Data Analysis and Decision Making – II
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 37
VIKOR

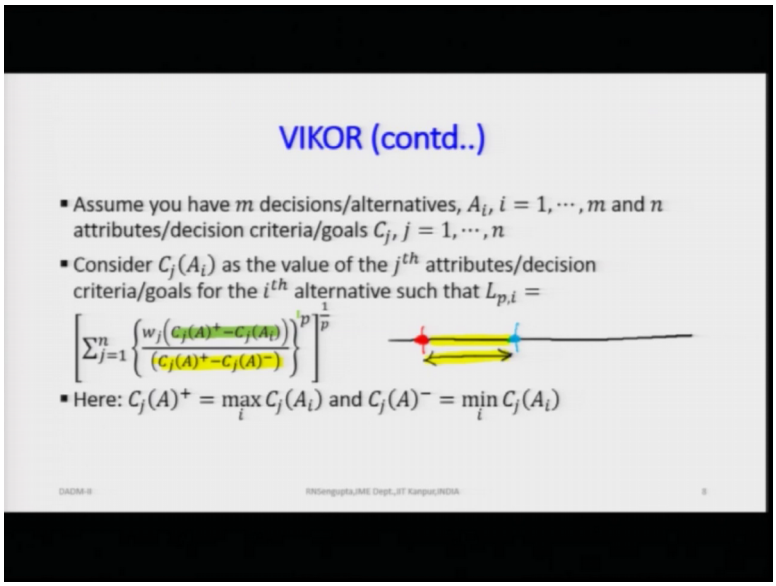
Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the world whether in India or Abroad. And as you know this is the DADM which is as you can see on the slide which is Data Analysis and Decision Making II course under the NPTEL MOOC series and this DADM II as DADM I was is basically for 12 weeks which is 30 hours; 30 hours being basically split into 60 lectures.

Each for half an hour and each week we have 5 lectures and after each week we have assignments and as you can see from the slide we are in the 8th week which is the 37th lecture which is the 2nd lecture for the 8th week and my good name is Raghu Nandan Sengupta from IME Department IIT Kanpur in India. You know if you remember we are discussing about the multi criteria decision making VIKOR and we have already discussed for one leg whole set of one week about ELEKTRA one set of one week for TOPSIS and then initially we have discussed about HP also.

Now, in VIKOR the similarity and dis similarity about TOPSIS and VIKOR had been discussed. One was basically linear normalization, one was vector normalization. We also discussed the concept that in VIKOR. We will consider the distance measures con considering the concept that what distance measure do you use? You will use the Lp norm and I will show you if you remember I did mention in the last lecture which was in 36 lecture that how the distance measure could be understood. And in the VIKOR method again I will go in the similar way state the algorithm take a very simple example.

And then solve it accordingly and I will mention the steps as we proceed. They would not be much of a dissimilarity apart from that how you calculate the so called best solution, worst solution which in many of the cases you have mentioned has concordance discordance set or the distance to the PIS or distance to the NIS, so, that different ways how you analyze the things.

(Refer Slide Time: 02:36)



So, we will assume this slide is basically the essence the one and I did mention about that about how you will cons con consider the distance measure. So, as usual consider the nomenclature you have m number of total alternatives or goals to achieve and for each goal you consider different criteria or attributes, characteristics and there are n in number. So, the suffix i changes from 1 to m as in Mangalore or mango and n change j changes from 1 to n as in nose are at Nagpur. Now remember one thing that in many of the methods or almost I should say in all of the matters of MCDM whether a HP TOPSIS, ELEKTRA and specifically in VIKORA I did mention that will consider the attributes of the characteristics.

As the main what I should say the quality based on which you will try to analyze each and every alternative against each other, corresponding the fact that the criterias are there. And in VIKOR method is simple to analyze that even if the units for that it was are different. We will be able to form or bring some semblance of comparison between the criteria even as the units are different.

So, we will consider which I have already said we will consider m number of alternatives and n number of criterias and we will consider the distance function of the mapping or the functional form C j A i where C implies the criteria, A implies the alternative. So, a such that C j and A i would mean the overall characteristics or the criteria or the priority

value which accrues between the relationship coming out from the j th criteria for the i th alternative. So, we will consider the distance function based on this the criterias was.

