

**Data Analysis and Decision Making - II**  
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**Lecture - 38**

**VIKOR**

Welcome back my dear friends; a very good morning, good afternoon, good evening to all of you wherever you are in this part of this world whether in India or abroad. And, as you know this is the DADM-II which is Data Analysis and Decision Making-II course under the NPTEL MOOC series. This course total duration is for 12 weeks which is 30 hours and 30 hours baking being split into 60 lectures each lecture is for half an hour and we have as I said is 12 week.

So, each week we have about 5 lectures each being for half an hour and after each week we have assignments. So, as you can see from the slides we are in the lecture number 38 which is the 8th week; that means, you already have taken 7 assignments and we are in the verge of trying to finish this 8 week. And, you will basically have the 8th assignment for assignment number 8 and my good name is Raghu Nandan Sengupta from the IME Department at IIT Kanpur.

So, if you remember we are discussing about the VIKOR method the steps. So, I did discuss the initial part of this pseudocode and I will try to wrap up the pseudocode and discuss because, I did not finish the whole part so, I will add it here and. So, it will be easy for all of us to do it accordingly.

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### VIKOR Algorithm (contd..)

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1. DEFINE:  $X_{m \times n}$  (matrix consisting of priority scores assigned to decisions/alternatives,  $A_i$ , based on attributes/decision criteria/goals,  $C_j$ , wj (weights for the attributes/decision criteria/goals) such that:
 $\sum_{j=1}^n w_j = 1$ ,  $C_j(A_i)$  (function-relationship between attributes/decision criteria/goals for each decisions/alternatives),  $C_j(A_i)^+ = \max C_j(A_i)$ ,  $C_j(A_i)^- = \min C_j(A_i)$ ,  $S_{pi} = \left[ \frac{\sum_{j=1}^n \left( \frac{C_j(A_i)^+ - C_j(A_i)}{C_j(A_i)^+ - C_j(A_i)^-} \right)^p}{n} \right]^{\frac{1}{p}}$ 
Here  $i = 1, \dots, m$ ,  $j = 1, \dots, n$ , and  $p = 1, 2, \dots, \infty$  (distance norms)

2. INPUT:  $X_{m \times n}$  (matrix consisting of priority scores assigned to decisions/alternatives,  $A_i$ , based on attributes/decision criteria/goals,  $C_j$ , wj (weights for the attributes/decision criteria/goals) such that:
 $\sum_{j=1}^n w_j = 1$ ,  $C_j(A_i)$  (function-relationship between attributes/decision criteria/goals for each decisions/alternatives), here  $i = 1, \dots, m$  and  $j = 1, \dots, n$ .

3. START IF  $i = 1$  to  $m$ :
4.   START IF  $j = 1$  to  $n$ :
5.     CALCULATE:  $C_j(A_i)C_j(A_i)^+ = \max C_j(A_i)$ ,  $C_j(A_i)C_j(A_i)^- = \min C_j(A_i)$ ,  $S_{pi} = \left[ \frac{\sum_{j=1}^n \left( \frac{C_j(A_i)^+ - C_j(A_i)}{C_j(A_i)^+ - C_j(A_i)^-} \right)^p}{n} \right]^{\frac{1}{p}}$  where  $i = 1, \dots, m$ ,  $j = 1, \dots, n$  and  $p = 1, 2, \dots, \infty$  (distance norms)
6.   END IF
7. END IF
8. CALCULATE:  $C_j(A_i)^+$ ,  $C_j(A_i)^-$ ,  $S_{pi}$ 
9. REPORT:  $C_j(A_i)^+$ ,  $C_j(A_i)^-$ ,  $S_{pi}$ 
10. END

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So as we were discussing. So, you will basically have the ranking system based on a normalized rank; that means, in the 1 p norm for each and every i th alternative will be based on a scaling system where, the denominator is the difference between the maximum and the minimum. And, in the numerator you will have basically the difference between the maximum and the actual distance which you have or the weightage which you have the priorities which you have which is  $C_j A_i$ .

And, this factor can be multiplied by the weights depending on the level of importance you want to put for each and every alternative. So, if the alternative weights are basically 1 by n depending on the number alternatives which is the criteria 1 by n. So, each would basically get the same weightages and then you will basically multiply find out the p th power, then sum them up find out the 1 by p th power corresponding to the norms which you have. And, we have already used the concept of the norms; to illustrate then as p increases how the difference between the norms for 1 30, 1 31, 1 32 so on and so forth decreases. I am giving 30, 31, 32 as an example.

