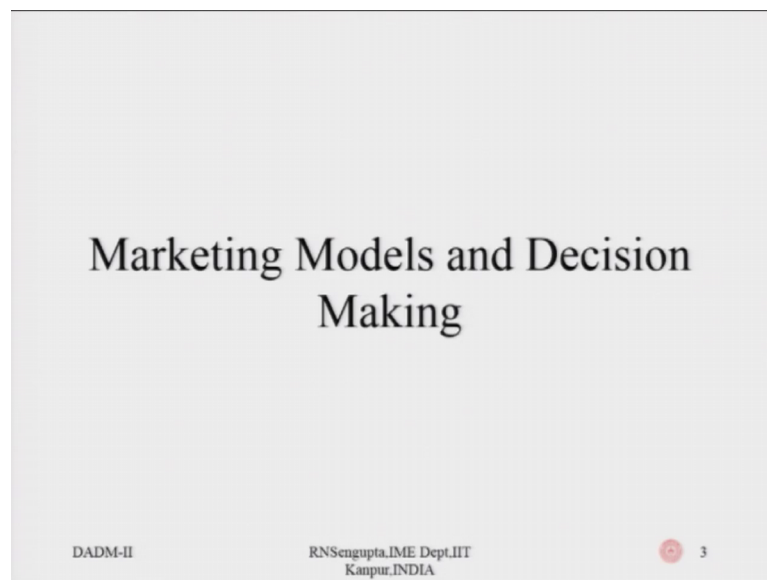


**Data Analysis and Decision Making - II**  
**Prof. Raghu Nandan Sengupta**  
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**Department of Industrial & Management Engineering**

**Indian Institute of Technology, Kanpur**  
**Lecture – 50**  
**Bass Model**

Good evening everyone. I am Priyanka Sharma, PhD student, IME, IIT Kanpur. I am working under Prof. Raghu Nandan Sengupta who is the instructor of this three module course and this is DADM part 2.

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So far sir has discussed different decision making models which come from statistics, probability, utility theory and other decision making techniques. What we are going to cover in this lecture is how quantitative methods are utilized in making some of the marketing decisions. So, in the previous lecture, we discussed about using Bayesian approach in predicting demand in making predictions about what should be the optimal pricing strategy before we launch a product in the market. And we also touched upon some of the purchase and incidents related models as far as choice models are concerned or brand choice models are concerned in the marketing domain.

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**New Product Launch**

- Before a new product launch a firm estimates its future sales, profits, and impact on the firm's objectives.
- A new products is adopted by customers/ people who initially do not know about it
- *Diffusion Process*
- *Adoption Process*

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This particular lecture we are going to cover new product launch. And to understand how the demand will vary for any particular product which is launched in the market through a model which is very widely known as the Bass model. It was developed by Norton and Bass and hence henceforth we it is termed as Bass model after the person who originated that.

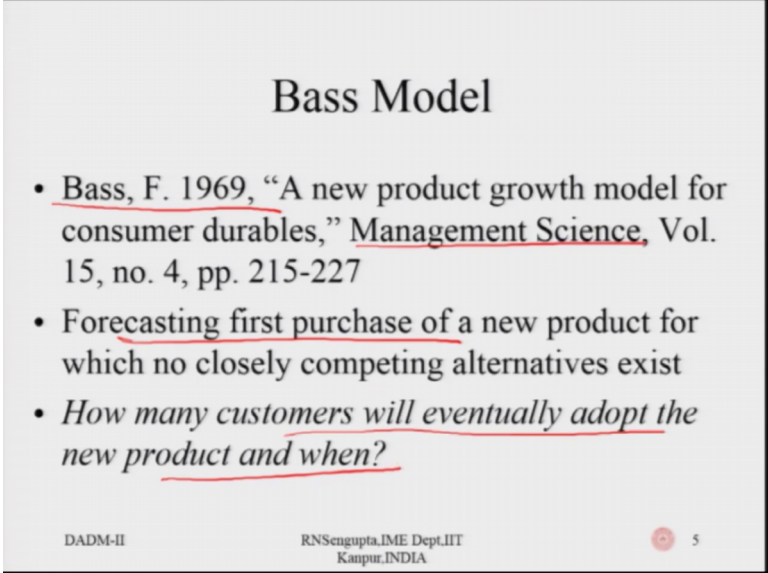
So, before a firm launch a product, they need to estimates its future sales, profits and what is the impact on the firm's objectives. So, every firm has a particular vision and mission so, the product that they are going to launch in the market or the services that they offer it has to be in line with those objectives. Because people associate a company with certain values, certain attributes, certain associations, and if the product reflect something else, then there is a dissonance in the minds of the consumers that this product is authentic, it will perform well as per the associations that they make with respect to the corporate name and the management team and other similar products that the company might have launched in the past.

