

**Data Analysis and Decision Making – II**  
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**Lecture – 34**  
**Weighted Normalised Matrix, Proximity Measure**

Welcome back my dear friends, a very good morning, good afternoon good evening to all of you wherever you are in this part of this world. And as you know this is the DADM which is Data Analysis in Decision Making II course under the NPTEL MOOC series course. Duration is for 12 weeks which is basically 30 hours spread over 60 lectures each lecture being for half an hour and each week we have 5 lectures and we are in the 7th week and that 34th lecture; that means, with this one and the 35th one we should be able to wrap up TOPSIS in all the details.

Now, if you remember. So, I will now move from the slide to the excel sheet.

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So, if we remember let me remove this one just. So, if you remember we had done the normalization along the column. So, now, let us change the first case the type of normalization we are going to do. So, let me take I will go a little bit slow here and then speed it up as we proceed.

So, consider I am using the venue very arbitrarily log this divided by this sum log, this second one plus log on the third one plus forth one is I am talking about the cell position in the column. So, I close it. So, these are the values. Now so, obviously, keep a look at this value because that should turn up to be 1 now. So, this is 1 because normalization if you see again second cell third cell forth cell. I am not going to go do it for the others for the time being.

So, it can be done. So obviously, the values have changed. Now, if I do normalization along the row. So, I will do it for the simple case. So, this would be the first one divided by sum of all of them its row wise remember. So, I should basically have the sum here as 1. So, let me first put it. So, you can basically appreciate. So, this is done. So, I copy it see if you check the values comes out to be 1. So, this can be implemented each.

So, if you follow the normalization along the row or the column, for it throw out and again you can use the different utility function to do that. Let me again come back to the slide.

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**TOPSIS: Step # 01 (Construct the normalized decision matrix) (contd..)**

$$R = \begin{bmatrix} 0.133333 & 0.263158 & 0.143678 & 0.156522 & 0.500000 \\ 0.033333 & 0.094737 & 0.281609 & 0.278261 & 0.222222 \\ 0.300000 & 0.263158 & 0.367816 & 0.352174 & 0.222222 \\ 0.533333 & 0.378947 & 0.206897 & 0.213043 & 0.055556 \end{bmatrix}$$

▪ Check each column adds up to 1 as it should be

So, taking some normalization concept I have these do not be too much bothered about the values in the cells be bothered about the concept and used excel sheet to understand that. Double check the second bullet point that check each column adds up to 1 and it should be.

Because I am doing it column wise it should have been could have been also row wise which I just showed. So, technically what would these values mean these values would mean that they are a normalized scale on one; that means, the bed total benefit if you consider the first column total benefit coming from criteria 1 is 100. Out of 100 ultimately 1 is getting about 13 percent, alternative 2 is getting about 3 percent alternative 2 and 3 is getting about 30 percent and alternative 4 is getting about 53 percent.

So, if there are resources they are being distributed in the ratio of 13 is to 3 is to 30 is to 53. Similarly if I come to the last column for the 5th criteria this criteria then the corresponding so, called weightages or the values which are accruing to alternative 1 2 3 4 are correspondingly given as and just I should use the pen color to on making add these 50 percent for first 22 percent for the second 22 percent for the third and 5 percent for the fourth I am not reading the decimal values. So, this is the normalized matrix which I have.

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**TOPSIS: Step # 02 (Construct the weighted normalized decision matrix)**

- If the decision maker decides on the set of weights, depending on his/her preference, then the weight,  $W = \begin{bmatrix} w_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & w_n \end{bmatrix}$ , such that  $\sum_{j=1}^n w_j = 1$
- Consider,  $W = \begin{bmatrix} 0.20 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.10 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.15 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.30 \end{bmatrix}$
- Calculate  $V = RW$

Now, the decision maker has to make a decision about the weights; that means, the weights which he or she thinks each and every criteria would accrue the importance relative importance of the weights by themselves within themselves such that the values with they are giving for each and every alternative would be multiplied by the corresponding weights to make a ranking corresponding to that. Because still now I am

only considering the values which are there in the matrix  $r$ , are for each and every criteria on a standalone basis.

So, if I only consider the first column I am not considering the other 4 criteria if I am considering the third column I am not considering the criteria 1 2 4 5. If I am considering the fifth column I am not considering criteria 1 2 3 4, but now with the weights I will basically weighed them up and give a priority. So, these weights would basically give you the priority amongst the criteria themselves.