So, you will start for each. So, once this ranking system is known to you how will rank them based on the distance concept, you will take for each i is equal to 1 to m, m you will take each and every criteria's. For i is equal to 1 you will take the criteria 1 to n, find out those corresponding 1 p 1 norm put it those value such that you can rank them. Then

change i is equal to 2 find out for j is equal to 1 to n, again the l p 2 norm put it; l p remains fixed remember that 2 means the 2nd alternative.

Continue doing so, till we have l p 1 to l p m and based on that we will proceed. So, you will calculate the values of C j plus A plus and C j A minus and use them for the banking. So, I will start going through slides and do that in an excel sheet also.

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**VIKOR: Steps # 01 (Construct the normalized decision matrix)**

- Assume the decision matrix,  $X = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{bmatrix}$
- Convert the entries in  $X$  into scaled **normalized** values,  $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^m x_{kj}^2}}$ , which has no dimension
- Thus we get  $R = \begin{bmatrix} r_{11} & \dots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \dots & r_{mn} \end{bmatrix} = \begin{bmatrix} \frac{x_{11}}{\sqrt{\sum_{k=1}^m x_{k1}^2}} & \dots & \frac{x_{1n}}{\sqrt{\sum_{k=1}^m x_{kn}^2}} \\ \vdots & \ddots & \vdots \\ \frac{x_{m1}}{\sqrt{\sum_{k=1}^m x_{k1}^2}} & \dots & \frac{x_{mn}}{\sqrt{\sum_{k=1}^m x_{kn}^2}} \end{bmatrix}$

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So, assume the; you will basically construct the normalized decision matrix. So, assume the decision matrix is  $X$ , size is  $m$  cross  $n$ . This is same which I have been repeating time and again, but I will still do it and without being too repetitive in order to make you understand that you are trying to basically compare. And, find out the overall priority weights or priority values for each and every criteria and each every alternatives. So, the size as I said is  $m$  cross  $n$ . So, the 1st row would be  $x_{11}$  till  $x_{1n}$  where, the  $x_{11}$  means the so called for a actual non-minus normalized priority values for the 1st criterion on the 1st alternative.

And, while  $x_{1n}$  basically being the priority weight or the value for the  $n$  th criteria on the 1st alternative. Similarly,  $x_{21}$ ,  $x_{22}$  till  $x_{2n}$  would with the corresponding values of the priority scores say for example, or values whatever you say. For the 1st, 2<sup>nd</sup>, 3<sup>rd</sup>, 4th till the  $n$  th criteria for the second alternative only. Similarly, the last row  $x_{m1}$  till  $x_{mn}$  is basically the priority values, criteria values being given for the  $m$  th alternatives corresponding to the fact, that I am considering the 1st criteria till the last criteria.

Now, you can basically use the normalization concept; the nomination concept again I am considering the very simple, this is a very simple concept which I which I considered where you take the values corresponding to the fact that you are for trying to find out the normalized scores as the sum is 1. So, if whether you is just cube square whatever it does not matter. I will show you an example which may be a little bit variant, but you will understand that.

Then once you find out the normalized values you have the R and then these and again this is of size m cross n which is the normalized scores or the weights or the values corresponding to the  $j$  th criteria for the  $i$  th alternative. So, let us basically consider value and I will go a little bit slow in order to make you understand.

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		C1	C2	C3	C4	C5	
19		0.11594	0.26087	0.26087	0.35507	0.00725	1
20							
21							
22		C1	C2	C3	C4	C5	
23	A1	30	0.4	120	4	0.01	
24	A2	50	0.1	110	9	0.08	
25	A3	70	0.8	190	2	0.04	
26	A4	40	0.3	160	6	0.07	
27							
28		C1	C2	C3	C4	C5	
29	A1	0.01538	3.6E-08	0.98458	3.6E-05	5.7E-13	1
30	A2	0.08581	6.9E-10	0.91369	0.0005	3.5E-10	1
31	A3	0.04763	7.1E-08	0.95237	1.1E-06	8.9E-12	1
32	A4	0.01538	6.5E-09	0.98456	5.2E-05	8.2E-11	1
33							
34							
35							

So, this I will mention as X. So, I will consider A 1 I am writing it so, that is everything is easy for us to understand. I will take four alternatives and then take a little bit more number of criteria. So, even though it will take time, but I am sure you will understand so, I have five criterias. They are values are given, but the corresponding units may be different which I said that they are attributes so, the units may be different.

