Data Analysis and Decision Making – II
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Lecture 36
VIKOR

Welcome back my dear friends and dear students, a very good morning, good afternoon and good evening to all of you and wherever you are. And as you know this is the DADM, which is Data Analysis and Decision Making course, II under the NPTEL MOOC series and this total course duration is basically for 12 weeks, which is 30 hours. And 30 hours gets converted into 60 lectures each lecture being for half an hour. And each week we have 5 letters, each for half an hour and we have already completed and as you can see from the slides, we have completed 35 lectures, which is 7 weeks completed and 7 assignments.

When you are going to take this 36 lecture, 7 assignments have already been completed by you. Now, in the last two lectures, two sets of lectures in the weeks, it was basically in the TOPSIS and the ELECTRE, if first the ELECTRE and then in the TOPSIS method. And this week we will try to cover and try to cover a little bit before and within 3 an or about lectures being 3 and half about the method of VIKOR method.

And mine good name is Raghu Nandan Sengupta from IME department at IIT Kanpur. Now, the concept of VIKOR, TOPSIS, ELECTRE, MACBETH, whatever methods we are going to consider they are almost the same is basically, either based on distance function is this, based on affinity, either based on concordance, liking, discordance disliking. And; obviously, here distance being and that how far and how close they are with respect your ideal solution, best solution, the natural solution say would be calculated and you will basically find out the ratios accordingly.

If you remember that in my ELECTRE and more so in TOPSIS I mention that many I trying to find out the distance with respect to the best ideal or best negative solution best positive solution best negative solution which is PIS and NIS, positive ideal solution and negative address solution. I kept mentioning that you will basically have four different combinations of ranking.

One based on that how close it is to the PIS, second would be how far it is from the PIS, third case would be how close it is to NIS and the third fourth case would be how far it is with respect to NIS. Now, technically it means that if it is closer to PIS; obviously, it will give us an information that is further form NIS. So; obviously, the first and the fourth ranking should be the same, but is not so. As I mentioned that we are going to consider the concept of asymmetric functions and how they can be analyzed, would be given preference in the practical examples.

A similarly, when you are considering that how far it is from PIS and then how close it use to NIS; obviously, they should be same, but they are not. So, again they would be an asymmetricity in the ranking system, in both the methods. So, technically we will have this four columns, which I again mentioning closeness to PIS, then it will be how far it is from PIS, third column would be basically, how far it is from NIS and the fourth so, it will be how close it is to an NIS and basically, how far it is from NIS would be combined together.

We will try utilize some of this concept, but the distance measures would be changed. Now, distance concept I have discussed, mentioned about Manhattan distance, which is l 1 norm, mentioned about l 2 norm within Cartesian coordinates, mentioned about the l infinity norm, mentioned about Hamming distance, mentioned about Mahalanobis distance, those who are in the two slides which were there and I did mentioned about Bhattacharjee distance and others, other things I do not think so I just mentioned it. So, these are different about distance measures, which are used you can use the weighted concept also on the distance.

We will try to use the very simple case of Lp norm and what is the Lp norm I will try to a solved given actual values, not for the problem for the VIKOR give actual values how that Lp norm can be calculated and also visualized if possible in and maximum in the three dimension. You can at least understand in the two dimension case and the three-dimension case, but in the higher dimension it need, would not be possible. So, I will try to give us simple example before we solve the concept of VIKOR. Now, this distance measures which I am going to mention would technically be applicable for say for example, for the TOPSIS method, may be applicable for the MACBETH method whatever method.

Maybe in this measure when you are doing VIKOR given example will be, we can much clearer to you such that you can appreciate, whatever we have covered in the ELECTRE, whatever we have covered in TOPSIS, whatever we have covered in AHP. Even though AHP did not consider the distance measure, but the point ranking system of 1 3 5 7 9 and correspondingly, if it is further away and; that means, you will getting some dis utility, it would be one-third, one-fifth, one-seventh, one-nine.

They can be just accordingly with respect to the distance measure and more affinity, less infinity; obviously, it is something to do with distance. More affinity, less affinity would also have something to do with utility function. So, they are not directly related, but there is definitely some a huge amount of underlying relationship or in the concept wise with respect to distance, affinity, concordance, discordant concept utility function and so on and so forth.