So, if the decision maker decides on the sets of weights depending on his or her preference the weights would be given by the matrix of size  $n$  cross  $n$  where the principal diagonal with there would be the weights  $W_1$  to  $W_n$  all the prince other than principal diagonal the value should be 0. And one should remember that the sum of the weights is 1 as it should be 1 or 100 200, but it should basically be normalized.

So, if it is 200, I can you divide by 200 it gets comes back to 1. So, the weights are given which are for this problem 20 percent for criteria 1 10 percent for criteria 2. So, let me mark the principal diagonal. So, third and then the third value 15 percent for criteria 3 25 percent for criteria 4 and 30 percent for criteria 5.

So, now see here this matrix is a size 5 cross 5s which is basically  $n$  cross  $n$  which corresponds to the number of criteria which you have nothing to do with the  $m$  value which is the number of alternatives.

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**TOPSIS: Step # 02 (Construct the weighted normalized decision matrix) (contd..)**

• Thus  $V = RW =$   $\begin{bmatrix} 0.133333 & 0.263158 & 0.143678 & 0.156522 & 0.500000 \\ 0.033333 & 0.094737 & 0.281609 & 0.278261 & 0.222222 \\ 0.300000 & 0.263158 & 0.367816 & 0.352174 & 0.222222 \\ 0.533333 & 0.378947 & 0.206897 & 0.213043 & 0.055556 \end{bmatrix} \times \begin{bmatrix} 0.20 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.10 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.15 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.30 \end{bmatrix}$

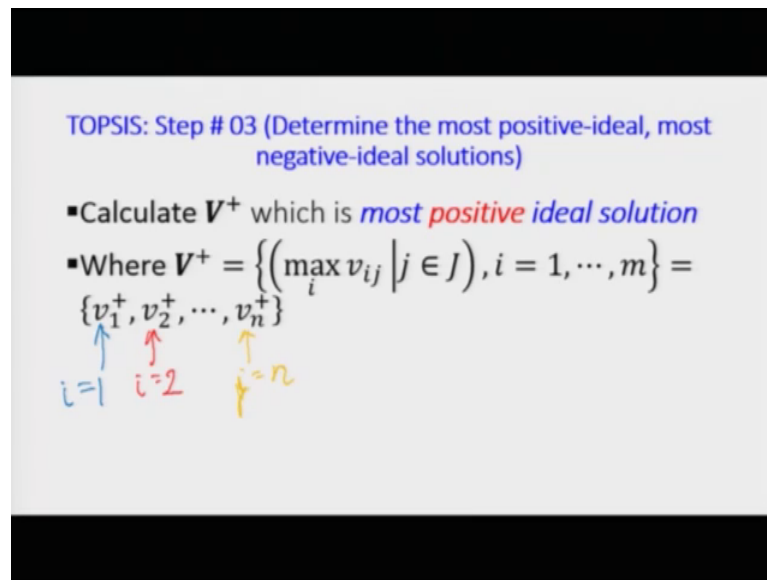
• Hence  $V = RW =$   $\begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{14} & v_{15} \\ v_{12} & v_{22} & v_{23} & v_{24} & v_{25} \\ v_{13} & v_{23} & v_{33} & v_{43} & v_{53} \\ v_{14} & v_{24} & v_{34} & v_{44} & v_{54} \end{bmatrix} = \begin{bmatrix} 0.026667 & 0.026316 & 0.021552 & 0.039130 & 0.150000 \\ 0.006667 & 0.009474 & 0.042241 & 0.069565 & 0.066667 \\ 0.060000 & 0.026316 & 0.055172 & 0.088043 & 0.066667 \\ 0.106667 & 0.037895 & 0.031034 & 0.053261 & 0.016667 \end{bmatrix}$

$R_{m \times n} \quad W_{n \times n} = V_{m \times n}$

Now, I multiply the value of R into W; that means, I am trying to now bring a parity on a normalized scale for each and every criteria along with the concept of the alternatives; that means, alternatives weights have been found or normalized then I am also ranking the criteria such that I find out a ranked normalized score or a matrix for each and every alternative with respect to the criteria.

So, I multiply this m cross n multiplied by this n cross n. So, the actual value which I will have I will I mentioned it time and again, but I still I write it. So, when you do that matrix R is m cross n it will be multiplied by W which is n cross n the actual value V would be m cross n. So, as it should be. So, this R is given here, this W is basically given here and this say for example, V once you find out is given here. So, you find it out accordingly.

