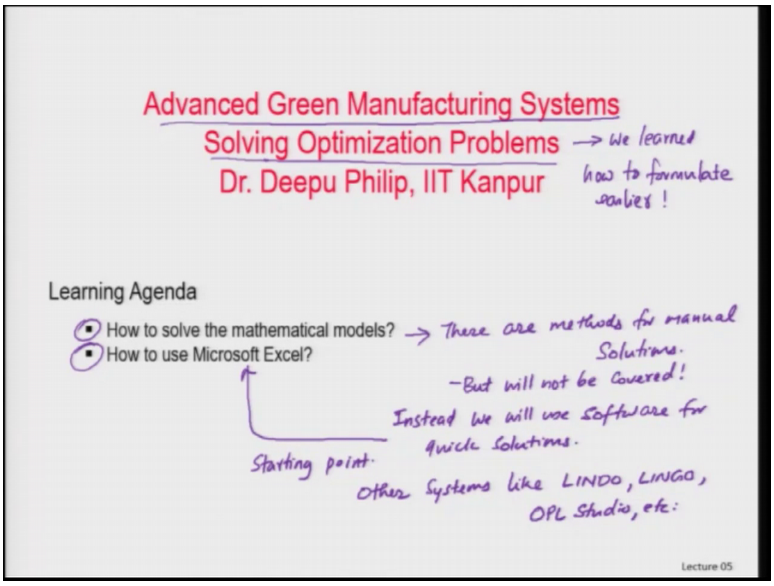

So, today we are going to talk about a topic in advanced green in manufacturing system and this topic is how do are we going to Solve this Optimization Problem that we have been creating. So, we learned how to formulate these problems earlier. Now today we are going to see, how are we going to solve them. So, the two things is how do we solve them and how do we use excel to solve these things. There are methods by hand; methods for manual solutions , but will not be covered. The reason is that the manual solutions are cumbersome and that is not part of this course.

So, instead we will use software for quick solutions. We will start with Excel; Microsoft Excel will be your first starting point. This will be a starting point and you may look into other systems like LINDO, LINGO, OPL Studio etcetera. You may see the mass time permits.

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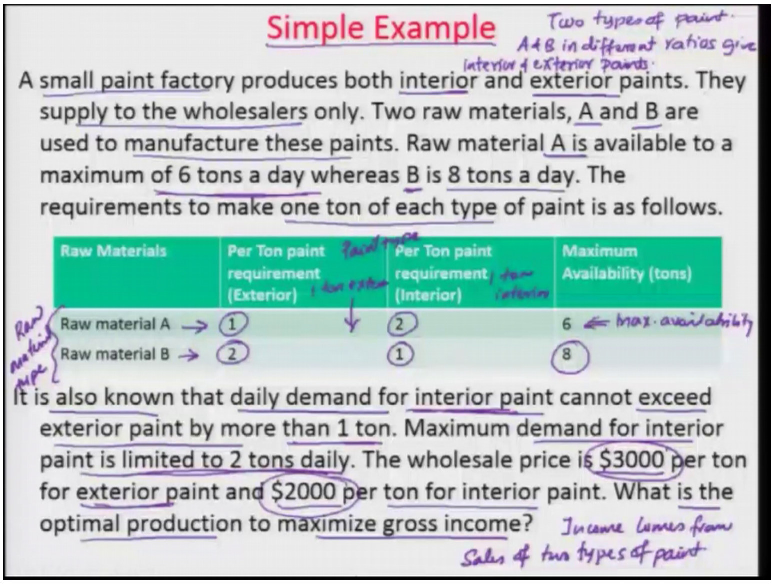

So, let us look at a simple example today; I would like to refresh you guys with a very simple example. The example problem states that a small paint factory produces both interior and exterior paints. So, there are two type of paints. Paint is the one that we use for painting houses and stuff like that. We have interior paints and exterior paints and this company supplies to wholesalers only so, they are wholesale they are not selling in the market.

Two raw materials; two types of raw materials A and B are used to manufacture these paints. So, to making these both type of paints you require raw materials A and B, but in different quantities. So, A and B in different ratios give interior and exterior paints, both interior and exterior paints can be manufactured by mixing A and B in different ratios.

