Data Analysis and Decision Making – II
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Lecture – 41
MAUT

Welcome back my dear friends. A very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe and as you know this is the DADM, which is Data Analysis and Decision Making two lecture series under the NPTEL MOOC. And this course total duration, as you all know is for twelve weeks, which is for 60 lectures, which is converted into 30 hours of lecture and each week, we have five lectures, each lecture being for half an hour

So, each lecture being half an hour so, 12 into 5 converts to 60 lectures and after each week we have assignments. So, as you know that we have already completed 8 weeks that is a 40 lectures are over and we will start the forty first lecture today, which is the 9th week and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, this is lecture number 41 on the 9th week. Now, we will consider the concepts .

So, this lecture, this half an hour lecture and most probably the next one also will be more to do, we consider just a conceptual background in that. What we mean by multi attribute utility theory some concept of, very simple concept of Pareto optimality multi criteria decision making. Those nonparametric methods I have already discussed. So, I will try to go into simple problems of multi objective in the tenth class, tenth week and try to basically wrap up with the metaheuristic techniques.

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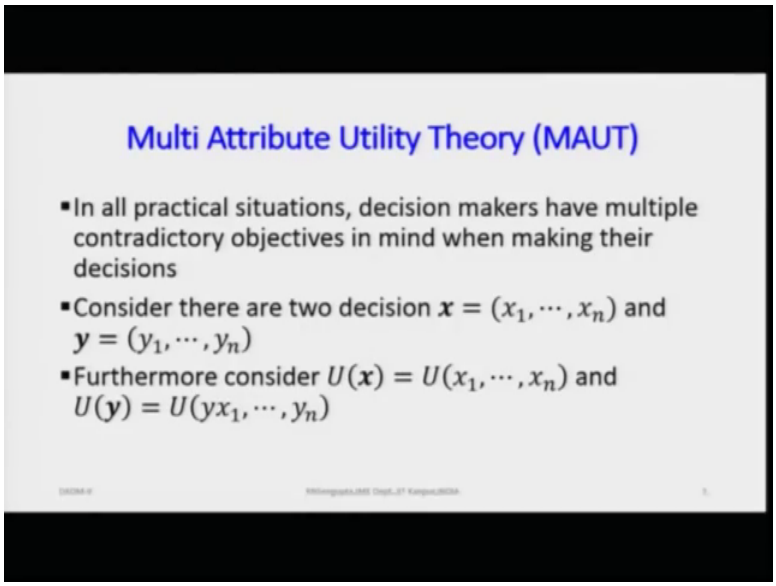

So, whenever as you remember, we have discussed utility theory. So, in all practical purposes whenever I am a decision maker; if you remember, we considered the concept of utility theory and those who are in a very simple sense, the univariate case. That means; there is wealth and there is utility; there is a decision, there is a utility and we considered very simply in the univariate case.

The four different utility functions in very simplistic sense; the quadratic logarithm, logarithmic, then the power utility function and the exponential utility function and I did mention about the safety first principle, different concepts of that. Now, when you are considering or a decision in it's whole gamut of all the nuances, there are many cases where you have to consider both the attributes which are the characteristics as well as the quantitative variables. That means, I have attribute characteristics which can be coloured, which can be taste, which can be styled, which can be how good or bad you feel off of by owning that or possessing that and characteristics with respect to quantitative variables can be price, can be quantity, can be weight; so, something which has quantitative values.

Now, when I am considering attributes, the attributes can be as I mentioned, it can be buying a car, where the color is important; buying the car where the, where the style is important. While buying the house you want to know how good the localities in a sense, where characteristics in the quantitative framework cannot be assigned to that. While in

the characteristics when there are as quantitative variables which you can assign, the utility can be found out based on that, can be the price of the car, can be the EMI, can be the running cost on a monthly basis, can be I am talking about the car, can be the maintenance cost.

Similarly, for the house gate can be the price, it can be say for example, how what is the EMI to be paid, what is the interest which you get to buy that apartment the house depending on which banks are assigned to give loans for when the apartment is built, being built. Others can be say for example, I want to take a decision that here attributes the third example, which I am going to give that which would be more important say for example, you want to hire a person. I have given these examples and you consider that what is his knowledge, when the knowledge concept can be very subjective attributes wise in a scale where the background can be important. And considering then if salary structure is important then; obviously, quantifiable variables are there. So, you can be sealed quantify, that that decision.