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TOPSIS: Step # 03 (Determine the most positive-ideal, most negative-ideal solutions)

- Calculate  $V^+$  which is *most positive ideal solution*
- Where  $V^+ = \left\{ \left( \max_i v_{ij} \mid j \in J \right), i = 1, \dots, m \right\} = \{v_1^+, v_2^+, \dots, v_n^+\}$

Handwritten annotations below the set notation: a blue arrow points from  $i=1$  to  $v_1^+$ , a red arrow points from  $i=2$  to  $v_2^+$ , and a yellow arrow points from  $i=n$  to  $v_n^+$ .

Now, what you need to do is that you need to find out the values of positive concept and the negative concept, I am only considering the remember in the last lecture which was in the 33rd lecture I did mention that I will only consider the concepts of 4 columns which is positive negative benefit corresponding to how close it is to the positive Eigen solution and then again positive negative corresponding the fact how close it is to the negative solution.

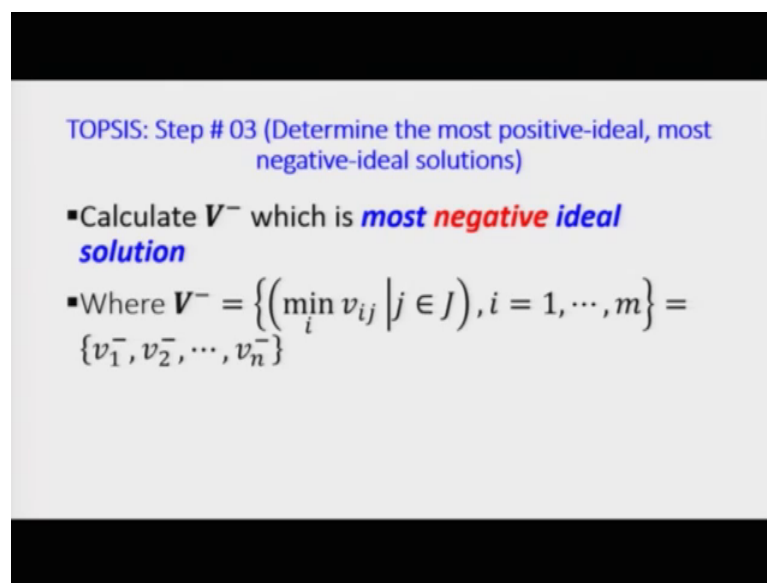
So, I will keep repeating it in order to make you understand. So, I will only consider the first the third column, I am not going to consider the fourth and the fifth they can mean, but they are the steps of calculation will be too much so, I will proceed accordingly. So, we will consider  $v$  plus means the ideal solution which says as it is. So, which is the most positive ideal solution? So, we are now determining the most positive ideal solution and also the most negative ideal solution we will come to that. So,  $V$  plus would be the maximum. So, if you consider either the row or the column you will consider the maximum of these values.

So, the concept whether you will consider either the row or the column would depend on the fact that which type of normalization have you done; that means, either the row 1 or the column one. So, follow one general rule says this parity. So, for each  $m$ ; that means, I am doing for each of these alternatives I try to find out the maximum for all the number

of criteria's which are there and I will write these values. So, write these values for the  $i$  is equal to 1 I have  $v_1$  plus.

So, this would be for  $i$  is equal to 1. When I go to  $i$  is equal to 2 is equal to  $v_2$  plus similarly when I go to  $i$  is equal to  $n$  as sorry in for each values I am calculating. So, I am getting the  $j$  value. So, I have basically the values corresponding to  $i$  is equal to the  $j$  value would be coming out. So, they would meet because technically should be  $j$ . So, I find out  $v_n$  plus.

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TOPSIS: Step # 03 (Determine the most positive-ideal, most negative-ideal solutions)

- Calculate  $V^-$  which is **most negative ideal solution**
- Where  $V^- = \left\{ \left( \min_j v_{ij} \mid j \in J \right), i = 1, \dots, m \right\} = \{v_1^-, v_2^-, \dots, v_n^-\}$

Similarly, I will calculate the  $V$  minus negative values corresponding to the most negative ideal solution. So, it will be the minimum.

So, I have the maximum the minimum corresponding to the ideal set this is where the maximum minimum corresponding to the non ideal set, but I am only considering the first and the third column which I mentioned. So, here I find all the minimum values corresponding to  $j_1$  to after all the sets of  $j_s$ . So, it will be done warning to all the values.