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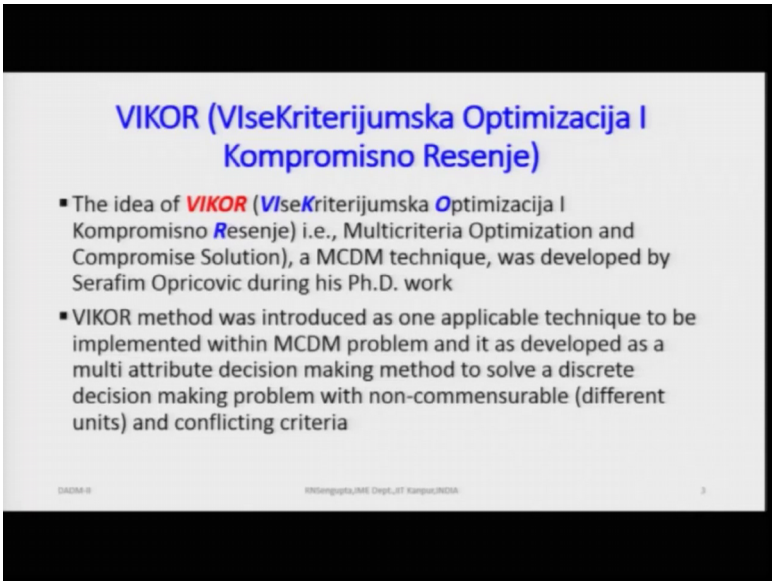

So, let us start the VIKOR method. The VIKOR method is basically, this was developed in Europe by during the work of the Ph.D work or Serafim Opricovic, in most probably in the 1990s and 2000. So, if this is the multi criteria optimization based on compromise solution. So, you are trying to compromise, find a best compromise solution between the best and the worst.

See this is an MCDM technique, as I said it was a part and parcel of the Ph.D work of one of the persons whose name is mentioned. So, VIKOR method was introduced as one

applicable technique, which could be utilized within, there are different types of MCDM techniques it can be optimization, nonparametric, parametric methods. So, it is one of those methods and it is as developed as a multi attribute decision method. So, here characteristics are important rather than variability, like in variability of Mathematical function in order to denote the how good or how bad the optimization concept is. So, if you want to maximize f of x corresponding to some subject to some constraints you will find out that the objective function has some decision, variable, some constant, some may be some deterministic one, some stochastic one.

And using the constraints we find out the boundary area of the feasible solution and try to find out the best feasible solution in that feasible set of solutions in that feasible solution space, which meets the criteria and gives you the optimized objective function, it can be maximization, minimization combination of maximization and minimization depending on the multi objective optimization ideas. Now, you in order to use this multi attribute. So, attribute would be characteristics like it would be good, bad, red colour, white colour, it can be very cold, cold, warm, hot, it can be based on marks, where marks are basically given side legs.

Like say for example, you want to compare how good, the bad the teacher was or how he has been or she has been able to deliver the lectures, corresponding with the problems which have been solving class then; obviously, some ranking system can be utilized. So, in that ranking system you are basically being trying to basically be a subjective of possible with and objective view in the sense that you are giving some marks based on your idea of what is the ranking should be.

So, attributes are characteristic. So, it is a multi attribute decision making method to solve a district decision problem. So, district is that either you rank alternative a l greater than a k or you rank a k, better than a l depending on different type of criteria's which are, which you have. So, in this decision making problem, which have non comment suitable or different units and conflicting criteria. So, units may be different, some maybe in liters, kgs, rupees, yens, dollars. So, the units are different they may be some numbers and you want to combine them in such a way that some parities or is there between the criteria such that comprising or comparing the alternate is based on the criteria's and then when you compare the criteria among themselves becomes much easy.

So, if we remember in this TOPSIS method, we had consider the criteria matrix, which was based on the utility function, if you remember and then we considered some weight. So, in the initial case the priority matrix was basically m cross n, where m is the number of alternators n is the number of criteria then we are basically the matrix W capital W, where the principal diagonals are given by weights W 1 to W n. So, which means that is a size of n cross n where the off the diagonal elements are 0. So, this W 1 W 2 till W n are the corresponding weights you will assigned to the criteria's depending on what importance do you think the criteria's have the final say in trying to help you make a decision for the alternatives.