So, raw material A is available to a maximum of 6 tons a day. So, every day you can get at the maximum of 6 tons of raw material A and the raw material B, you can get a maximum of 8 tons a day. So, the maximum available raw material is 8 tons. So, the requirement to make 1 ton of paint; 1 ton of each type of paint, so if you have to make 1 ton of exterior paint so, this is 1 ton exterior, this is 1 ton interior paint. So, to make 1 ton exterior paint, what we require is? We require a raw material, a 1 unit of raw material A and 2 units of raw material B. And in the case of making 1 ton of interior paint, you require 2 tons of raw material, 2 tons of raw material B, ok.

So, for raw material A, it is required in 1 unit; so, if you look at it this fashion, so this is the paint type and this is the raw material type. Raw material A is required in 1 unit; 1 ton for exterior, 2 terms for interior and this is the maximum availability. It is available to a maximum of 6 tons same each with the raw material B and this is the maximum you can get for the raw material B. Then it is also known that or it is also given that the daily demand for the interior paints; the daily demand for interior paints cannot exceed the exterior paint by more than 1 ton.

So, the if the demand for the interior paint is some X value, then X plus 1 ton is the maximum that we can have by the, for the interior paint. The maximum demand for interior paint is limited to 2 tons daily. So, we would not go any more than 2 tons of interior paints per day, that is the maximum we can do. The wholesale price is 3000 dollars per ton of exterior paint and 2000 dollars per ton of interior paints. So, for 1 ton of exterior paints, we will have to pay dollar 3000 and for 1 ton of interior paint, you have to pay 2000 dollars, ok. So, what you are supposed to find is , what is the optimal production to maximize the gross income. So, the company's gross income is; so income comes from sale of paints, sale of two types of paints two types of paint. So, both interior and exterior paints are sold and from there you are able to get this much of money.

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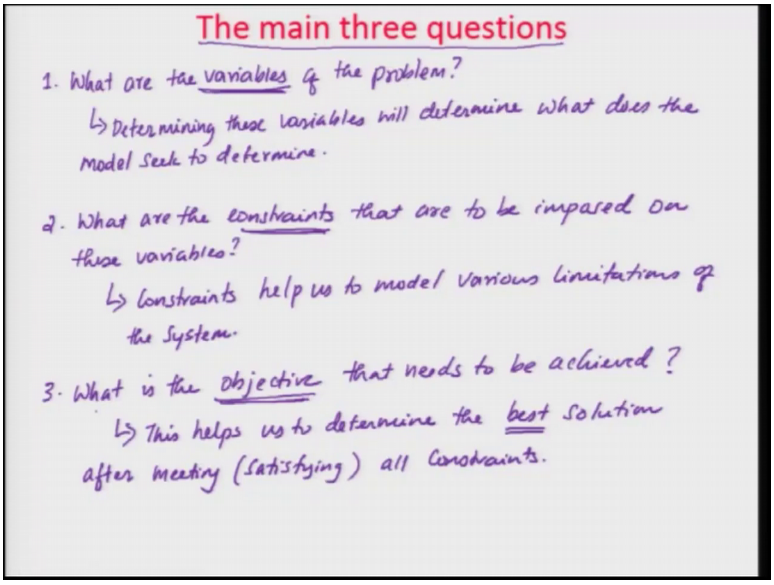

So in this regard, we have two there are 3 main questions I remember telling you guys and what are the 3 main questions that we are going to ask in this problem? The first question we are is what are the variables of the problem ? So, remember we always said the variables is the first thing that you need to decide. Determining these variables will determine; if you determine the variables, then it will determine what will help you determine, what does the model seek to determine.

So, first question is finding what the variables of the problems are or identifying what the variables are. Then the second question we have to answer is what are the constraints that are to be imposed on these variables? So, why do we need to impose these constraints, constraints are imposed, constraints help us to model various limitations of the system. So, what are the limitations within the system is modeled with the help of constraints.

So, what are the constraints that are to be imposed on the variables which will help us to model the limitations on the system and then, the third question what we are to do is, what is the objective; the next question is, what is objective that needs to be achieved and why do we do that? This helps us to determine; by deciding the objective it help us to determine the best solution. Some people call it as an optimal solution, but I would use the word best solution, after meeting all constraints or satisfying all constraints ok. So, what we are actually doing is the first part we need to find out is variables, second is the constraints and third is the objective and these three things actually help us to determine or solve this problem.