So, I will find it out and then wrote write it down as  $v_1$  minus  $v_2$  minus the minus sign on the top basically gives you the non ideal or sorry in the negative ideal one. So, if you remember PIS and then NIS positive ideal solution and negative ideal solution.

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TOPSIS: Step # 04 (Calculate the distance based on most positive-ideal, most negative-ideal solutions)

- Calculate  $S_i^+$  based on **most positive ideal solution**
- Where  $S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}$ ,  $i = 1, \dots, m$

So, once you have that you will basically find all the distance function. So, distance function for each would be for each of the cell those that matrix  $v_{ij}$  which I have which is of size  $m$  cross  $n$ . So, each cell value would give me the.

So, the point in the coordinate system, where it is placed with respect to each and every criteria and each and every alternative. So, you will try to find out the maximum the ideal one best one you will try to find out how close you are. So, say for example, I try to give a pictorial one um. So, considered best would be green. So, consider the best one which is maximum is green here and I am considering the real line.

So, what I have the real line this is the positive solution. So, this is positive and go further you go. Now we will consider each and every criteria. So, consider the I will write it using black. So, consider the first one is here the second one is here, I am not using the color I am using the numbers only third one is here fourth one is here. So, what I am trying to find out is that I am giving the finding of the weights.

So, I will find out the difference between first and the green solution square it up. Second then find out the difference between the second and the green solution green is the most positive square it up, find out the difference between the third and the green solution square it up, and find out the difference between the fourth and the green solution square it up.



So, these are the distance function, once you have that add them up find out the I have so, called average square root. So, it will gives you gives you a so called weighted average and weights corresponding to the fact would be equally weighted because you are not basically finalizing anything. So, you will be signifying it for all i is equal to 1 to m.

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TOPSIS: Step # 04 (Calculate the distance based on most positive-ideal, most negative-ideal solutions)

- Calculate  $S_i^-$  based on **most negative ideal solution**
- Where  $S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}$ ,  $i = 1, \dots, m$

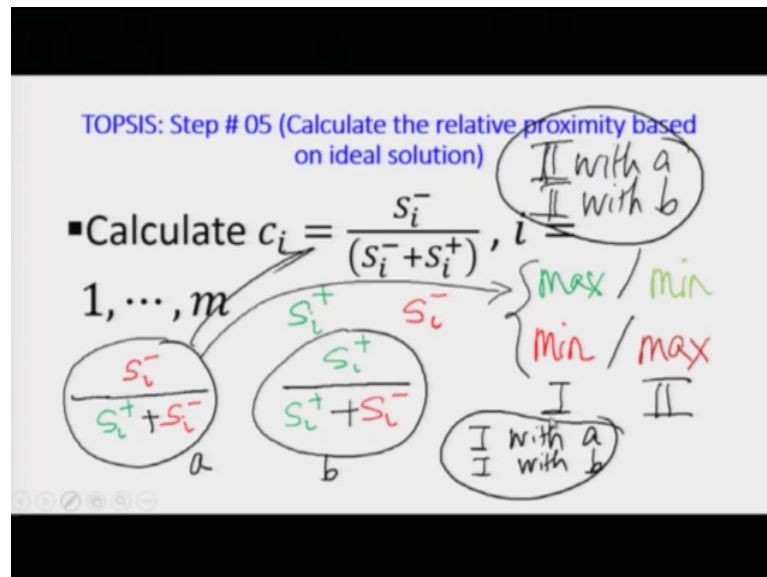
Now, as you have done for the positive one you will have the negative ideal solution also I need to find out the most net value which is  $S_i$  for each and every  $i = 1, 2, 3$  till  $m$ . So, it will basically have the number 9. So, I will first mark the negative one as red because I am trying to follow that policy of red being. So, consider the red one is here. So, I am not comparing the green point with the red point. So, if I have to do I have to draw the real line actually and for mark the green and red I am concerning them separately.

Consider 1 2 3 4 now in the earlier diagram when it was the green one those distances were nice to me good closer it is better for me, but now further I am better for me closer it is bad for me. So, I find out difference between the first and the red one square it up then find out the difference between the grade and the second one square it up, find the difference between the third and the red one square it up.

Find out the difference between fourth and the red one square it up. So, this is again equity penalized 1 finding all the differences squaring them within the Cartesian sense

and then adding up and finally, this is basically the simple distance which I am trying to find out. I do it for each and every this alternative and find out  $s_1$  till  $s_m$  plus similarly I find out from  $s_1$  to  $s_m$  minus; minus and plus are on the top.