So, a new product is adopted by customers or people who initially do not know about it, so that is the basic fundamental understanding about new product launch that people do not know about this product, they do not know how to use that products. So, it is the companies prerogative to teach them what this product is going to offer, what problems

that are customer faces will be solved by this particular product that the company launches in the market.

So, for new product adoption, there are two terms which are very widely used in the marketing literature, one is diffusion process and one is the adoption process. Diffusion process is from the firm side how the product diffuses or percolates or penetrates the market over a period of time. And adoption process is on the other side of the market which is the consumer side of the market which talks about how each and every consumer adopts the product in different time period.

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**Bass Model**

- Bass, F. 1969, "A new product growth model for consumer durables," *Management Science*, Vol. 15, no. 4, pp. 215-227
- Forecasting first purchase of a new product for which no closely competing alternatives exist
- How many customers will eventually adopt the new product and when?

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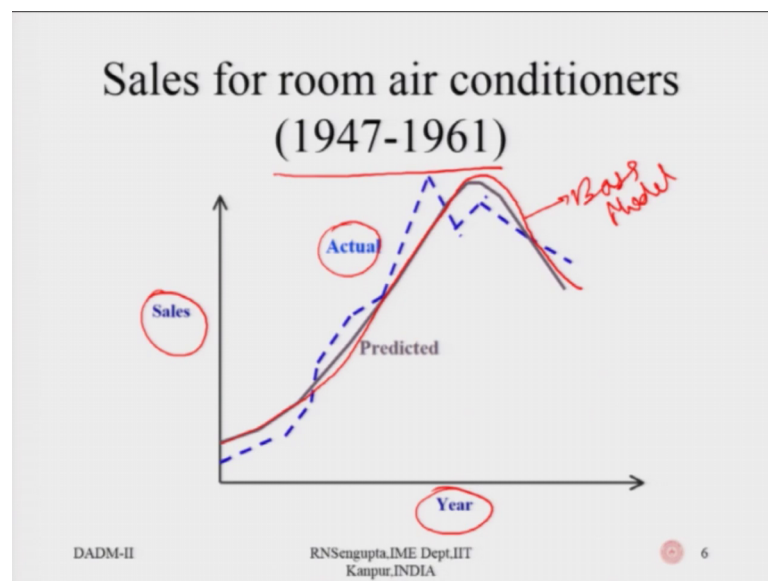
So, accordingly, we actually categorize consumers into different buckets, to make it easy to understand, we will keep it into two forms like innovators and imitators. Innovators are those set of consumers which are very innovative they are lightly to adopt the product based on the advertising or the marketing communication that firm has done, and they like to try out different new products in the market. So, they do not refer to any friend or they do not look for any experts to recommend that product, but they have their intrinsic motivation to try out new products which are launched in the market.

The other set of consumers which are known as imitator they actually bank on innovators plus other set of customers who would have tried that product, to give them a recommendation that, yes, this is worth trying and they should adopt and purchase this product.

So, Bass in 1969 came up with the paper in Management Science, which is of very highly cited paper and there he gave this model which is termed as Bass model. So, what it does it, it forecast the first purchase of a new product for which no closely competing alternatives exist. So, we assume that there are no substitutes for this particular products, yes; there are different variations which of now come up which also take into consideration the substitution effect, game theoretic effect and so on and so forth, but to make it easy to understand we are going to study the standardized Bass model.

And it aims to answer the question how many customers will eventually adopt a new product and when? Because as a firm you have to allocate budgets to your development and commercialization of a product; so unless and until you know that there is a sustained demand in the market, you may end up in a loss. Because a lot of these products which are launched in the market they fail to generate the necessary demand and then they are likely to be shelved by the companies.

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So, what is the outcome of a Bass model? For example, this is the sales for room air conditioners. Ok based on data I have taken this from a paper from literature, so the sales versus year. So, you can actually see the dotted lines are the actual sales and this smooth curve which you see this is actually predicted by the Bass model. So, you see this is more or less similar to the actual sales that the AC market saw in this particular time period.



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### Assumptions

- Diffusion process is binary ✓
- Constant maximum potential number of buyers ( $m$ )
- Eventually, all  $m$  will buy the product
- No repeat purchase, or replacement
- The impact of the word-of-mouth is independent of adoption time
- No substitute effect

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So, there are certain assumptions like in any other statistical or decision making models, we have certain assumption in the Bass models that diffusion process is binary. So, either you adopted or you do not adopt that particular product and there is a constant maximum potential number of buyers. So,  $m$  is the maximum number of buyers for that particular product, eventually all  $m$  will buy the product.