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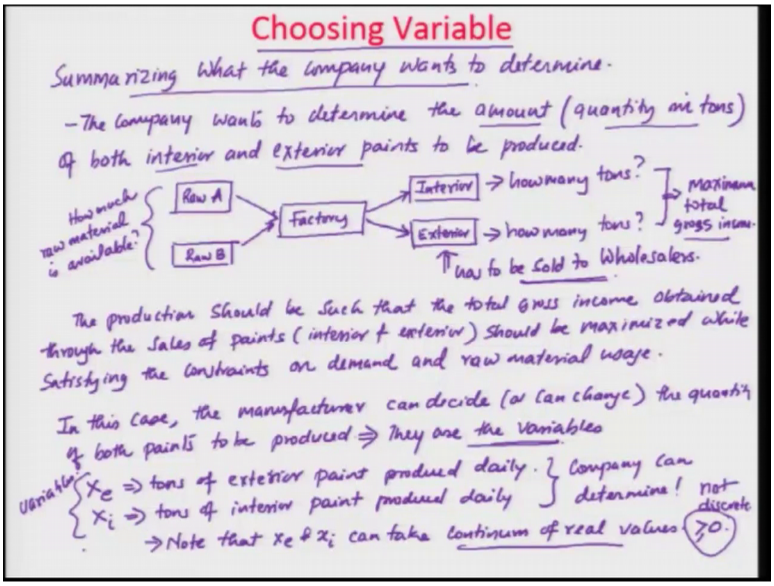

So, first thing first let us choose the variable. So, summarizing what the company wants to determine; that is the variable that we were talking about so, what is the company wanting to determine? The company wants to determine the amount. Another way to think about it is the quantity in tons, the amount means the quantity in tons of both interior and exterior paints to be produced ok. So, what is the company doing actually speaking, you have if you think about it, the company has raw material A, you have raw material B combine them. This is the factory of the company and you have interior paints and you have exterior paints.

The question is how many tons of interior paint, how many tons of exterior paint? So, if you do this, what are we going to do, so that you get the maximum total gross income. So, you want to maximize your total gross income by using these raw materials and then these paints what is that, whatever paint you build has to be sold to wholesalers. So, you how to ensure that you are also meeting the demand in this case, ok. So, you were to find out what is the paint to be produced and the production should be such that the total gross income obtained through the sales of paints, two paints interior and exterior, through the sales of paints, should be maximized while satisfying the constraints on demand and raw material usage ok.

So, you have a constraint here on, how much raw material is available, you know that there is a constraint on that right and then, also there is another thing is on demand that is whatever you produce you are to sell. If you do not sell, then nobody you will not get the money. So, that constraint also need to be satisfied. So, the company wants to know; want to know how much of what is the quantity, how much of the paint, how much of the both interior and exterior paints are to be produced. So, then in our case; then in this case the manufacturer can decide or can change the quantity of both paints to be produced so which implies they are the variables.

So, since this is the things, the two things that the manufacturer is free to do is change the quantities of them, it is easy to understand that they are the variables in this case. So, let us call the first variable as X sub e as the tons of exterior paint produced in it produced daily. So, X e variable determines the tons of exterior paint that is produced daily and X i denotes the tons of interior paint produced daily. So, this the company is free to choose, company can determine. So, since the company can determine that we know that this becomes our variables ok, X e and X i.

X e stands for the exterior paints, X i stands for the interior paints. Note that X e and X i can take continuum of real values ok, remember we suggested that the variables should be of continuous. So, these values X e and X i should be continuum of real values greater than or equal to 0, should be more than 0, ok. So, there helps you to understand and it actually makes there. So, this way it is not discrete, it is not a discrete problem to solve so it is a continuous problem to solve, alright. So, now I hope you guys understand how we decided the variables in this problem.