So, here the criteria's are conflicting and the weights; obviously, would be conflicting correspondingly, but we will not be considering the weights immediately for this problem.

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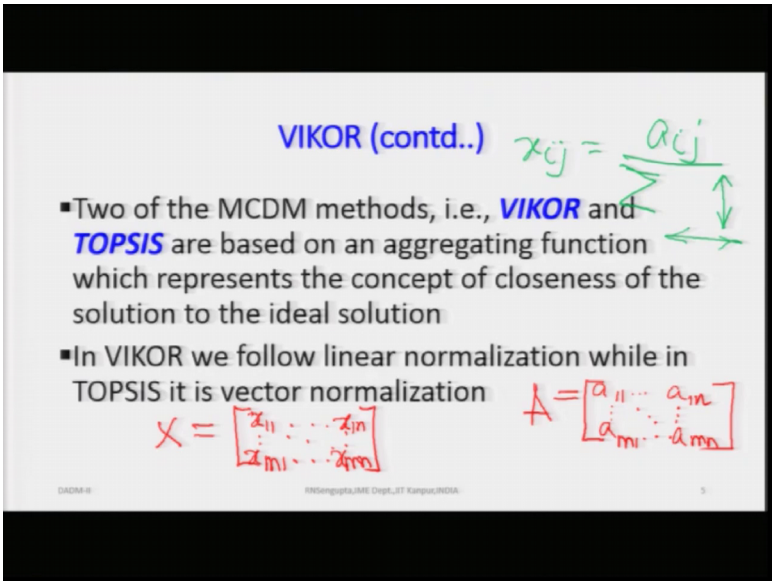

In this method, VIKOR method, decision makes a solution that is closest to the ideal solution. If you remember in the concept of concordance and discordance what we considered was that how close the decision was and you basically had this, those values of j, j means 1 to n number criteria which can be clubbed as a concordance set c or a discordance at d depending on concordance being the characteristics that how close it is and how like likings are similar. And in the concept of discordance we consider that how dislikings basically plays an important factor.

So, in this case we will consider the closest solution for the idle and; obviously, the further solution would be the non-ideal one how far it is exactly like what we have done in the TOPSIS method and hence the decision alternatives are evaluated compared ranked accordingly. So, you will consider the criteria and rank the alternatives which is as 1 to m based on the criteria c s which in from 1 to n, m as in mango n as in Nagpur. While ranking the decision alternatives rather than the best solution it is basically the target would be to find out the ideal solution which may not be most feasible.

So, you have to basically find the compromise and compromise would basically depend and what type of distance measures you are going to use. But rather the closest one to that compromise best solution will be utilized. Maybe it is not possible to find out the best solution. Consider this when you have dice and I am just giving a very simple example, you know order to make you understand that ideal solution or the best ideal solution may not be possible.

So, consider you have a dice, you playing a game of Ludo and consider the dice is unbiased. So, the faces are 1 to 6. Now, when you basically roll them the numbers which are coming are all discrete numbers. So, 1 will come within probability one-sixth, 2 will come with a probability of one-sixth, similarly 3 will come with a probability of one-sixth, 4 will come with a probability of one-sixth. Similarly 5 and 6 will come with a probability of one-sixth. Now, if I consider and ask you the question that what is the expected value, what is the no so called normal value, which you will expect, if you keep crossing this so, what will your answer would be, you will answer would be?

You will basically multiply the corresponding realize values with the corresponding probability. So, it will be 1 into one-sixth plus 2 into one-sixth till the last value which is 6 into one-sixth. So, it will be 1 plus 2 plus 3 plus 4 plus 5 plus 6 divided by 6. Now, the value which you have in many of the cases, as in this case may not be the integer value, which means the values, which you have in front of you depending on the actual realize values which is the ideal solution which you are trying to aim is not feasible. So, in that case you will try to basically be pick up value as close as possible depending on the example which you have do the expected value or the best ideal solution of the best possible solution.