Now, whenever you have attributes, which is subjectivity, I will use the word subjective in object in a very general sense and when you have characteristics, where each you can quantify objective, there are utility functions. So, and we have already considered that. So, we will consider that there are two and now, in this case we will consider the decisions sets consist of vectors x and y, which are elements of the larger set x.

So, the decision can be buying a car with the price of say for example, 3.5 lakhs, colour is red and then say for example, EMI you may have to pay 3000 rupees per week, say for example. The other car can be in that say for example, color red was a characteristics which is attribute price and the EMI's were quantifiable objective value. So, there are three. So, called characteristics both subject and object to combine.

So, it will be x 1 x 2 x 3. So, I will basically have utility based on or combined effect of x 1 x 2 x 3. Similarly, they can be other car, the whether price is 700000, say for example, the color is black and the EMI you have to pay per week say for example, 7000 rupees. So, again the second characteristics is an attribute, the first and the third are characteristics, which are objective, I will basically have those those set vector based on which I will find out the utility would be y 1 y 2 y 3, the initial car was x 1 x 2 x 2.

So, hence x y; that means, x 1 x 2 x 3 and y 1 y 2 y 3, which is basically x and y the sets of the total. So, called set sets of decisions, which you have under your consideration. So, let me continue reading, what is given in this slide. So, in all practical situations, decision makers have multiple contradictory objectives in mind when making their decisions. So, they can be contacting their means I want to buy the red car, but the cost is high.

I want to buy the flat in say for example, in outskirts in Delhi, but the price is high. I want to buy the flat in say for example, in outskirts of Delhi price is low, but safety is not good or say for example, I want to buy a car with the prices low, but the safety concerns are a major issue for me or say for example, I want to make a decision of sending my kid or whoever is basically taking a decision he or she wants to go to a certain college.

So, considering the college is very good, in somewhere in India, somewhere abroad wherever it is or the university institutes, but the costs are very high like per semester cost so; obviously, have to take a decision that way, they were, they are going to take a loan for education and send your son or your daughter to that place, because the values later on may be good, but initially the cost are very high or you may consider that considering the child has got an admission in either pharmacy or in medical profession or in engineering or sciences. In some college, which is very good, but the branch, which he wants or she wants is not up to what he or she expects or wanted.

And there can be another college, where it is not higher up on the ranking, but the branch, which he or she is getting is according to what his or her wish is, so; obviously, you will have different type of attributes and characteristics both quantitative and subjective to make a decision and you may be a little bit not sure the decision, which you are taking is what is the net worth of that. I am talking you means the child, who is taking the decision when I talk about this college or university or institute.

So, consider there are two decision and then the sets of decisions x is a vector is x 1 to xn, n are the total number of different type of so called criteria, which you have already considered attributes, criteria, characteristics and y is also a vector with characteristics criterias, variables, attributes whatever you name as y 1 to yn, some of these xs are quantifiable, some of these xs are not. You have to give attributes similarly for wise. So,

furthermore we consider the utility, which we get utility is the net worth of taking the decision, which you get by taking that, there if you choose that set x is U of x.

So; obviously, so, U of x will give you a utility of U x 1 2 U xn. Similarly, I have y, y will give me a utility of y 1 and y 2, just one minute, these two decisions on three.

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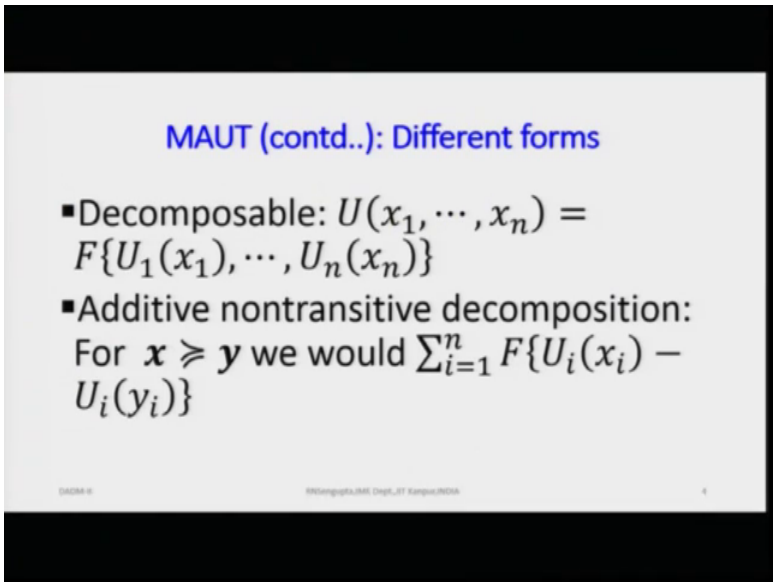