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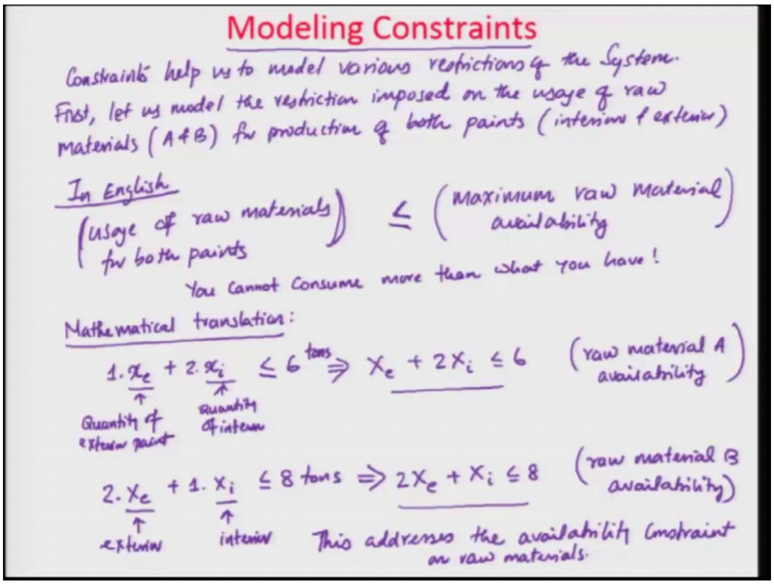

Now, we talk about the modeling of the constraints how are we going to model the constraints. So, there are many restrictions in this problem. So, constraints help us to model various restrictions of the system. So, there are many of them available here. So, first let us model the restrictions or a restriction imposed on the usage of raw materials A and B. Remember there are two type of raw materials for production of both type of paints. There are two type of paints interior and exterior.

So, if we think about this, if we go back in the previous slide, you can see that the raw material requirement for A is 1 ton per exterior paint and 2 tons per interior paints and there is a total of 6 tons available, raw material B is 2 tons per exterior paint, 1 ton per interior paint and there is a total of 8 tons that are available, right. So, if we are trying to model that constraints, then we are going to say that in English I can write it like this usage of raw materials for both paints ok. So, whatever the usage you have should be less than or equal to maximum raw material availability.

So, whatever the raw material is available to you, you cannot consume more than that or in a way, you are actually saying that you cannot consume more than what you have that is what you are actually trying to do. Then mathematical translation if you translate this to math or language of mathematics how will we do that. If you see that 1 unit of exterior paint is required, 1 unit of raw material A is required for exterior paint, 2 units of raw material A is required for interior paints. If you think about it that way and the total availability is 6 tons, if you can write it in a simple way, then you can write it as 1 unit times x e plus 2 units time x i. So, 1 unit is required for the exterior paints how many our tons of exterior paint you are manufacturing, all right. So, this x e is the quantity of exterior paint that you are wanting to manufacture, this is the quantity of interior paint ok.

Whatever you use it, it should be less than or equal to 6 which I can write it in a better fashion as X e plus 2 X i than or equal to 6 which is the raw material A availability ok. So, by modeling this constraint 1 unit of X e and 2 units of X i combined together should be less than or equal to 6 means, it should be less than or equal to 6 tons ok, this is tons ok. Similarly if I write the other one, if you go back you can see that there is 2 tons of exterior and 1 tons of interior paint. So, we understand right that we can cite it as two times X e plus one time X i should be less than or equal to 8 tons, 8 tons this is for the raw material B ok.

So, this is again quantity of exterior this is the quantity of interior. So, this is exterior; this is interior. So, depending upon whatever the quantity that you are going to produce, it is going to give you how to fulfill this constraint. So, then it becomes 2 times X e plus X sub i should be less than or equal to 8. this is the raw material B availability. So, now we know what is the availability of raw material B and we have modeled that is using two different constraints.

So, one of this addresses the availability constraint on raw materials, this is a simpler way problem to understand these kind of things.

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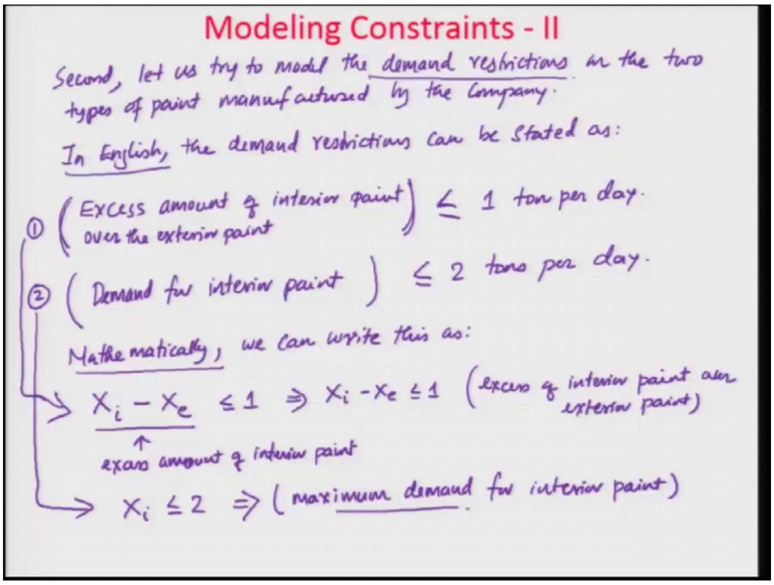