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Now, once I have that I find out the relative proximity. So, how far and close it is. So, this would be very simple in the sense I have some positive  $S_i$  pluses are already there, then I also have the  $S_i$  minus. So, I will calculate the proximity. So, I am only using 1 set of calculation I am using  $S_i$  minus it could have been done in different ways. So, it would have been done I am just mentioned I am using the concept of. So, this is what is given I will first write and then write all the different ways of trying to find out the proximity values.

So, this would be I am using. So, I will write it down and then compare  $S_i$  minus. So, if this is the calculation is done, the value is given here. Now if I do a separate type of calculation which would be noted on carefully  $S_i$  plus plus  $S_i$  plus plus minus. So, I will have a second set of proximity values.

Now, this I have done how close or far they are. So, I found out the maximum corresponding to that. Now if I want to find out the minimum. So, if you remember I found out the max of the closeness. So, I could have done for the positive one. So, the positive one we were doing max and for the negative 1 we were doing the min and write it in the colored font.

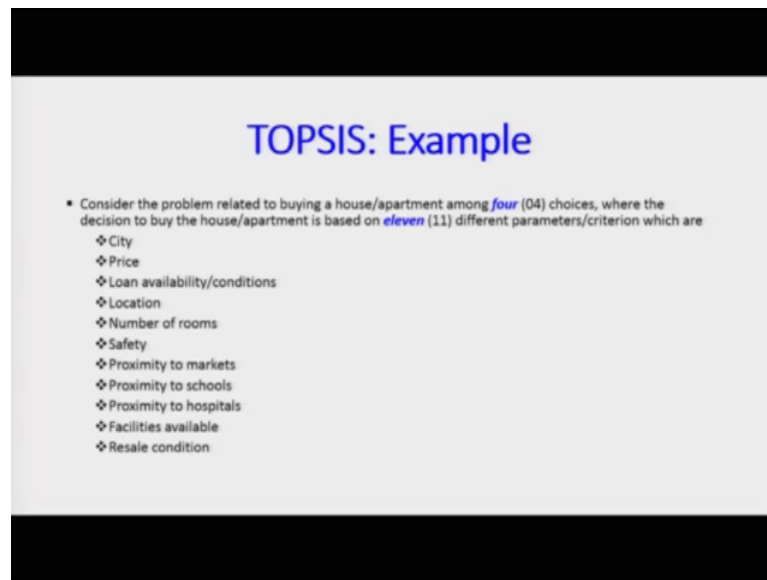
So, you doing the max positive benefit. So, based on this we will obtain these remember. The cell values which I have here are based on this concepts this has nothing to do with it we will change it you can find it out what you can do is that you try to find out the minimum and the maximum for these cases. Now these would be which is not done in the calculation for the problem, but I am just mentioning it. You find these values I am now using the coloured black.

So, it because it would be or I can use a different colour I will try to I try to a different shade. So, this would be minimum for the maximum case and this would be maximum or the minimum case. See if you find out technically. So, you will basically have one set of  $S_i$  minus divided by  $S_i$  plus plus  $S_i$  minus for the first comparison max min with coloured dark green and red black red then based on the dark green and dark red you can also have  $S_i$  plus divided by  $S_i$  plus plus  $S_i$  minus is the second methodology which is basically would give you the second column.

When I use the light green for meaning finding on the minimization and a dark brown sort of thing a blackish red for the color it will give you in the maximum. So, that will be in the maximum on the distance which is there from the negative sign that is further it is good, but whether it is positive I do not know. Negativity is decreasing whether it is giving positive values I do not know. So, based on that min and max which is on the second stage, I will basically have again  $S_i$  minus divided by  $S_i$  plus plus  $S_i$  minus again I have this fourth 1 which is the fourth column which will  $S_i$  plus divided by  $S_i$  plus plus  $S_i$  minus.

So, I can mention them as 1 2 mention them as a b and what I will have the combination is 1 with a 1 with b then also can I can have 2 with a and 2 would b. So, I have here and here. So, this will give me 4 different proximity values I am only considering one of them.

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### TOPSIS: Example

- Consider the problem related to buying a house/apartment among *four* (04) choices, where the decision to buy the house/apartment is based on *eleven* (11) different parameters/criterion which are
  - ❖ City
  - ❖ Price
  - ❖ Loan availability/conditions
  - ❖ Location
  - ❖ Number of rooms
  - ❖ Safety
  - ❖ Proximity to markets
  - ❖ Proximity to schools
  - ❖ Proximity to hospitals
  - ❖ Facilities available
  - ❖ Resale condition

Now, we will start a simple problem. Problem is if we give you the background consider the problem is related to buying an house an apartment among 4 choices in any city.