Now, there are different ways, how you can combine them. So, you can combine them considering the additive one; that means, you take Ux 1 plus U x 2 plus U x 3 combined and find out the overall utility function or it can be that you want to give weights or it can be, it can be joined combined function based on x 1 to xn or it can be say for example, a joint functional form of x 1 to x j, which is less than n and the rest j plus 1 till n is basically can be decomposed into linear function.

So, there are different ways, how you can do this. So, one of these this multi attribute utility theories concept. So, there are different forms, I will come to that one by one. So, one of the very simple ways how we can decompose the utility and this when I am mentioning about x 1 to xn, it means that is applicable for y 1 to yn also. I am taking x as in a very simplistic sense, that is it just a general element, vector element in this and the set capital X

So, there are many different combinations, I am taking one of them and the characteristics holds for all of them. So, in there decomposable form and basically have a

functional form of U 1 to U U n based on which I can find out the utility and based on the functional form, I take a decision whether I want to take x 1 till xn or I want to take y 1 till y n. So, when I mention either x, it means is a is a, thus the vector x 1 to x n, when I mentioned y, it is basically the vector or y 1 to yn. In the additive non transitive decomposition, you take a functional form, where you find out the difference between a utility of x 1 and x 2, utility of y 1 and y 2 and basically find out the functional form based on that.

I will come to those example in a very simple sense. First, let me go through the general characteristics.

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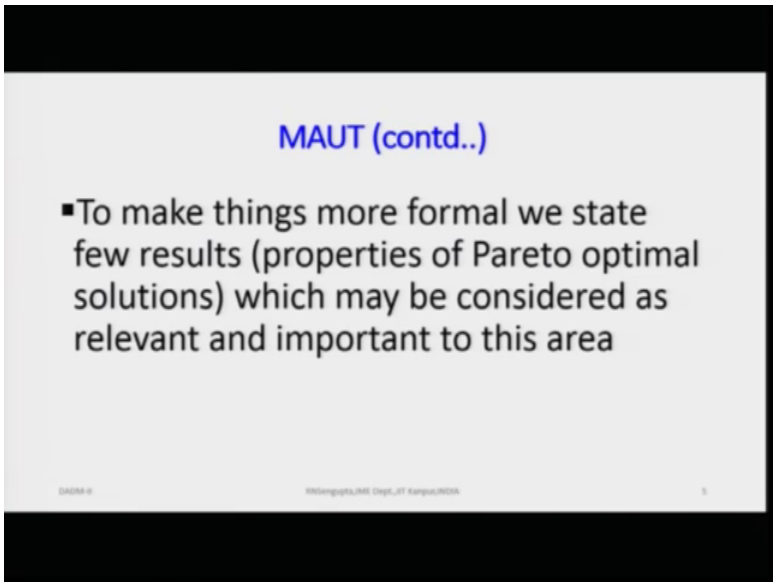

To make things more formal, I will come to the examples later. To make me things more formal, we state few results, which are the properties of Pareto optimality and I will give an example without solving it in the sense without solving it in front of you. I have the solution, I will state the solution and then basically give, how it can be analyzed.

So, to make things more formal we state few results, properties of Pareto optimality solutions, which may be considered as relevant and important to this area of MAUT or MCD M. MAUT is multi attribute to euclid theory and M CDM is multi criteria decision making.