Now, let us talk about the second modeling constraint. Second, let us try to model the demand restrictions on the two types of paint manufactured by the organization; manufactured by the company. So, the company manufactures two type of paints and there are some demand restrictions that are put on to this one.

In English the demand restrictions can be stated as if I say in English how do I write it? I can write it this way, excess amount of interior paint over the exterior paint, the excess amount of interior paint over the exterior paint.

So, if we go back we are trying to know this is the daily demand of interior paint cannot exceed the exterior paint by more than 1 ton, that is what we are trying to model. The interior paint demand cannot exceed more than the exterior paint by 1 ton. So, writing that in English we will say the excess amount of interior paint over the exterior paint should be less than or equal to 1 ton per day so, that is the first thing. So, the excess demand of interior paint over the exterior paint, it should be less than or equal to 1 ton per day, the maximum it can have is 1 ton.

If we go back again then says that the maximum demand for interior paint is limited to 2 tons in a day ok. It said that it is limited to 2 tons a day. So, if we go back and we can say that in English is, so this is one statement, the second statement in English says that demand for interior paint; interior paint should be less than or equal to 2 tons per day, that is the second constraint.

So, mathematically we can write this as; so, now we are writing this in English. Now, we are trying to write this in mathematical conditions. So, we know that the X e or X i is the interior paint. The excess amount of interior paint, how is the excess amount of interior paint we talked about that is X i minus X e. So, X I; the interior paint is larger X e, the exterior paint is smaller. The difference between them X i minus X e this is the excess amount of interior paint should be less than or equal to 1 ok.

So, we can say it as X i minus X e less than or equal to 1 which is the excess of interior paint over exterior paint so, that is the first constraint. So, this excess constraint translates to this one. Now, we write the second one, the demand for the interior paint. So, that is X i demand or whatever the X i that you are producing it should be less than or equal to 2 which means, if you produce more than this; which implies that or which is basically stating you that maximum demand for interior paint. So, that is one way to think about it. So, that means if you produce more than this, then you would not be able to sell because the demand is max than this point ok. So, now we have put two more constraints in this case. So, now we have all together 4 constraints.

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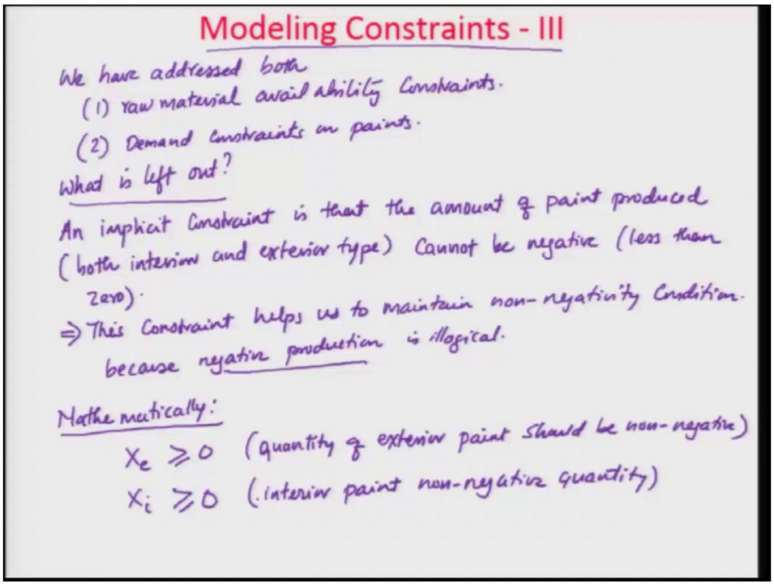

Now, let ue talk about modeling the third set of constraints. So, now we have addressed both one raw material availability constraints and then we have addressed the demand constraint on paints, you have done that, ok. So, what is left out then the problem said only these things right and it only was left out was the price of the paints. So, what is left out ? So, an implicit statement, implicit constraint is that; it is an implicit constraint in this and says that the amount of paint produced, whatever the amount of paint you are producing both interior and exterior type. Whatever the amount of paint you are producing both interior and exterior type cannot be negative or what you are saying less than 0.