Basically will have a weight, which have already discussed if I am repeat excuse me if I am repeating please bear with me we will consider the weights w j, j is equal to 1 to n and we will have the corresponding values. So, what you are going to take if you look at the numerator than a denominator. In the denominator you have basically difference between the max in the mean as it is stated. So, C j A plus this should be what should change it. So, there is no confusion A plus minus yes so, we have this is little bit. So, this is fine. So, C j A plus is basically you take either the row or the column.

Now, remember we will be taking the matrix m cross n where m and n have their own notions which has been already been mentioned time and again from my side. Now you will take the maximum corresponding to C j A i's and you will take the minimum corresponding C j A i's. For all the values of is; i's means the first alternative, second alterative, third alternative so on and so; that means, you are going row wise. Now the denominator is basically the difference; that means, you are normalizing or taking the ratios on the max and the min and based on that you are trying to find out where do they stand.

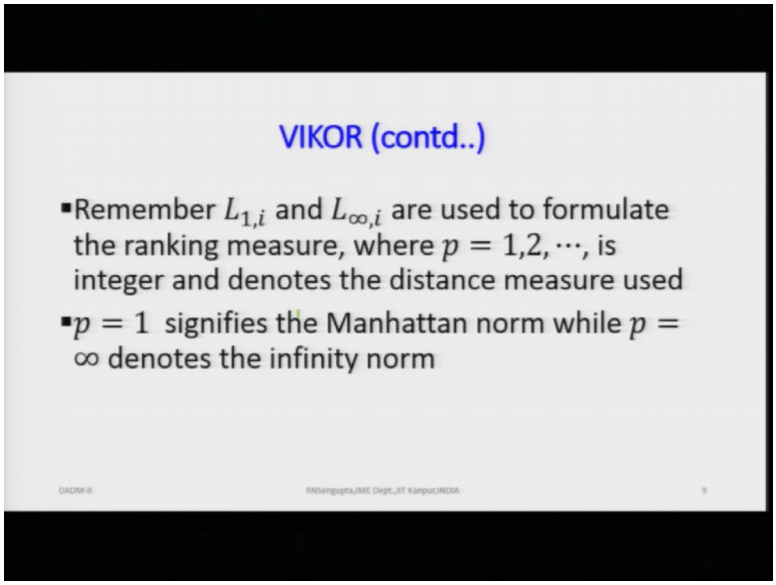
So, you have a real line. The max and means are has been normalized as the distance which is C j A plus minus C j A minus and based on that you will basically put the values where does C j A i stand and you will basically find out for each and every j is equal to 1 till n. So, say for example, for j is equal to 1 you will find out C j A i and then basically subtract that value from the maximum value. So, which is so you will draw the real line first. Now what I will take is I will take the max mean let me use the blue color. So, this is the max, this is the mean.

So, you will basically have the line has given this is mean, this is max. So, this distance what we have is basically the one which is, let me use the high back there. This is the one that I am see, I am normalizing a finding on anomalous scale maximum minimum. Now forget about w j see if I consider this. So, for each C j A i that particular row we have a maximum, we will try to find out the difference between them and then multiplied by the corresponding weights. So, consider the weights are all equal.

So, the weights are all equal, we will basically multiply the distance, which is occurring between the max and each and in individual value normalized by the difference between the max and the min and sum them up, but remembering that what type of norm which you are using. I will come to the norm within few minutes and explain that what the concept of norm would mean in a more details. Now if it is L 1 norm you will take the mod greater than the L infinity. Now we will tilt the mix in the max and corresponding to L 2 you will basically have the Cartesian distance and so on and so forth.

So, once you have that you rank them and utilize this ranking system to further find out the scoring for the max and the min and then find out the overall ranking of the A i 1 A i 2 so on and so forth for i 1 and i 2 being element of the total set of alternatives.