So, we assume that all of these customers are going to purchase that products. So, basically if you draw a cumulative sales diagram, so it would look like this. So, over time all of these customers will purchase. So, this is the probability of purchasing that particular product. There is no repeat purchase or replacement of the product, the impact of the word of mouth is independent of the adoption time and there is no substitute effect which I already discussed about.

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### Mathematical Formulation of the Bass Model

- $p$  = Coefficient of innovation (or coefficient of external influence)
- $q$  = Coefficient of imitation (or coefficient of internal influence).
- Then:

Number of customers who will purchase the product at time  $t$  =  $p \times \text{Remaining Potential} + q \times \text{Adopters} \times \text{Remaining Potential}$

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So, the mathematical formulation of the Bass model there are two important decision factors in developing the Bass model one is  $p$ , one is denoted as  $q$ .  $p$  is the coefficient of innovation or external influence and  $q$  is the coefficient of imitation or coefficient of internal influence. And the number of customers who will purchase the product at time  $t$  is  $p$  multiplied multiply by the remaining potential plus  $q$  multiplied by adopters multiplied by the remaining potentials. So,  $p$  are the ones who adopt on their own,  $q$  are the ones who are actually influence by other adopters and word of mouth to be able to purchase that product in the next time period.

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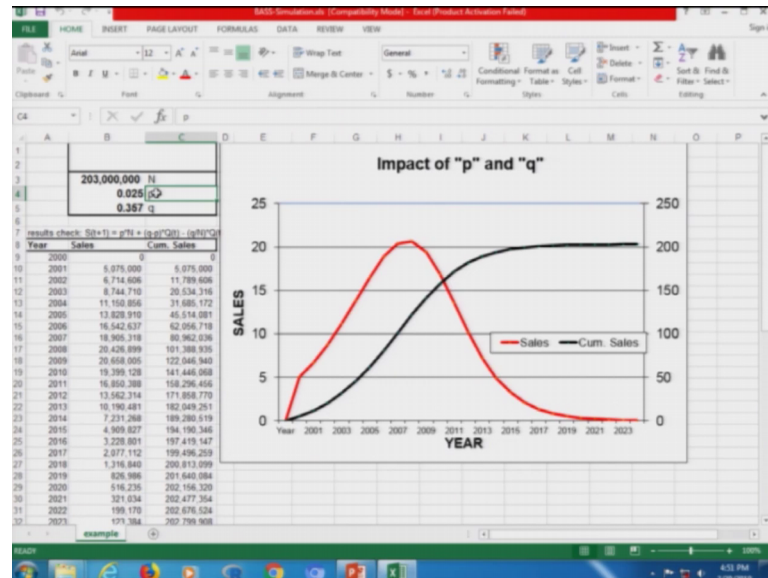
### Excel example

- [BASS-Simulation.xls](#)

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So, you can use programming in r you can use even excel sheet to actually come up with this model. So, I will show you how it is done in excel sheet.

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So, here I have this excel sheet this is a simulated data. So, we need not worry about the authenticity or it does it actually represent any product, but I just wanted to tell you how the model would look like or the outcome would be. So, I assume that there is market size, there is some number of customers which is the maximum number of customers that would ultimately purchase this particular product. And I will use for p and q.

In general the average values for p would be 0.38 and p would be somewhere around 0.02, but it actually varies with different sector. For example, for high tech sector for electronics and for say microprocessor chips the value of p could be 0.001 as well. So, it can vary from different values, but it is always greater than 0; it is not less than 0.

So, here you can see I have plotted the sales for different years; so, in 2000, I launched the product, so there was 0 sales cumulative sales is 0. But then as we move on I saw that the sales have increased. So, the their sales is actually the number of adopters in that particular time and this is coming from the equation of the Bass model that how; which is dependent on both p and q. So, if I change the value of q here say to 0.23 and enter then you can see that there is a change in the graph of sales versus year.

So, what it means is that, the firm plots this graph which is sales versus time and it looks something like this which we typically call as the S curve. So, this looks like S. So, we call it as the S curve. We want to understand at what time the sales would be maximum, because after that as you see the sales start falling. So, there would be decrease in the revenue for the firm, so that is the strategic decision point for the firm to understand whether they want to retain the product or there is a time that they come up with a new product.