So, you can decide not to produce the paint, but you cannot have a negative production right. This constraint helps us to maintain non-negativity condition because negative production is illogical. So, we can say that there is no logic in you saying that I am going to produce a negative quantity so which means mathematically you can say that, mathematically this constraints translate to your X sub e greater than or equal to 0. Amount or quantity of exterior paint should be non-negative that is what this constraint enforces. That whatever the exterior paint quantity that you are producing should be non-negative. The other one is X i should be greater than or equal to 0 which says it is the interior paint non-negative quantity.

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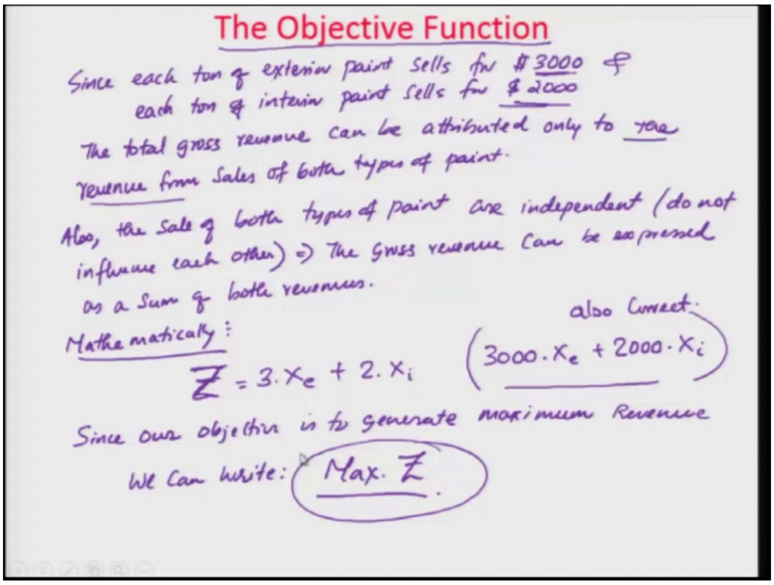

So, we have done both interior and the exterior paint conditions to be non-negative. So, this finishes off our constraints and we get into the last part of this one which is called as the objective function. So, since each ton of exterior paint sells for dollar 3000 and each ton of interior paint sells for dollar 2000.

So, you would like to simply put, you would like to produce more of the exterior paints because you get more money, but you know that there is a limitation to the exterior paint production because there is an availability of the raw material. So, if this is the money that you are going to get from both the paint sales. So then, the total gross revenue can be attributed only to the revenue from sales of both types of paint.

So, since we are only producing paint and we are selling it to wholesellers, this is the only one way for us to get revenue from the; revenue to the firm, right. Also the sale of both types of paint are independent what does that means? Do not influence each other. Since that is the case, the gross revenue can be expressed as a sum of both revenues.

So mathematically, what we are doing here is say it does Z equal to you are getting 3 times the revenue from the sale of exterior paints plus 2 times revenue from interior paints. You can also write it, if you want you can think about it just writing it as 3000 times X e plus 2000 times X i. This is per ton that we are talking about right.

Since in our previous case, we only did in if you think about it here we just used 6 and other kind of things per ton. So, it is the ratio it is fine, if you want to put 3000 this is also correct, but since to make it easy to solve we just simply can use it on these thousands can be taken out of it ok.

Since our objective is to generate maximum revenue, we can write; what can we write, maximize Z. So, your objective is to maximize the Z function which is 3 times the exterior paint plus 2 times interior paint, where we are getting it from the this value or it can be written also this fashion. So, the max Z is the objective, maximize the function so if you think about it, what we have done so far the complete formulation of this problem.