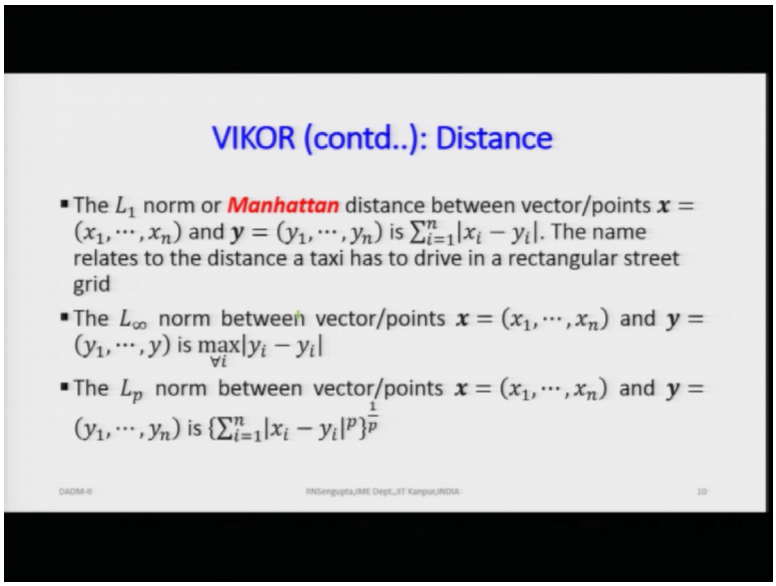
(Refer Slide Time: 09:34)



Remember L 1 and l infinity are, I am not talking about the i th one, so that we will basically be for i is equal to 1, 2, 3, 4 till m for each and every alternatives which you have. So, L 1 and L infinity are used to formula the ranking measures where p can be 1, 2, 3, 4 till infinity. It is basically an integer and you knows the distance measures used. So, what distance measures we will try to find out we will use that.

P is equal to 1 denotes as we have already discussed that in the TOPSIS method it will you knows the an Manhattan norm while p is equal to infinity denotes the in the infinity norm which is the max 1. Now we have done it, still I will repeat it for your own convenience.

(Refer Slide Time: 10:21)

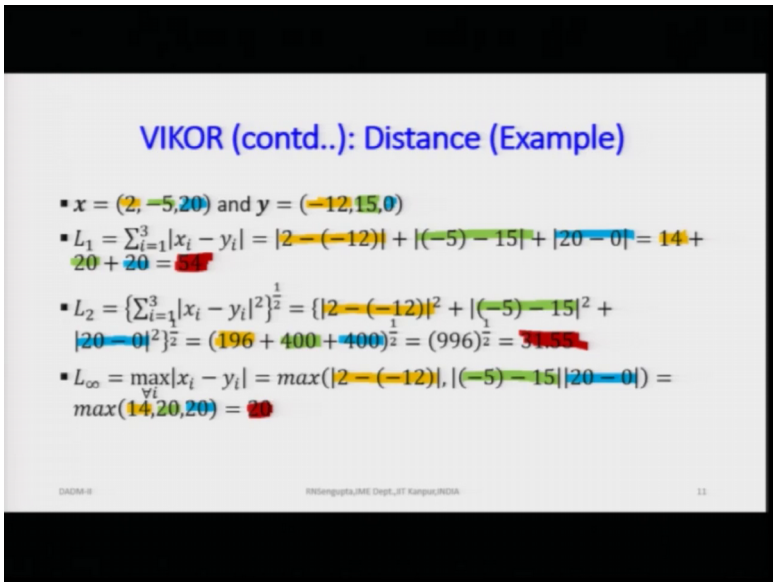


So, we will basically have different norms, L 1 norm would the Manhattan distance norm between vectors and points when the vectors are x vector is x 1 to x n, this n has nothing to do with the n number of criterias. I have I am just taking in arbitrarily, it could be anything. So, this is just a nomenclature concept I am using. This I use that in the case of when we are discussing about the TOPSIS also. So, I did not mention then there.

But I am again mentioning it here in order to make it clear. So, this n which is here I am not highlighting I am just putting my pointer here this n has nothing to do with the concept of criteria. Similarly y vector y bolt will be y 1 to y n and if I want to find out the L 1 norm it will basically be the mod of the difference and sum them up from i is equal to 1 to n. Again this n is the number of elements which you have in x vector and y vector. The name relates the distance which you have already discussed as the taxi has to drive in the rectangular street grid in the city of New York, in the area of Manhattan.

While n infinity norm would give me again for x vector x 1 to x n y, vector y 1 to y n. It will give me the maximum values for each and these are which I have max of x i minus y i and L p norm would basically be between 2 vectors again x vector x 1 to x n, y vector y 1 to y n be the difference between the mods to the power p th one and then basically sum them up and find out the 1 to the power p power of that means, you are trying to scale them up and then bring on the scale depending on the p th power which you have.