So, we will try to utilize the same concept in the VIKOR method. Now, we will basically devote some from few minutes about the comparison of the VIKOR and the TOPSIS method and; obviously, you will have similar comparison between the VIKOR TOPISIS, VIKOR ELECTRE, ELECTRE TOPSIS and all this comparison can be done. So, two of the MCDM method, which is the VIKOR and the TOPSIS are based on aggregate function; that means, you aggregate the so called scores. Scores are dependent on the closeness.

So, if you remember we found out the C set, we found out the D set, any of the matters, then we found out the function which was basically the combination. So, s plus and s minus s i plus and s i minus would be give you the closeness in the positive sense and other would be the closeness in the negative sense how far it is. And you will try to find out the ratios of si minus divided by si minus plus si plus that is one way and another can be si plus in the numerator divided by si plus plus and s i minus. So, when you are basically considering those.

So, they are basically aggregating, the characteristics of the functions which will represent the concept of closeness of the solution to the ideal solution. So, closeness and how far it is basically would give you some concept. So, when you are considering in the concordance, it will give you the closeness when you considering the discordance, it will give you how far it is.

So, in VIKOR we follow the linear normalization while in TOPSIS it is the vector normalization. So, when you are trying to follow the normalization either along the column along the row. So, we are basically trying to follow in the TOPSIS method, vector normalization. Vector normalization can be based on the fact what is the logarithmic utility function which you have, the quadratic utility function which you have.

The power utility function which you have, even though I did not discuss it in the problem solving, but I did not mention it time and again and in the VIKOR method will use the linear normalization concept, which would be just the simple concept of distance function. So, it can be say for example, if you have the matrix x, which is the criteria values. So, you will basically have x i j divided by the sum of either the columns or the rows.

So, if I am basically considering this, so you will basically have X, which is the vector x 1 1, x 1 n, because you remember the number of rows and columns are accordingly x m 1 x m n and each of these a s, which will find out. So, consider now, let me put the corresponding.

So, it will be like this, this will be a a a, this is capital A a and we have X which will be the normalized case. So, when I am taking the normalized case it will be x 11, x 1 n, x m 1, x m n. So, any value which I have, I will write it using different colour. So, if I have basically x i j it will be a i j

Now the summation which you are doing would be basically along the column, it is either this way. I am using just pictorial ways or another we along the rows. So, in this case when you are considering the rows or the columns remember I am just repeating it. Remember the normalization concept, we such that the sum should be one. So, here just its I did not divert.

But I just want to mention that in the TOPSIS the vector nomination depending on whichever method is used was applicable, but for the VIKOR one we have the linear normalization depending on the distance.

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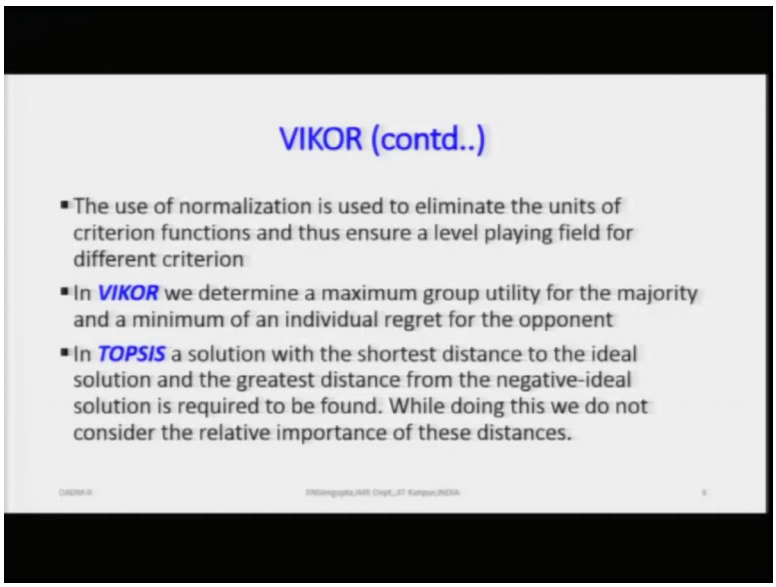

The use of normalization, you use to eliminate the units of the criteria functions. So, you have different term unit. So, if a, remember that we are using the concepts of can be

weight the litres length. I am just giving a very simple example, so it can be manpower rupees, electricity amount of water used.