So, you can start with actually another S curve. So, a firm which is like an innovator type of firm, you will find actually multiple S curves originating at different times as soon as they understand that this is the time at which I have reached the maximum sales for this particular product. So, that is the time that you come up with something new, a new variant of either in the same category or in the different category to stay in the business.

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## Bass Model

- Suppose  $N(t)$  = cumulative number of adopters,
- $m$  is the ceiling for market size,
- $p$  is the coefficient of innovation, and
- $q$  is the coefficient of imitation

$$\frac{dN(t)}{dt} = \left( p + \frac{q}{m} N(t) \right) (m - N(t))$$

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So, now we try to understand the mathematical derivation for this particular model. So, if  $N(t)$  is the cumulative number of adopters and  $m$  is the ceiling for the market size, and  $p$  and  $q$  we have already understood these are the different coefficients of innovation and imitation. So, the rate of adoption at some time  $t$  is given as an equation  $p$  plus  $q$  by  $m$   $N(t)$  multiplied by  $m$  minus  $N(t)$ . This was given by Bass in that management science paper.

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### Bass Model (Contd...)

- Considering  $F(t) = N(t)/m$ , where  $F(t)$  is the fraction of potential adopters who adopt the product by time  $t$

$$\frac{dF(t)}{dt} = (p + qF(t))(1 - F(t))$$

- $N(t = t_0 = 0) = 0$

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And to solve it further, we consider that  $F(t)$  is given as  $N(t)$  by  $m$  and  $F(t)$  is the fraction of potential adopters who adopt this product by time  $t$  so, based on that they come up with this equation.

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$$N(t) = m \left( \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \right)$$

- $[dN(t)/dt]$  gives point of inflexion/max demand

$$N(t^*) = m \left( \frac{1}{2} - \frac{p}{2q} \right),$$

$$t^* = -\frac{1}{p+q} \log \left( \frac{p}{q} \right),$$

$$f(t^*) = \frac{dN(t^*)}{dt} = m \left( \frac{q}{4} + \frac{p}{2} + \frac{p^2}{4q} \right)$$

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If you solve that equation finally you can find out the number of adopters at time  $t$  which is equivalent to the sales at that point  $t$ . And as you see it is dependent on the total number of customers that would ultimately purchase that product, coefficient of innovation, coefficient of imitation and the time.  $dN(t)/dt$  gives you the point of

inflexion or maximum demand. So, if you differentiate that and put it to 0, so this gives you the maximum demand at that particular time  $t^*$ , this is the value of  $t^*$ .

What is this  $t^*$ ? When you draw this, the point at which the demand is maximum, so this is the  $t^*$  value and this makes the  $N t^*$  value. So, the firm is interested in finding out at what time from now the product is going to edge or it is going to decline or it is going to lose start losing market share. So, I have to plan my advertising and budgeting accordingly.

Because if you see if for any S curve, this set of customers where you see a very sluggish growth in the market share, these are the innovator type of customers. This set of customers will fall under the imitator or follower type of customers and to stay in the market, you need a very high chunk of imitator type of customer, so that you can stay in the market for a long time.

However, their adoption is contingent on the innovator type of customer. So, you need a thrust from both the ends. And there are different marketing strategies which are adopted. For this particular part, when you have actually launch just launch the product to the point where you are seeing a growth in the lifecycle of that particular product.

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**p and q – Practical Implication**

- For low values of  $x=q/p$ , an increase in q increases the time to sales peak, and for high values of  $q/p$ , the time decreases with increasing q.

$$t^* = \frac{-1}{p+q} \ln(p/q) = \frac{\ln(q/p)}{p+q} = \frac{1}{p} \frac{\ln(q/p)}{1+q/p} = \frac{1}{p} \frac{\ln(x)}{1+x}$$

$$\frac{\partial t^*}{\partial x} = \frac{1}{p} \left[ \frac{1}{x(1+x)} - \frac{\ln x}{(1+x)^2} \right] = \frac{1+x-x \ln x}{px(1+x)^2}$$

3.59  
 $x > 1$   
 $1+x-x \ln x$   
 $x=3.59$

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So, what are the practical implications of  $p$  and  $q$  for the firm? So, we calculated the value of  $t^*$  right earlier. Now, what we do is that I am just replacing this  $q$  by  $p$  with

the value  $x$ , and I am trying to differentiate this with respect to  $x$  and I get this equation. So, this particular equation  $1 + x - x \ln x$ , where  $x$  is greater than 1; the root for this equation is  $x$  equal to 3.59. So, basically what it means is that for low values of  $x$  or  $q$  by  $p$  ratio, an increase in  $q$  increases the time to sales peak, and for high values of this the time decreases with increasing  $q$ .