(Refer Slide Time: 12:25)



Now, let us consider an example I am considering in a very simple case of the 3 dimension one and you will understand why this distance measures would change as we change the norm, consider the vector is 2 minus 5, 20 and y vector is minus 12 15 and 0. So; that means, the x point is we go travel plus 2 along the x axis, x I am using as a vector. So, and then I travel minus 5 in the y axis, axis I am talking about and then I travel 20 plus in the z axis. So, that is the coordinate system point I have. Similarly for the point y vector I travel minus 2 L along the x axis.

Then I travel plus 15 along the y axis and in the z axis I do not travel because it is 0. So, if I want to find out the L 1 norm again as per the formula I add up all the values i is equal to 1 to n. So, let me I think it would be easy for you to understand. So, rather than writing n I should write it down. So, it is easy for all of us to understand. So, let us go a little bit slow. So, three value would come here let me change this also, let me change this also. So, for i is equal to 1 2 3 you find out the difference which is I will put the color scheme here which is much easier for me too.

So, this is the first one is 2 and minus 2, 12. So, this is 2 minus or minus 12 mod of that which is 14 which is here then I take minus 5 or minus 12. So, minus 5 a minus 12 is minus 5 and plus sorry plus 15. So, minus 5 or minus 15 the value comes out to be 20. Similarly I have 20 in the z direction 0 in the z direction for x and y vector values 20

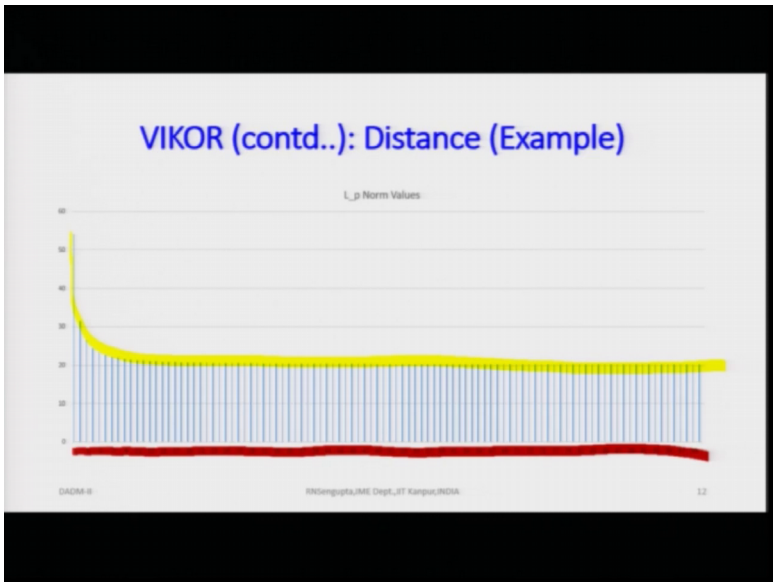
minus 0 which is 20 and the total value of L 1 comes out to me 54, I will just circle it. I will use this value so in order to differentiate, so, I get a value of 54.

Then when I come to L 2 norm again I have this 2 value minus 12. So, is 2 minus or minus to a whole square as I did as it should be because p is now 2 when I can come to the y axis the colors code I am using is green. So, is minus 5 of minus 15 whole square which is basically 20 square, so, this is 400. This value I have not ma highlighted for our convenience, it is 14 square which is 196 and finally, for movement in the z direction which is blue color which is 20 and 0. So, it is 20 minus 0 whole square which comes out to 400.

So, square them up, add them up find out the square root it comes out to 996 square root and the actual value comes out to be 31.55. There is let us consider at 31.6. So, when I mentioned I will be basically talking about 31.6 until unless required. So, I will highlighted 31.5. Note down one thing 54 has now decreased to 31.5. Now let me come to L infinity norm again I use the same color scheme for x direction movement 2 n minus 12, 2 of minus of minus 12 that is 14.

Then the second value is green in color which is minus 5 of minus and minus 15 which is 20 and the last value I am using the color blue which is 20 and 0, 20 of mine minus 0 which comes out to be 20 and actual value comes out to be 20. So, the values are given as 5431.5; obviously, they would be L 3, L 4 alpha is so on and so forth till the last one which is 20.

(Refer Slide Time: 17:28)



Now, in this graph I have mentioned very simply that how does the Lp norm value change as I change the values of p. So, on along the x axis I have and you I have the values of p 1, 2, 3, 4 till 100 I have taken till 100 and along the y axis I have taken the values of L p.