So, you will try to eliminate the difference in the units. So, you will eliminate the units of the criteria function and thus ensure a level playing field for the different criteria's; such that when you find out the weightages or find out the effects of the criteria's on the alternative. So, it will be the day term should be such that you can compare them and make a decisions accordingly.

In VIKOR we determine and a maximum group utility for the majority and a minimum for of an individual regret for the opponent. So, basically you will have regret, is basically something to do with the discordance. So, you will use the closeness, said to basically group them for those criteria's and the alternatives and such a way that how close they are positive benefit. And if you find out some of the criteria's have negative effect on the on the alternatives you will group them and find out the dis utility, which is in some way the discordance concept. In TOPSIS method of solution, TOPSIS means, whatever we have done in the seventh week. In TOPSIS solution with the shortest distance to the ideal solution and the greatest distance from the negative ideal solution is required to be found.

So, PIS and NIS who are being repeated time and again which I said, while doing this we do not consider the relative importance of the distances, we just find out the positive and a negative PIS and NIS, the distance, we do not give any weightages to them. So, we could at a say for example, if the weightage is as close they are, they would be given more weightages. Like say for example, you are doing the forecasting method. Forecasting method I did consider in a very simple way in DA DM 1.

So, if you have the forecasting method and you are trying to find out that how good or bad the forecast is, and you doing say for example, exponential smoothing which means that closer it is. So, if you are basically predicting for t using the data for t minus 1, t minus 2, t minus 3, then; obviously, more weightages would be given to t minus 1, a little bit less where is to t minus 2 and so on and so forth. So; obviously, here the weightages and playing an an important fact so in the similar way if you are, if you are doing the conjuring, the distance functions as being weighted, then; obviously, it will have a different implications that closer or further would have the weights.

Such; obviously, you can have the weights in such a way that you want to basically ensure the sum of the weights should be one, as we have done in the case of forecasting method.

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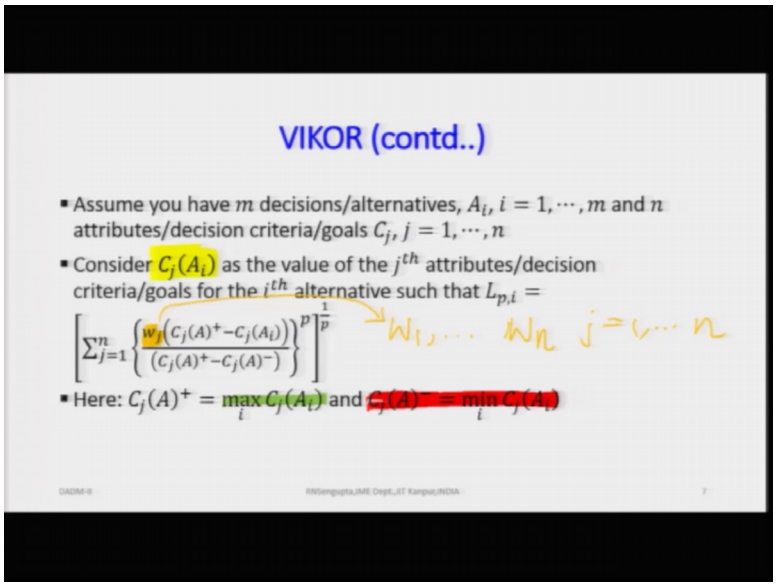

So, assume now we will just go through the simple nomenclature. So, assume there are m decisions or alternatives. So, A 1, A 2, A 3 till A m this they are the suffix, while you have n attributes or or decision criteria of the goals, based on which you are trying to basically analyze the alternative. So, they are given as C js, j is equal to 1 to n, and consider the functional form of what is the relationship between the jth attribute of the decision, criteria of the goals with the ith alternative is given by that L p norm distance. So; obviously, if the third criteria with respect to the fourth alternative has some relationship that will be given by the distant functions and that would be based on the Lp norm.