What it means for the firm is that the product that you are going to launch and the category in which it is going to compete, you need to understand what is the relative values of  $q$  versus  $p$ . Because accordingly the slope will change and how sooner the product will reach the maximum demand or how much more time is it going to reach the maximum demand depends on the relative values of  $p$  and  $q$ .

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### Parameter estimation

- The ordinary least squares procedure

$$\frac{dN(t)}{dt} = \left( p + \frac{q}{m} N(t) \right) (m - N(t)),$$

$$N(t_i) - N(t_{i-1}) = pm + (q - p)N(t_{i-1}) - \frac{q}{m} N^2(t_{i-1}),$$

$$X(i) = \alpha_1 + \alpha_2 N(t_{i-1}) + \alpha_3 N^2(t_{i-1}),$$

$$\alpha_1 = pm, \alpha_2 = q - p, \text{ and } \alpha_3 = -q/m$$

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Now, how do you estimate these parameters? You can estimate these parameters through ordinary least square procedures, maximum likelihood and means; few other techniques. So, this is the basic Bass model equation for the rate of adoption and based on that you calculate the number of adoption at any point  $t$  which is given by  $N(t)$ . You can reformulate this equation to come up with this point of quadratic equation, which is squared in  $N(t_{i-1})$ . And you just substitute say  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  for  $p$  into  $pm$  minus  $p$  and minus  $q$  by  $m$ .



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$$\hat{p} = \frac{-\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2},$$

$$\hat{q} = \frac{\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2}, \text{ and}$$

$$\hat{m} = \frac{-\hat{\alpha}_2 - \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2\hat{\alpha}_3}.$$

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And finally, you can just solve it as a quadratic equation to find out p hat, q hat and m hat. So, this is one way to solve this equation to find out the p, q and m for the product that you are going to launch.

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	Quarter	Sales	Cumulative Sales
Market Size = 16,000	0	0	0
	1	160	160
Innovation Rate = 0.01 ✓ (Parameter p)	4	425	1,118
	8	1,234	4,678
Imitation Rate = 0.41 ✓ (Parameter q)	12	1,646	11,166
	16	555	15,106
	20	78	15,890
	24	9	15,987
	28	1	15,999
	32	0	16,000
	36	0	16,000

*Example computations*

$n(t) = [p + (q/m)N(t-I)][m - N(t-I)] = pm + (q-p)N(t-I) - qN^2(t-I)/m$

Sales in Quarter 1 =  $0.01 \times 16,000 + (0.41 - 0.01) \times 0 - (0.41/16,000) \times (0)^2 = 160$

Sales in Quarter 2 =  $0.01 \times 16,000 + (0.40) \times 160 - (0.41/16,000) \times (160)^2 = 223.35$

Just see for example, you have different yearly, quarterly sales and cumulative sales for a particular product that you have observed for certain time. And you know that innovation rate is this and imitation rate is this, and you assume certain market size. So, what you can do is, you can calculate the number of adopters at some time t using this equation.



So, in this equation, you know  $p$ , you know maximum market potential, you know how many adopters were there prior to that time period. So, you can utilize this equation to actually find out the  $N_t$ .

So, for example, here sales in quarter 1 is 0.01, which is  $p$  multiplied by 16000 which is your  $m$  and then you have 0.41 which is  $q$  and then minus 0.01 this is  $p$ . And similarly, so if you get a value of 160. Similarly, you can calculate the sales for different quarters, and this is how you make predictions as to how the demand would be for this particular product for say 10 years or 15 years and what you can do is you can accordingly plan your marketing and budgeting skills.

And you can also see that if a product is going to reach the maximum in demand in say just 5 years of time, so is it advisable to invest into that product development and that is also something question that the firm needs to answer. So, there has to be a good time horizon for the product to stay in the market and earn revenues for the firm.

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**Determine  $p$  and  $q$**

- 1. Calculate  $p$  and  $q$  from an analogous product's past sales
- 2. Calculate  $p$  and  $q$  from various analogous products whose  $p$  and  $q$  estimates are provided.

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So, there are other different value a ways in which one can calculate  $p$  and  $q$  you can calculate  $p$  and  $q$  from an analogous products past sales. And you can calculate  $p$  and  $q$  from various analogous products whose  $p$  and  $q$  estimates are provided. So, when we say analogous products, it could be a product in the similar category which other firm has launched earlier or if it other firms; so, several other could be many other firms have

launched that products similar product and you are the firm which is launching that product for the first time. So, you can take the p and q values of these other products.