So, we are able to consider that in the VIKOR, I am not go into the detail discussion and again I will just solve the problem. So, consider I am go column wise so, it will be easy for us to understand. So, consider these are in on a say for example, on a unit of 2 units

places 30, 40, 70, 40, 30, 50, 70, 40. And, the units and the tenth values are there others I will consider this decimal.

This I will consider these values I will consider on a scale of say for example, yes I am consider; this is I am taking arbitrarily values in order to make you understand. So, these are for each and every criterias and the weight. So, I will mark these criterias with a color yellow these as; so, this is done. Now, weights I would not take now so, let us go one by one.

So, I am basically finding the; consider it is basically row wise. So, let me use the 1st concept of normalization, normalization number 1. So, I am using different normalization which you can change accordingly, but let us proceed. So, if I use the simple let us consider I am using the 1 3 so, let us proceed by 1 3. So, this is done, now I will basically freeze the values and at each cell.

So, this so, they do not want to be any change as I copy the values. So, let us consider, so sum should be 1, this is 1. So now, I normalize the 2nd value, so it will be now C 24, D 24, E 24, F 24, G 24. So, this one is done again 1, then I go two values, this is 25; this is 25; 25; 25 and the last row would be 20's.

So, let us I will double check and do not worry, then when I come to 26; 26; 26; 26. So, let us check whether the values the same, I could have done the (Refer Time: 12:44) goes. Now, once it is done what does the so, these part is done, so I have normalized them.

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**VIKOR: Step # 02 (Construct the weighted normalized decision matrix)**

- If the decision maker decides on the set of weights, depending on his/her preference, then the weight,
$$W = \begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{bmatrix}, \text{ such that } \sum_{j=1}^n w_j = 1$$
- Calculate  $F = RW = \begin{bmatrix} f_{1,1} & \cdots & f_{1,n} \\ \vdots & \ddots & \vdots \\ f_{m,1} & \cdots & f_{m,n} \end{bmatrix}_{m \times n}$

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So, once normalized I go to the next step, which is basically find out the weights; give the weights, so let us go and for give a some weights, so let us.

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	C1	C2	C3	C4	C5
C1	0.2	0	0	0	0
C2	0	0.2	0	0	0
C3	0	0	0.2	0	0
C4	0	0	0	0.2	0
C5	0	0	0	0	0.2

So, this will be of 5 cross 5, so I will write the value as, so it will be let us consider them of equal weightage. So, it is one-fifth. So, these are the; so, these are the overall matrix which I saw, if you want I can shift it and write the corresponding weights.

So, these are the criteria weights which I have. So, this becomes 2, this becomes 3, this becomes 4, this becomes 5. So, this you have the weights now. Once the weights are

done what does it give; I have to find out the multiplicative value. So, I will deduce the view and multiply them. So, I need to find out F, so let me put it F.

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
26															
27															
28															
29															
30															
31															
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53															

So, normalization step 1 I can change, so this is of size 4 cross 5, this is 5 cross 5 if 4 cross 5 into 5 cross 5 would give you 4 cross 5 matrix. So, this would be normalize, so called normalized values for the 4 alternatives and 5. So, this is the F value.

So, this is m m u l t of the 1st row and the 1st column. So, obviously, this row will get fixed here showing. So, freeze it, then I kept it for all others; let me check whether I am right, so I am considering all the different columns respect to the 1st row. Then I multiply, this is the second row what is for A 2 was going to C 1, so again I freeze the values.

So, let us check whether they are the same, so this is; the columns are changing, the rows is the second one, then I go to their 3rd row which is for A 3 Alternate 3 multiplied by corresponding C 1, C 2, C 3, C 4 for each cell it will come.

So, let me freeze the values; let us see they are frozen. So, it is basically the row remains the same, the columns are changing. Finally, it is the last row multiplied by each corresponding column; last row is basically the 4th alternate. Let us freeze the values.

So, it is basically; now these F values are obtained which is multiplication of each and every normalized vector which was basically R into weight which is W. So, normalize

So, it will be L 3, so we are taking ps 3. Now we have found out f 1 1 till f 1 n which is the overall normalized weights; weighted criteria for the 1st criteria for the 1st alternative. Similarly f 1 n would basically be the weighted normalized priority for the n th criteria for the 1st alternative.