So; obviously, we know that we can change the distance function of L 1 L 2 L 3 L infinity norm, based on what is the importance you want to place on the concept of distance affinity characteristics. So, we will consider the L infinity norm. So, what we do? We will give some weightages. Now this weightages would be given late I will come to this later on. They have something to do with the concept of the weightages matrix and process and which we consider. So, if you remember here those n cross n. So, I will

also highlight it here. So, this a W j, actually this means I should use a different colour, so it will be easier.

So, this W j which is there, so if I use this so this will mean I have W 1 till W n, because g is equal to changing from 1 to n. Now let us look at the concept of the relationship. So, what is actually being done if you look at this, great way. If you look at this function, this is general function. It means that when I am trying to find out the characteristics it does not give the difference between the alternatives based on a criteria remember that.

So, what I am taking is that for any alternative based on any criteria what is the overall weightages or the, not weightages the so, those so called the criteria's weightages you will give based on the alternative. So, as A changes so considerate for A 1 you have, if the function is basically C 1 A 1; that is for the first criteria and the first alternative that will give me so called the criteria's. Characteristics in the positive and the negative sense, it will come out when you are trying to analyze alternative 1. Similarly C 1 A 2 would give me the characteristics of what is the weightages, when the positive sense and negative sense which criteria, one would have one alternative to.

Similarly, C 1 A 3 C 1 A 4 till the last one which is C 1 A m. When you go to the next criteria, it will be C 2 A 1 I A m 2 n 1. All this numeric values which I am stating are in the suffix position, C 2 A 1, C 2 A 2, C 2 A 3 till C 2 A m. Similarly you will have C 3 A 1, C 3 A 2 till C 3 A m and the last would be C n A 1, C n A 2 till the last one which is C n A m.

So, once you have that you will basically try to find out the maximum of the minimum. Now here where the so the actual concept comes, so say for example, I am trying to, I will use colours definitely. So, positive I will use the green colour, and negative I will use the red colour. I will already highlight them and then basically discusses recording. So, when you are trying to find out the maximum of Cj Ai for all the ''s. So, for give fixed j as 3, say for example, and I find out the functional form of C 3 A 1, means C 3 A m and find out the maximum of that.

So, I will basically place it for that j th criteria, what is the maximum for, which alternative is coming. Similarly when I just change j is equal to 4, j is equal to 5, j is equal to 6, till i go to j in n. I find out the maximum corresponding to js, I find out the maximum for each and every alternative market. So, this will give me all the positive

concepts. Then when I go to the negative one, again I will take the same row or the same column wherever; however, you have basically able to define in those concept of season s I take the minimum.

So, once I have the minimum maximum, it means that for that particular criteria what is the maximum concept which I am getting and what is the minimum concepts which I am getting, and basically I will wait them with the functional form that what is the weightages for those criteria's which I have now. If you remember these weights are exactly, the weights which you have consider for W so W the principle diagram was found W 1 to W n and the off the diagonal element was 0.

So, these are the basically the importance which are going to assign for the criteria. So, once you have them you try to find out the corresponding L p norm for that, based on the fact that you are taking the, the distance function as L p norm, find them out the values for each and every corresponding concept of the criteria's.

So, for W 1 it will basically multiply the weights in the numerator as W 1 and find out the ratio, then you do it for W 2 W 3 till W n. That means, for j is equal to 1 to n add them up and then when you find, use the concept of L p norm you will basically have that so called weightages of the distance based on which you can rank them. So, with this I will end the first lecture for the 8th week and consider in the more discussion about the VIKOR method in the last 4 lectures.

But one thing I will try to do is that in this VIKOR method I will try to give you an example initially for the distance measures within the very simple concept of the two dimensional and the three dimension one, which I did mention when I started the class. And then take up that simple example of the VIKOR, such that it will be much easier for you to understand and appreciate that as you change the linearity of the functional form because if you remember in VIKOR method it was mentioned when you trying to compare the TOPSIS.

In the VIKOR method I mentioned that VIKOR method is the linear normalization, while TOPSIS method is vector normalization. So, we will try to basically use different type of linear functions and try to make one to one correspondence that how you change the linear functions, you will get different results. So, thank you for your attention and have a nice day.

Thank you very much.