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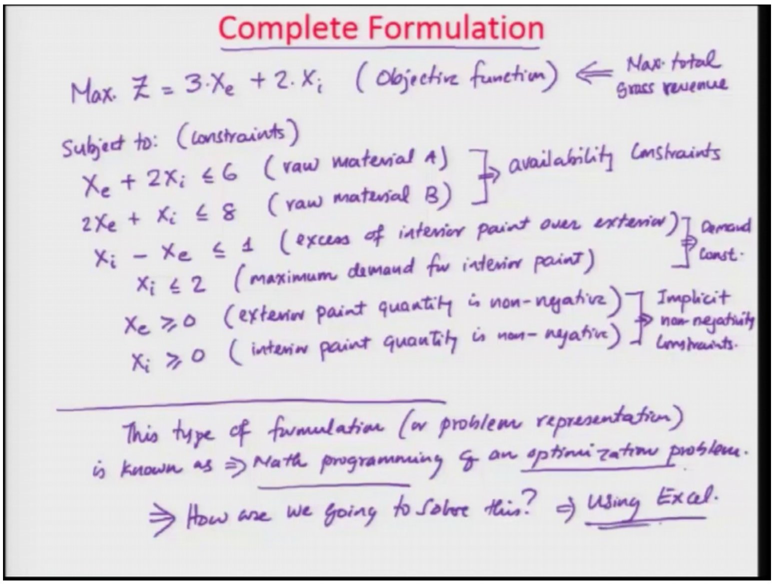

So, let us write the complete formulation. The maximize Z which is equal to 3 times X e plus 2 times X i this was our objective function. And we are saying that subject to our aim is to maximize this so, subject to implies the constraints; what are the constraints of this model ideally speaking. So, the first constraint let us put it is, so if we go back in this scenario we had these two constraints; the raw material usage constraints, the quantity of raw material A and quantity of raw material B.

So, if you write that then we say that X e plus 2 times X i will be less than or equal to 6 which was for the raw material A, then 2 times X e plus X i less than or equal to 8, this was the raw material B constraint. So, this constraint was modeled as the initial modeling that we did where the usage of the raw materials for both paints should be less than the maximum availability of the raw material. So, that is the first set of constraint.

Second set of constraint that we modeled was a demand constraint, where is the difference in the demands X i minus X e should be less than or equal to 1 and the maximum demand of interior paint cannot be more than or equal to 2. If we do that, then we can write that also as the next set of constraints which we say X i minus X e should be less than or equal to 1 which is the excess of interior which is of interior paint over exterior. So, that is the first part that we are trying to say, then we are saying X i is less than or equal to 2 where we are saying maximum demand for interior paint that constraint. So, where we are actually saying is that the maximum demand for interior paint cannot be more than 2 tons per day and the excess amount of interior paint over the exterior paint should be less than or equal to 1 ton per day.

So, in each day the interior paint can the maximum amount you can have on an interior paint is only to 1 ton per day and the interior paint demand is limited to upper limited to 2 tons per day. So, we think about it then that are the two constraints that we put it right. Then the last set of constraints that we are going to add is the X e greater than or equal to 0 which means exterior paint quantity is non-negative.

And you are also saying that X i greater than or equal to 0 is the interior paint quantity is non-negative. You can understand that, we mentioned these constraints which are the implicit constraints. We saying that there is no the reason this is an implicit constraint is because the negative production; quantity of producing a negative quantity of paint does not make any sense. So, that is your non-negativity constraints and that those are the ones that are added here.

So, you can say that there are altogether you know this total thing that we just wrote it down is called as a people this type of formulation or problem representation; we just formulated or we represented the problem representation is known as what we call this is known as math programming, some people call this as mathematical programming of a of an optimization problem.

So, how do we take an optimization problem and do the math programming say, again a simple problem compared to the other things. This problem is pretty simple where we have an objective function. The objective function is to maximize the revenue; maximize total gross revenue and we told about this total gross revenue where we are getting it and subject to the constraints where the raw material A and raw material B, these were the availability constraints and these two are the, what we call as the demand constraints and implicit non-negativity constraints.

So, these three set of constraints is what we are actually looking into this problem. So, we giving this, we are all together have six set of constraints. Now the question is and we said this whole thing is called math programming. Now we are going to ask is how are we going to solve this. There is graphical methods and many other things and the answer to this is using Excel; Microsoft Excel. This is one way to solve this problem and how do we use Excel to solve this is the next part of this lecture. So, we will continue this lecture through show you how we use Excel to solve this problem.

Thank you.