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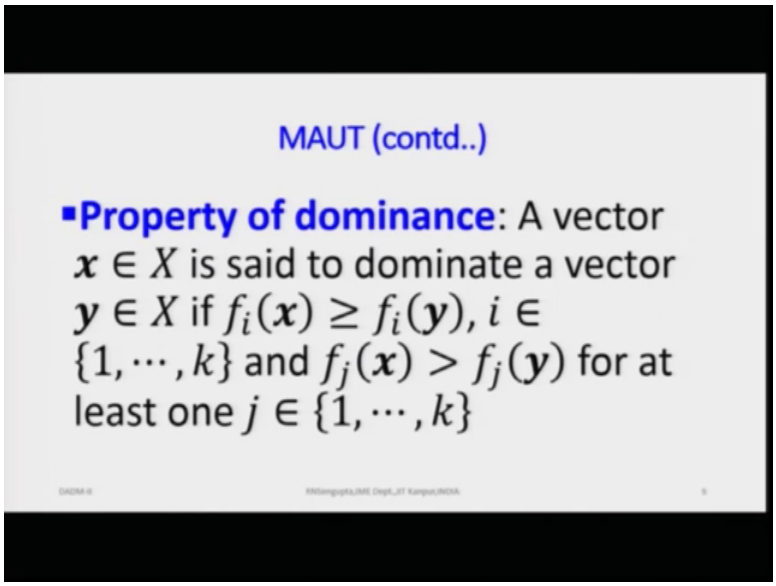

So, there are three properties of Pareto optimality, Pareto optimality, it is like this say for example, I will give an example case, considering of 100 rupees and you want to basically get some worth consider forget a utility function. Consider you want to buy something and that 100 is the maximum.

So, you can either buy say for example, some pencil and some erasers. Either you have spent all the 100 rupees to buy a pencil or you spend all the amount to buy, I need eraser or a combination of that. So, whether you buy all pencils or eraser or some combination of that the net worth, which comes to you is the same. So, we will consider that net worth, when you are considering in the combination of two, here is an example of two different decision variables pencil and the eraser.

The net worth corresponding that fact if it is equal, it can be extended to a higher dimension also, three dimension, four dimension, five fifth dimension so on. And so, it is considering n increases, which means by finding out a combination of the utilization of these two variables, in this case the pencil and the eraser or higher dimension. I am trying to achieve a level of satisfaction or level of goal, which can be met by combinations of different levels of the decision variables.

Now, if that is the case, you basically have an optimality case, where the networth, where it is very difficult for you to choose that whether you want to take the set x or the set y, because there may be cases, many of these cases where the overall value of x 1 is more

than y 1. But they can be cases where the value of see for example, xn is less than y n considering, there are n 1 to n number of such characteristics which accrue to the utility based on finding out either x or y. Or there may be cases where say for example, that the combinations of x and y is such that some of these characteristics, whether attributes or quantifiable characteristics are more for individual x.

And in some of the cases they are more for individual ys, such that the combined value which you have for x and y considering there are vectors, they give you the net worth the same. I will I will come with an example in the finance. Now, when you are considering that you will basically have the concepts of dominance. Dominance means to what level they nominate each other and we will consider two sets of stochastic dominance characteristics to happen, which is the first order stochastic dominance and the second order stochastic dominance; so, before that the property of dominance, a vector x.

Now, here the x vector means x 1 to xn, which is an element of x, the total set of decisions and x can be an infinite set, depending infinite combinations are there. We will say that x is said to dominate a vector y, if for some functional form of x the value accrued to you based on taking the decision x is always greater than equal to the value accrued to you for taking the decision y. So, if it is equal they are same for one of them, as I mentioned you f of x is greater, it is better to take the decision x.

Now, here what I consider that, I am considering I is equal to 1 to k consider there are different sets of decisions. Now, these different sets of decisions can be 3, number it is nothing to do with the dimension. So, for each set say for example, I consider there are n number of variables and the characteristics based on which I want to find out and compare these variables are 4 in number. So, first I consider f 1 for x f 2 for y. Similarly, I do it for f 1 for y then I do it for f 2 x f 2 y then I do it for f 3 x f 3 y and then I do it for f 3 f 4 x and f 4 y.

So, if any one of them the value is accruing for x is more; obviously, we will stay state the claim that the dominance property holds for this decision x with respect to y considering the functional form is f and we will consider based on the fact that the Pareto optimality conditions would be hold true, such that we can take the decision, such that we had equally disposed equally inclined in taking decision whether x or y gives us the same worth.

Now Pareto optimality, this is the dominance fact factor can have two flavours; one is the strong Pareto optimality characteristics and another we will see the weak Pareto optimality characteristics.