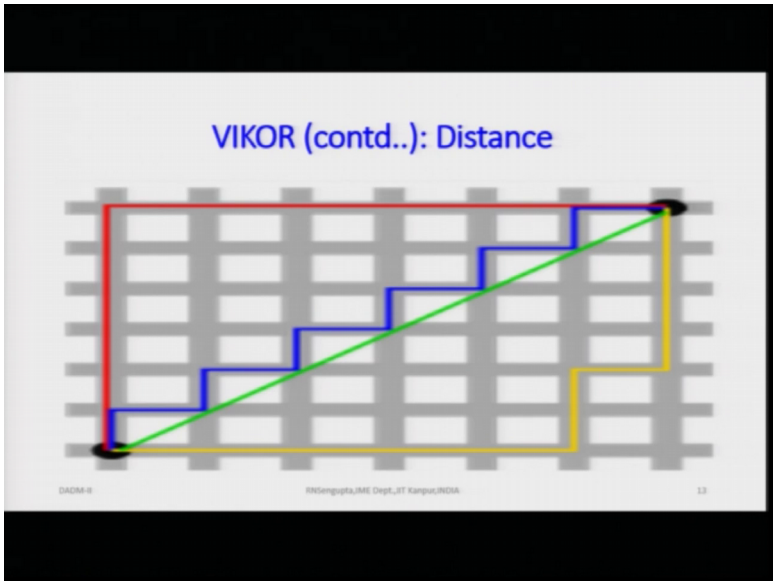
So, for one it is 54, if you remember for 2 it is basically 30.55 whatever or 3 so on and so till it goes to L infinity which was if you remember the value which you found for alpha in infinity which was 20. It is almost coming out to be 20. Slowly it will become 20. So, if you consider the values it goes like this slowly asymptotically it becomes 20. So, if you increase the dimension.

So, here we are 3 dimension in the x axis, y axis and z axis. See for example, we consider x 1 axis, x 2 axis, x 3 axis, x 4 axis so on and so forth and then we find out these values which was n dimension. So, here n is 3 that is why its 3 dimension. If you have n dimension you can find out that the L 1, L 2, L 3 and same similarly and infinity norm would have these characteristics where you can find an asymptotic curve based on the value of p changing which will have an effect on the L p norm value changing.

This is I just thought i will mention it to what you are under string. So, as you change it you will have different ranking system and ranking system in the L infinity norm would almost be equal to each other because closer the values of p it is towards one higher the

distances values if you notice it here. So, this I thought I will should explain it to you such that you understand when you do the problem.

(Refer Slide Time: 19:31)

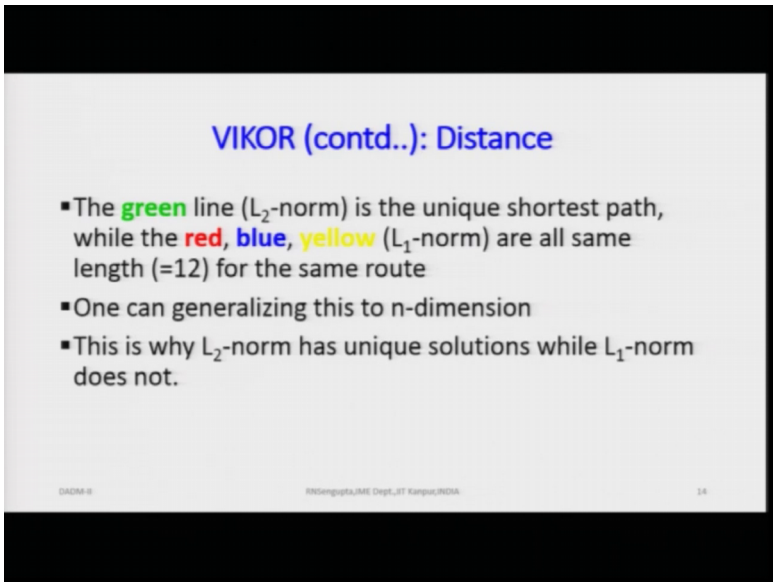


Now I will illustrate another graph which will make things much easier. So, consider the grid system as shown in this slide and the grid system is in a Cartesian coordinate 2 dimension. So, here we have 2 points, the first one and market is let me market is some color because already we have this color would be different.

So, I have x vector point, similarly y vector point whatever they are and it is 2 dimension. So, it will be x 1 y x 2 for x vector and similarly you have basically y 1 and y 2. We are not considering z 1 and so on and so forth in third dimension. Now look at these curves. So, this green one would basically make you simply understand that we are taking the L 2 norm and similarly blue, red and yellow would give you a concept that we are basically used the L 1 norm different ways you take it L 1 norm.