I would not say these are the alternatives or these are the substitutes, because when we say a product is new to the market we assume that there is no perfect competitive product which is there in the market, but you take hints from other similar products. For example, if you are launching CT scanner then maybe you can take help of say how the ultrasound machines have done in the market. So, if there was no other CT scanner prior to this and you are trying to launch it now then you need to take hint from other similar medical equipments which are there in the market to make predictions for your case.

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Year	Sales per year	Cumulative sales	Average price per unit (\$)	% change average price per unit	Sales revenue (\$ millions)
1998-1999	2711	2711	9700	1	26.3
1999-2000	3556	6267	11950	1.188284519	42.5
2000-2001	3893	10160	12251	1.2082279	47.7
2001-2002	4400	14930	14500	1.331034483	63.8
2002-2003	6600	22085	14500	1.331034483	95.7
2003-2004	6925	28712	14150	1.314487633	98
2004-2005	5680	35093	15900	1.389937107	90.3
2005-2006	6792	41885	17000	1.429411765	108
2006-2007	6693	48578	17000	1.429411765	111.1
2007-2008	4495	54642	21000	1.538095238	94.5
2008-2009	6613	61255	17374	1.441694486	114.9
2009-2010	7069	68324	19000	1.489473684	135.1
2010-2011	7906	76230	21000	1.538095238	161.2

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So, for example, this again I have taken from a literature. So, there is some system for which the sales and revenue data is available for different years, what is the sales cumulative sales and revenue which is the product has earned over the years.

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- Draw the regression line
$$S = 3495 + 0.1153C - 0.000001C^2$$
- where  $S$  is the sales per year, and  $C$  is the cumulative sales per year.
- Equate this to Bass model
$$n(t) = a + bN(t-1) - c(N(t-1))^2$$
- coefficient  $a = 3495$ , the coefficient  $b = 0.1153$ , and the coefficient  $c = -0.000001$

$$p = \frac{a}{m}$$
$$m = \frac{-b - \sqrt{b^2 - 4ac}}{2c}$$
$$q = p + b$$

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So, what you do is you draw a regression line. So, you draw regression line based on this where  $S$  is the sales per year and  $C$  is the cumulative sales per year. It would come out to be something like this. And you equate it to the Bass model equation which is given as a plus  $bN(t-1) - c(N(t-1))^2$ . And you can compare that the similarity between the different coefficients over here.

So, once you know these coefficients, you can actually calculate  $p$ ,  $q$  and the maximum market potential which the product will have based on these equations. So, this is one way to calculate  $p$  and  $q$  once you have the sales data for a similar product in the same or a different category.

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**Calculate  $p$  and  $q$  from analogous products whose  $p$  and  $q$  estimates are provided**

- three different products whose functions and features were similar to our system were identified

Medical device	Analysis period	Weight	$p$	$q$
Ultrasound imaging	1965-1977	1	0.001 ✓	0.51 ✓
Mammography	1965-1976	1	0.000 ✓	0.738 ✓
CT scanners (>100 beds)	1974-1985	2	0.034 ✓	0.254 ✓
<b>Weighted average</b>			<b>0.017</b>	<b>0.439</b>

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If you have a many such products whose  $p$  and  $q$  and you know then what you can do you can do a weighted averaging to find out the  $p$  and  $q$  for your particular case. So, in this particular case, I have a ultrasound imaging, mammography, CT scanners, I have  $p$  and  $q$  value for each of these three products which are already there in the market. And I give certain weights to these  $p$  and  $q$  values and I calculated a weighted average.

Now, how I give a weight 2 to CT scanners, because it is a much more recent product. So, I tend to believe that the behaviour of product which I am going to launch in the market is more closely related with the behaviour of this particular product which is recently launched. And as you see the other two products which were launched much earlier than this particular product, so I am more inclined to give a higher weight to this recent product.

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## Extensions to Bass Model

- So far a constant ceiling for the number of potential adopters  $m$
- Sharif and Ramanathan (1981)
- Dynamic model

$$\frac{dN(t)}{dt} = \left( p + \frac{q}{m(t)} N(t) \right) (m(t) - N(t)),$$

$$m(t) = m_0 e^{gt}.$$

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Now, there are certain extensions to the Bass model. We first of all we consider the constant ceiling for the maximum number of customers which is  $m$ , then later on there were certain modifications and certain researchers said that market potential is a function of time and it should not be considered as a constant number.