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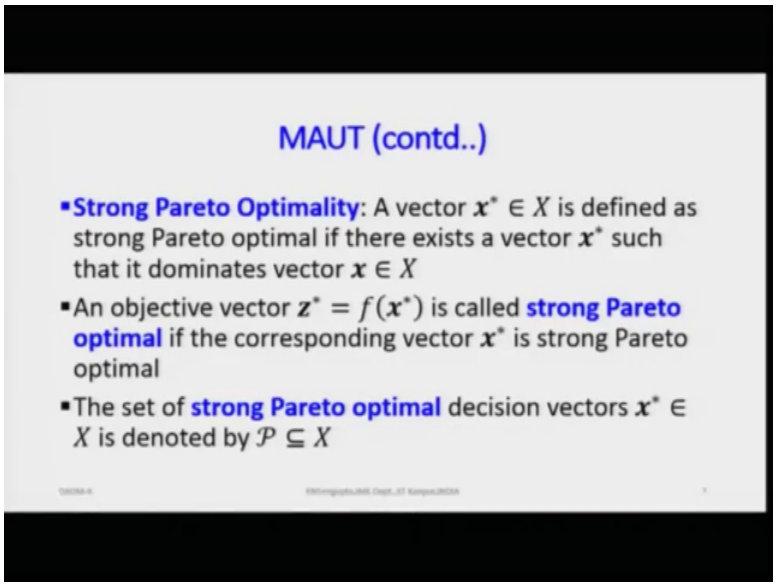

So, we will say that x star. Now, x star is any particular vector in x, which is any element in x it would be defined as strong Pareto optimal. Optimal if there exists a vector x star, such that it dominates all of the rest of x.

So, consider that say for example, x are vectors 1 to n and consider there are for our time being consider it is a 5 number of xs. So, capital X for 1, capital X for 2, capital X for 3, capital X 4 pack 2 X 4 5 each has n dimension, consider third one dominant. So, it will dominate 1 and 2 as a as well as it will dominate 4 and 5, then we will say that the vector x star strongly Pareto optimal with respect to the rest of the values x 1 x 2 x 4 x 5.

Now, the vector x z star, because based on each of these utilities for xs x 1 2 x 5, we will basically have a network, consider that network that is z. So, in that case when x attends a value of x star, the net value which will get for x attaining that x star will be z star. So, an objective function z star would be called a strong Pareto optimal solution. If the corresponding vector x star is strong Pareto optimal based on the fact that the net value net worth which is coming out from x star such that z star would be always greater to any of these z values corresponding to those x 1 x 2 x 4 x 5, for this example which I

mentioned. The set of strong Pareto optimal decisions x star would be denoted, It can be, they can be many things.

So, what the example which I had given is 4 x 3. It can be say for example, for that is x 1 to x 5 and x 2 and x 3 are very strongly Pareto optimal. So; obviously, this x star would basically will now be two elements of vectors x 2 and x 3, which would basically fall in the subset in x, where they strongly dominate, rest of this values x 1 x 4 and x 5 the characteristics of weak Pareto optimality would be a vector x star is would be defined as weak Pareto optimal, if there exists, no other vectors x such that x, f of x for those I is equal to 1 to n, they are not dominating in the sense that in the whole range of these x values I will have some extras values which absolutely dominate the rest of the x and there would be another set of values which do not dominate x star and hence, we will basically club them as strong and weak.

Strong would be they would basic be greater than x 1 x 2 x 3 x 4 for all of the values of the dimension of n we have and in the case when we have weak Pareto optimality it will be only dominating in some of the dimension based on which you are trying to consider maybe. say for example, colour is better for one other card not price or maybe say for example, boot space is better not say for example, style, but in the case if all of these are better price, the boot space, the safety features the EMI. So now; obviously, it would be strong optimal optimality conditions would hold Pareto optimality conditions would hold such that we say that value of x for which we get the decision as the best on all accounts would be taken, because as it is strong Pareto optimal.

An objective function z star would be called weakly Pareto optimal, if the corresponding vector x star is weakly Pareto optimal, the set of weakly Pareto optimal solutions would be denoted by P suffix w. So, you will basically have P, which will be a subset of x, which is strongly optimal Pareto optimal and P suffix w would be weakly Pareto optimal based on the fact that that greater than equal to and less than sign would hold for the strong Pareto optimality point and the weak Pareto optimality point.