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VIKOR: Step # 03 (Determine the maximum and minimum of the criterion values)

- Determine the **best**:  $f_j^* = \max_{\forall i} f_{i,j}$
- Determine the **worst**:  $f_j^- = \min_{\forall i} f_{i,j}$

So, we need to basically find out  $f_j^*$  and  $f_j^-$  which are basically for each and every row or column depending on how we have mentioned that; we will find the best and the worst. Now remember  $f_j^*$  stars; that means, for  $j$  is equal to 1 you will take along each and every alternatives, so  $f_j^*$  is there. So, let us go to, so we will put it at  $f_j^+$ , so  $f_j^+$ . So, this you can find out for each criteria.



So, you will take, so this will be for the 2nd criteria, this will be for the 3rd criteria, this will be for the 4th criteria, this will be for the 5th criteria. Similarly I will have  $f_j$  minus let me write it. So, minus value will go constant; I will put the minus values as this. So, star is basically minus, so this is minus for the 3rd criteria and this is for the 3rd; this is for the 3rd, then I come to the 4th, then I come to the 5th. Now I need to find out the maximum; here the maximum and here the minimum. So, for all the  $i$ 's, so remember for I will take for all the  $i$ 's, so it is going to max for this. So, I take max, so this is the max values.

Now, I take the min for the worst criteria. So, the mins are calculated. So, this is the max value and the min value. Now remember here the max and min would be utilized depending on the weights which you have already considered. So, giving the max on the min.

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**VIKOR: Step # 04 (Determine the relative ratios of maximum and minimum based on weights)**

- Compute:  $S_i = \sum_{j=1}^n \left\{ \frac{w_j(f_j^* - f_{i,j})}{(f_j^* - f_j^-)} \right\}$
- Compute:  $R_i = \max_{\forall j} \left[ \sum_{j=1}^n \left\{ \frac{w_j(f_j^* - f_{i,j})}{(f_j^* - f_j^-)} \right\} \right]$

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We want to find out now the weights are already given, so you want to find out for  $f_j$  minus the each and every alternate multiply the weights. So let us go; so let us consider the normalized denominator. So, the  $f_j$  minus plus star minus  $f_j$  minus; so this is what I have. So, this would be  $f_j^* - f_j^-$ .

So these are the values, so let me and this is the 2nd, this is the 3rd, this is the 4th, this is the 5th. So, here I have the 1st criteria, this is for the 2nd, this is for the 3rd and this is for the 4th, this is for the 5th for each and every criteria we have already done that.

So, let us find out; this will come in the denominator, so this will be. Now I go to the ratios; that means,  $f_j$  plus minus that  $i$ th one, so for each and every  $j$  is equal to 1 to  $n$  and sum them up, so either sum up for at 1. So, I am going to multiply the weights.

So, it would be the weights multiplied, so it we will put weight multiplied by max value minus the 1 which is the max value this minus each and every cell value plus I am taking equal weightages. So, max value minus the second plus the weights multiplied sorry I should put the multiplication sign or else it would not come.

So, this would be max minus the 3rd cell bracket closed plus the 4th weight  $W_4$  multiplied by the max minus the 35, 36, 37, 38. So, you will basically have let me check you are going to go by the sorry, so I have just the  $j$  and then I am think is equal to 1 2 ok. So, weights minus the this would be 40 D;  $D_{41}$  minus  $j$  is equal to 1, so it will be 2nd cell value which will be  $D_{35}$ .

So, it will be; this will be become E and this will become F. So, F finally, you will basically have the last cell multiplied by I have not done it, so it will be F it will be  $g_j$ . So, I have basically taken all the values along for the 1st1st one. Similarly I need to find out the corresponding, so the values are being, so I need to copy them.

So, once I copy them I need to basically; so the weights are to be fixed because they are not changing. So, I again the max values are also to be fixed. So, if I go there and just to one one of them and it will become much easier for us to solve. So, once I have this again I go for the; this sum change wait. So, it will C.

So, I have these values for each and every, now I can find out one of the 2nd one. So, these values which I will get for the maximum minimum I will do it accordingly for the F minus of 1, 2, 3, 4, 5. I will I will solve this example and also show, so continue this in the last two classes and try to basically give the overall picture that how you basically rank in the concept of VIKOR method. With this and I will end this class and continue the discussion of what the solving step by step, so it will be easier for you to understand have a nice day and

Thank you very much.