But I also consider that the city is preference is also under my jurisdiction whether the decision to buy in the house an apartment is based on the fact there are eleven different parameters criteria's which are which city you want to buy, what is the price of the house, what is the loan availability conditions of loan number where is the location in that city, number of rooms which are there for the house how many bedrooms kitchen drying room so, on and so, forth.

Guestroom what is the safety of that apartment how safe is the apartment? Whether they are security guards for that to taking care whether the security services their CCTV is there people are definitely would be bothered about that. but this is a gated community whether the people who were come strangers are asked where they want to go at a steady maintained for that

So, also you will be interested to know to know when you buy that house an apartment what is the proximity to the markets of that apartment house. So, you want to buy a few things, daily consumption rice, sugar, coffee, tea any vegetables potato, tomatoes whatever you how close it is you want to immediately get it. Then say for example, the person is buying that house and he or she has kids who are in school or in colleges.

So, you want the schools and the colleges to be near. So, that they do not spend too much time in commuting. So, you will also find out the proximity of that apartment on the house to the schools. I am just using the word school it may be the education hub tuition, coaching center, the kid go to say for example, to learn a different music some from Hindustan classical music or say for example, Odissi dance, Katak whatever then you will try to find out how close or far is it the proximity from the hospitals.

So, because old parents are there with you need medication or say for example, you some you suffer from fever because the weather change affects you. So, you are you are your family members. So, you want to be closer as far as possible close to the good hospitals or doctors whatever it is what are the facilities available if it is a guarded communities.

So, obviously, you would like to have nowadays people want a swimming pool and a super small auditorium, a community center, they should be a walking space there would be a space for the old age old people to sit down and talk amongst themselves. So, you like that like that to happen. And say for example, you also find want to find out what is the resale value conditions is it that if you sell that house you have to pay some amount of money as a as a caution money or something of the sort to the cooperative or to the broker.

So; obviously, those can be managed, but you would also like to know about that. So, now, here the apartments numbers is 4 which is  $m$  is equal to 4; that means, 1 2 3 4 number of alternatives are there  $a_1 a_2 a_3 a_4$ , and the criteria's are 11 in numbers  $c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8 c_9 c_{10}$  and  $c_{11}$ ; 11 of them are there which is  $n$  is 11. So, you based on that you want to proceed and find out the ranking system.

Now, let us the matrix of priority one is given to you; obviously, you can do that as if you remember for the concepts of a hp or analytical hierarchy process one can ask questions and then find out the values. So, I am just considering the values as it is given. So, they are if you consider the I am only read one of the row which is basically the first row.

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**TOPSIS: Example (contd..)**

▪  $X = \begin{bmatrix} 45.5 & 40.0 & 66.7 & 09.2 & 1.8 & 0.0 & 1.3 & 2.3 & 1.6 & 4.8 & 3.1 \\ 50.0 & 33.3 & 83.3 & 08.5 & 1.5 & 1.3 & 0.0 & 2.4 & 2.4 & 3.3 & 1.6 \\ 40.0 & 42.9 & 62.5 & 06.8 & 1.3 & 0.8 & 0.0 & 1.2 & 1.8 & 4.2 & 2.2 \\ 41.7 & 25.0 & 77.8 & 10.6 & 2.3 & 1.8 & 1.9 & 3.0 & 3.2 & 3.8 & 1.3 \end{bmatrix}$

▪ Use the normalization formulae as  $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^n x_{ik}^2}}$

So, values are 45.5 40 66.7 9.2 1.8 0 1.3 2.3 1.6 4.8 and 3.1 which means that if I take if I take this value of 1.3 it means for the first house the seventh important factor.

So, let us see let me go back to the first line 70 point of 1 2 3 4 5 6 7 possibility to the markets gives you a value so, called value of 1.3. So, if similarly maybe if I consider let us consider the 0 0 may be the 7th one sorry 7th one for house number 2, which is proximity of markets and house number 3 are does not matter they give you a 0 value.

Consider they are very far off; obviously, you will not be getting in value and closer it is you are getting the benefit. So, these values are given and you want to normalize them the normal factor again, I will use the same concept that is any sale value divided by the square root of the sum of the squares. And with this I will I will end this 34th lecture and try to wrap up with the 35th lecture the overall problem of how when you bring to buy a house how we will rank them have a nice day and.

Thank you very much for your attention.