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## Ordinary Least Squares Estimation

$$N(t_i) - N(t_{i-1}) = pm + (q - p)N(t_{i-1}) - \frac{q}{m}N^2(t_{i-1}),$$

$$X(i) = \alpha_1 + \alpha_2 N(t_{i-1}) + \alpha_3 N^2(t_{i-1}),$$

$$\hat{p} = \frac{-\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2},$$

$$\hat{q} = \frac{\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2},$$

$$\hat{m} = \frac{-\hat{\alpha}_2 - \sqrt{\hat{\alpha}_2^2 - 4\hat{\alpha}_1\hat{\alpha}_3}}{2\hat{\alpha}_3}.$$

where  $\alpha_1 = pm$ ,  $\alpha_2 = q - p$ , and  $\alpha_3 = -q/m$ .

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So, if you plug in that value of  $m$ , you get a different set of equations which if you use ordinary least square estimation, you can again find out the  $\hat{p}$ ,  $\hat{q}$  and the  $\hat{m}$  hat considering that, so that was one modification that you can do.

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### Adding Marketing Variables

$$X(t) = \int_0^t x(\theta) d\theta.$$

$$F(t) = \frac{1 - e^{-(p+q)X(t)}}{(q/p)e^{-(p+q)X(t)} + 1}, \text{ and}$$

$$f(t) = \frac{(p+q)^2}{p} x(t) \frac{e^{-(p+q)X(t)}}{[(q/p)e^{-(p+q)X(t)} + 1]^2}.$$

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Another modification that came recently is use of marketing other marketing variables. For example, pricing, advertising or any other management decision variable that can affect the demand and adoption of that particular product. So, we take that capital  $X(t)$  is actually that particular decision variable; and it is a summation of many such decision variables.

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### Bass and Srinivasan (2002)

- Generalized Bass Model (GBM) proposed by Bass, Krishnan and Jain (1994)
- Hazard function:
- $f(t)/[1 - F(t)] = x(t)[p + qF(t)]$
- We consider price and advertising
- Where  $Pr(t)$  is price at time  $t$  and  $ADV(t)$  is advertising at time  $t$

$$x(t) = 1 + [\Delta Pr(t)/Pr(t-1)]\beta_1 + [\Delta ADV(t)/ADV(t-1)]\beta_2$$

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So, this is something termed as Generalized Bass Model the one we discussed earlier where we had constant market potential that was the standardized Bass model that was

the first one to be launched in 1969. And later on in 1994, Bass along with Krishnan and Jain, he came up with this generalized with Bass model idea. So, he says is that it is the likelihood that a customer will purchase product that sometime  $t$  given that he has not purchased that product until that time  $t$  is given by this equation, where  $x_t$  is the management decision variable. And in for example, to understand, we consider price and advertising as two such decision variables. So in that case, this  $x_t$  will be given as rate of change of price multiplied by beta 1 and rate of change of advertising multiplied by some other beta 2.

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### GBM

- For continuous time  

$$x(t) = 1 + [dPr(t)/dt/Pr(t)]\beta_1 + [dADV(t)/ADV(t)]\beta_2.$$

$$X(t) = \sum_{\tau=0}^t x(\tau).$$

$$X(t) = t + [Ln(Pr(t)/Pr(0))]\beta_1 + [Ln(ADV(t)/LnADV(0))]\beta_2$$
- The adoption rate (Sales at time  $t$ ) for GBM is  

$$S(t) = m((p+q)^2/p)x(t)e^{-(p+q)X(t)}/(1 + (q/p)e^{-(p+q)X(t)})^2$$

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We use this particular  $x_t$  and we come with the adoption rate for generalized Bass model as this where,  $x_t$  is given as the rate of change of price and rate of advertising. And this  $X_t$  is nothing but a summation of all such expenses over a period of time.



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## Fruchter and Van den Bulte (2011)

- GBM

$$\dot{F}(t) = [p + qF(t)][1 - F(t)]x(t), \quad F(0) = 0$$

$$x(t) = 1 + \alpha \frac{p(t)}{p(t)} + \beta \frac{\dot{a}(t)}{a(t)}, \quad \alpha \leq 0, \beta > 0,$$

$$p(t) = ve^{-\gamma t}, \quad \gamma \geq 0.$$

$$x(t) = 1 - \alpha\gamma + \beta \frac{\dot{a}(t)}{a(t)} = k + \beta \frac{\dot{a}(t)}{a(t)}, \quad k = 1 - \alpha\gamma \geq 1.$$

$$\dot{a}(t) = R(t)a(t)$$

$$a(t) = a(0) \exp\left(\int_0^t R(\tau) d\tau\right)$$

$$x(t) = k + \beta R(t).$$

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Then later on another set of researchers they came up with a different concept where they said that this price is also a function of time. So, now, there is another extension. So, first of all this adoption rate depends on management decision variable  $x$   $t$  which depends on rate of change in price and rate of change in advertising. Now, we say that pricing is also a function of time, and this rate of change of advertising is also a function of time given by this equation.