So, I will pause here have a look such that it makes life easy for us to understand. So, you have x vector which is a point and a Cartesian coordinate y vector and also another point of the Cartesian coordinate green line is the L 2 norm between x and y. Red one, blue one and yellow one on the different ways you can reach from x to y and these are the L 1 norm.

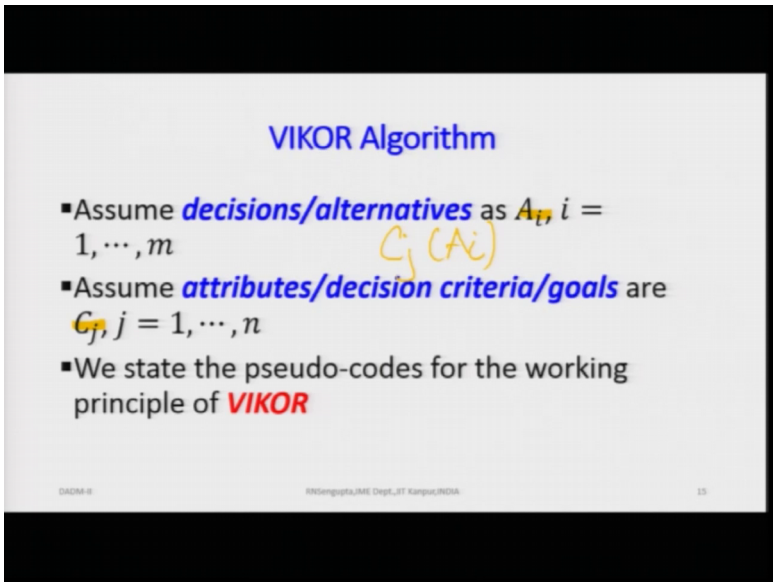
(Refer Slide Time: 21:22)



So, now it states, the green line which is the L 2 norm is the unique shortest distance while the red, blue and yellow L non norm are all same and the length of 12. So, if you remember what we have seen in the L 2-norm which was 54 in for that calculation which we did. So, it will be same in whichever direction in the 3 dimensional case you are going. So, in this case is the Cartesian coordinate we have moving from x to y in different ways and in that other example where it was 54 it was basically would have been cuboid with some x and y values whatever it was stated and you can basically take different ways of reaching from x to y.

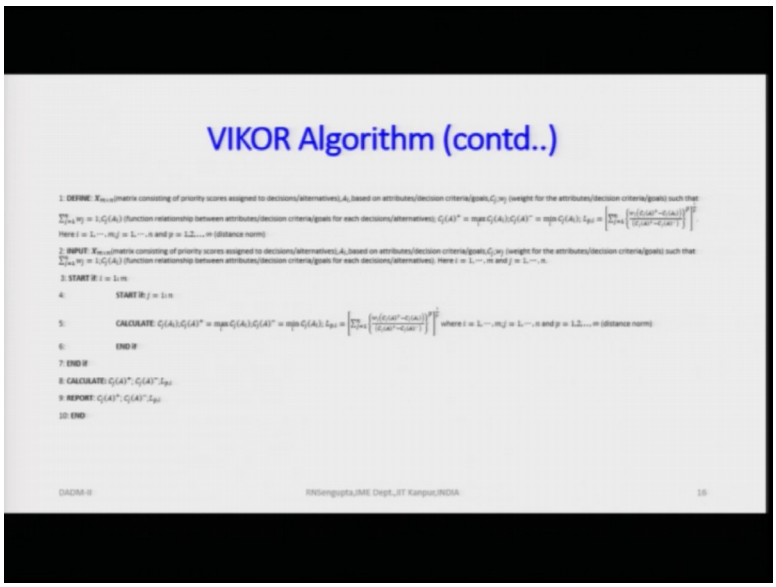
One can generalize this in the end dimension case which I mentioned. So, this is why the L 2-norm has a unique solution. Well the L 1-norm would not have the unique solution. This would have an implication that when you are trying to utilize the L 1-norm for the VIKOR method you may get different results, but further on for L 2, L 3, L 4, L p, till infinity your answer would be unique 0.1 and obviously, the difference between the answers would start decreasing as you go for the L infinity norm. This we will try to explain through the example. So, let us go through the VIKOR algorithm and the steps and solve it using the excel sheet for a problem. We will solve it here as we did in the last example.