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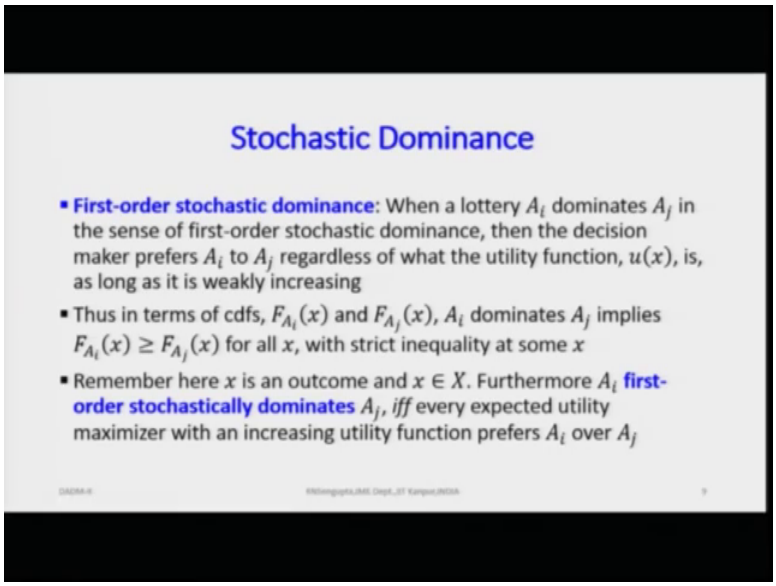

We will consider next the concept of stochastic dominance. Now, stochastic dominance I will give one examples after the slides which will basically give you the concept that what does this first order and second order stochastic dominance mean the first order stochastic dominance would mean, consider you are basically playing a gamble, where you are basically buying a lottery. So, when a lottery Ai dominates Aj. So, there are different alternatives A 1 to Am

If you remember I have considered 1 i is equal to 1 to m and j is equal to 1 to n. So, that j value, what I have been talking about the dimension. So, those are the criterias, based on which you are taking the decision. So, if Ai and Aj are both elements of A 1 to A m the whole set. So, Ai will in they will dominate the decision on the lottery Aj in the sense of the first order stochastic dominance. In the case the decision or the decision maker may prefer Ai to Aj regardless of what the utility function Ux is as long that is weakly increasing.

So, as the values of x change, if the concept of utility is weakly increasing in the sense, some of them are more or some of them and some of them are less and something of they may be equal also. So, as long as whatever the combination is if I always check, if I had a decision maker I always check Ai with respect to Aj, whatever the two alternatives are then I will always say that the A ith alternative, A ith decision, A ith lottery would strongly with would dominate the A jth alternative A j th decision in the first order case.

Consider this as the combinations consider, there are two cards one is brand A 1 is brand B, let me put it at brand say for example, 1 and brand 2. So, in case if brand 1 and brand 2 are there and different values of characteristics are there, some which are good for brand 1 and some are not good for brand new 1 with respect to brand 2, but as you keep changing the values of those characteristics whether attributes or objective variables that I am not going to consider, it can be anything

As brand one always is better than brand 2s and; obviously, we will say that the first order stochastic dominance property will hold for brand one with respect to brand two depending on whatever combination of the characteristic it is take. May be the price is increased slightly so; obviously, it will happen that brand two prices also in increasing. May be the fuel efficiency is in increasing or decreasing for both the case, may be the boot space is increasing decreasing for both the case.

These are the political concept, which you will trying to consider when you are making the decision based on that when we when we consider brand will take brand 1 to be first order, stochastic dominance based on the fact. Thus, in terms of the cumulative distribution function. Now, here the overall net worth when we get Ai Ai and Aj energy, we are considering the utility is based on the fact that we have some pdf or a pmf for the net worth of the decisions, which we have

So, say for example, I take a decision and there is a probability of outcome as it is in a lottery so; obviously, there would be a pdf and a pmf. Now, if I consider the cdf values of both the alternatives that is f of a 1 and with an f of A 2 where A 1 and A 2 are basically Ai and Aj, if Ai dominate in the first order stochastic dominant sense Aj, it will always be implied that for whatever the value of x said, whatever the combination of x is the cumulative distribution function value for Ai would always be greater than equal to that cumulative distribution function value of Aj.

So, whatever the combinations we take, that will always hold true remember here that x is an outcome, which is an element of the whole set of decisions, which we have x furthermore Ai, which is the first order stochastic dominance will dominate Aj, if and only if the expected utility maximize their properties of the increasing utility property would hold true for Ai and Aj; that means, as I am increasing the decisions I am getting more and more worth. So, this will consider to be true for the case of Ai and Aj

considering that acts as x changes it's values. So, with this I will end this, the first lecture for the ninth week, I am considering the concept of MCDM and M A routine more details later on.

Thank you very much and have a nice day.