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## Profits

- With the sales process given by the GBM and assuming a unit cost of zero, the profit  $\pi(t)$  at each instant  $t$  is given by

$$\pi(t) = w(t) \dot{F}(t) - a(t) = w(t)[p + qF(t)][1 - F(t)][k + \beta R(t)] - a(t)$$

$$w(t) = p(t) = ve^{-\gamma t}, \quad \gamma \geq 0$$

- $w(t)$  is the contribution margin per unit sold (i.e., per adoption)

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So, with the sales process given by GBM and assuming that unit cost of 0, the profit at each instant is given by this. This is basically how you are adopting it. So, this is rate of adoption minus all the advertising related cost that you have; based on that you can calculate the profits of that particular product.

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### Objective function

- The firm's problem is choosing the advertising expenditure over time,  $a(t)$ , to maximize the discounted profits over a time horizon  $[0, T]$ , where  $T$  may (but need not) go to infinity.

$$\Pi = \int_0^T \{w(t)[p - qF(t)][1 - F(t)][k + \beta R(t)] - a(t)\} e^{-rt} dt$$

- the discounted profits from the adoptions during the period  $[0, T]$ .

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So, the objective function of the firm is to choosing the advertising expenditure to maximize the discounted profits over a time horizon say 0 to T. So, you have to sum that profit from 0 to T with respect to p, q, adoption rate and other advertising expense that you have made in launching that product.

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**Objective function**

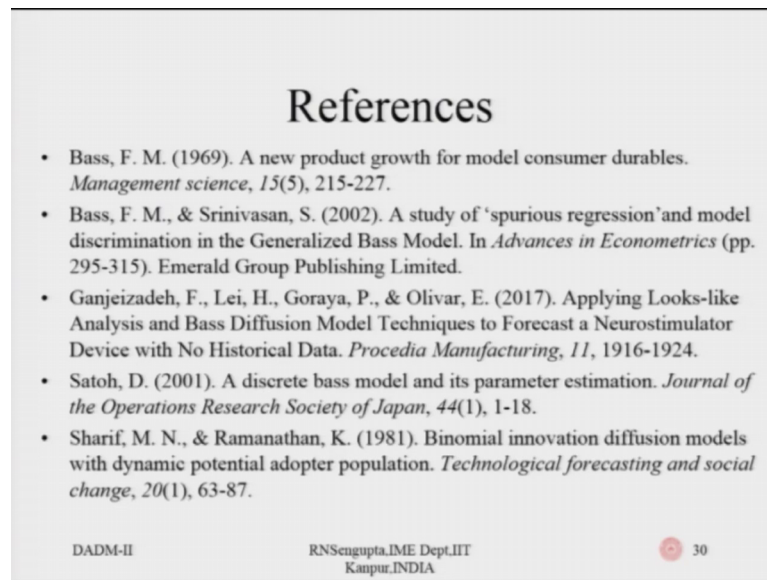
$$\begin{aligned} \text{Max}_{a(0), R(t)} \Pi &= \int_0^T \{w(t)[p + qF(t)][1 - F(t)][k + \beta R(t)] - a(t)\} e^{-rt} dt \\ a(t) &= a(0) \exp\left(\int_0^t R(\tau) d\tau\right) \\ \underline{R} &\leq R(t) \leq \bar{R} \\ a(0) &\geq \underline{a}_0 > 0 \\ \dot{F}(t) &= [p + qF(t)][1 - F(t)][k + \beta R(t)], \quad F(0) = 0 \end{aligned}$$

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And you want to maximize that profit given certain conditions, given that the advertising follows a particular model, here in this case advertising is an exponentially varying with respect to certain rate of advertising. And the rate of advertising is falling between certain numbers. So, there is a bound for the rate of advertisement at any point in time and this is the basic Bass equation.

So, the firms they use generalized Bass model to maximize their profits and identify what could be the optimal pricing strategy. So, you optimize this and you find out what would be the optimal pricing strategy. So, what would be the values of price coefficients in the Bass model, so that you can get the maximum profits?

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These are some of the references that you must consider if you want to understand more about generalized Bass model. And I hope that this could add some value to your understanding of how you make predictions about new product launch and sales and how quantitative methods are used in marketing decision models.

Thank you.