(Refer Slide Time: 22:50)



Assume decision on alternatives as A i, i is equal to 1 to m and we also assumed the attributes on the decision criteria goals are given by C j; j is equal to 1 to n. So, these as are being affected by the weights of the criterias and the effects of the criterias which they accrue to the alternatives. So, this is basically when you find out the C j along with A i you had the functional form as C j A i based on that we proceeded. We state the pseudo code for the working principle of the VIKOR method and how it works we will state it very simply as we did for the TOPSIS method.

(Refer Slide Time: 23:41)



Now, it looks complicated, its not you have to just assume the steps and we will basically utilize this accordingly. So, I would not go to immediately the definition define step which is the step one, I will only go to in the input and then go back to the once I finish the overall algorithm on the flow process. I will go back to the defining state which is the first one. So, as usual you will have 2 sets of matrices; one is the x 1 which is of dimension m cross n, which are the priority values which you have for each and every criteria and each and every alternative.

So, as it is of dimension m cross n as it states. So, x is a m cross n matrix consisting of priority scores assigned to decision alternatives A i corresponding to the fact that the criteria goals are given by C j and the weights are given by w j. I am going to come to the weights again even though I have repeated it, but its just way of discussion such that the logical flow remains.

Now, in this case again the value which you will have in the matrix x non normalized one. So, say for example, I have x 3, 4 it would mean that for the fourth criteria what is the weight or the priorities accrues to the third alternative, similarly x i j would basically be the g th criterias priority weights or priority values accruing to the i th alternative. Now these are for the different attributes.

So, they need to be normalized for the normalization case we have considered that if we use for each and every decision maker the case that what would basically be the utility function, but here we are considering the fact that the in distance measure should be used. Now again I will repeat few things which I have done that in the ELEKRA process in the TOPSIS process. Remember one thing that if you consider the relative distance measures to be fixed continue using that calculations for each and every steps for the same decision maker point one.

Point number 2, it has it may be true that the distance measures being utilized by decision maker 1 or decision maker 2 or 3 or 4 considering. There are 5 decision makers may change, but for simplicity we will keep the decision may distinct majors for each and every decision makers to be the same. So, if it is basically L 3; that means, p is equal to 3 we will continue utilizing that for each and every decision maker. As I said we will take the values of the weights which I did mention fittingly. So, weights again are basically the priority which you are trying to put for each and every criteria amongst

themselves. So, if you have a weight matrix of size n cross n, it means the principle diagonals are values corresponding to w 1, w 2 till w n and off the diagonal elements are 0.

Which means that w 1 is the overall weight which I can accrue when I compare the first or my criteria with respect to the rest simulate w 2 would be the weight which I can give to the second criteria with respect to the rest. Similarly for the last one which will w n and again we will very simply consider the weight should be add up to 1 or they are basically being normalized on a scale to 1 or 100 whichever you want to do. Now once you have this x and w you will basically try to find out that considering the values of x which you have which have different type of attributes priorities you need to normalize them. When you need to normalize them again the question would come whether you need to basically normalize along the row or along the column.

The question would again I will give the answer is that if you follow the principle or normalizing along the row continuing that same concept, if you try to basically normalizing along the column continue to utilize the same concept. Now when there are many decision makers for simplicity we will consider that as utility functions are same as discipline measures are same as the distance measures are being utilized by any decision maker for step by step are same we will also consider the normalization concept also to be the same between different type of decision makers.

So, once you basically have x you normalize them along the row or along the column and once you do that you will basically are at the stage where you can basically start off the process of trying to find out the distance measures.

Now, as required we will consider that the actual matrix, based on which we will try to proceed and compare the different type of distance would basically be the multiplication of the priority weights, which are normalize multiplied by the weights, which means that we have been able to subsume or consider both the priority weights or a nominal scale plus the corresponding weights which accrue to each and every criteria. So, utilizing that f matrix so these are the generally nomenclature I am trying to utilize, we will utilize this f matrix in order to compare the ranking system considering the L p norm, so p would can be changed.

So, as I showed you into the graph the asymptotic one as you change the values of p you can have different type of rankings, but the ranking differences would basically start decreasing as p increases. With this I will close this lecture which is the 2nd lecture in the 8th week and consider more about the VIKOR method in the subsequent 3 lectures which are therefore, this week.

Have a nice day and thank you very much.