Data Analysis and Decision Making - II
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Lecture – 03
Utility Analysis

Welcome back my dear friends. A very good morning good afternoon good afternoon good evening to all my dear friends and students for this DADM to course which is Data Analysis and Decision Makin - II. And we are in the 3rd lecture as you can see from the slide and this total course DADM – II is for; 12 weeks which is 30 hours. And each week will have 5 lectures each being for half an hour. And as you know that after each week we will have one assignment. So, total in total we will have 12 assignments plus there will be one final end term examination. And each week assignments would be based on that week which has just been covered.

So, if you remember we are discussing about utility functions you are discussing also that; depending on the outcomes. Now I did mention different connotations of outcomes and how the utility functions would change depending on the outcomes. They can be the number of outcomes which are favorable to one decisions. They can be the utility function them sell or they can be just numbers without mentioning anything about UW. And we then consider two different neutral functions two different decisions and considering different combinations of 2 into 2 4 we saw that how the decisions can change depending on the total value of the utility. We are always trying to find out the maximum of the utility.

And I also did mention without solving that if the utility is the same we will consider the variance to be used to rank the decisions and if both the utilities expected value and the variances are given we will use a combination of them where we rank the ratio of if we take the ratio of expected value to variance will rank them from the highest to the lowest. If we take the inverse of that that is the variance with respect to the expected value will rank them from the lowest to the highest. So, they can be different ways of trying to ranking depending on what your statistic is based on which you are trying to take a decision.

Now further continuing utility analysis so in the problem which we are considered where there was a value of minus four the function and if we remember we did mention if the utility function is negative we will purposefully consider them to be 0. Even though that may not be true, but in practicality for problem solving for this course we will take m 0 until unless mentioned in the problem. So, now the question is that with the and then we also considered a problem where you had a government bond of 1000000s and the utility was W to the power half which is a power utility.

And other hand you have three different options with probabilities of 20 percent 40 percent 40 percent and the outcomes were given, mm the government one was not 1000000it was 600000 sorry my mistake. And outcomes for the other non deterministic decisions what 10 5 and 1 and then we found out the corresponding expected value and found out the that the on that the government one was given a higher expected value than the probabilistic one hence we take the government one.

So, they can be different variants of the problems the probabilities can change the values can change that the rupees lakhs wealth can change the utility function change they can be different ways of trying to understand. Rather than using the probability as probability mass function we can have rode attends a fractional function also then you need to find out the expected value corresponding to the fact that we integrate the values rather than sum them up.

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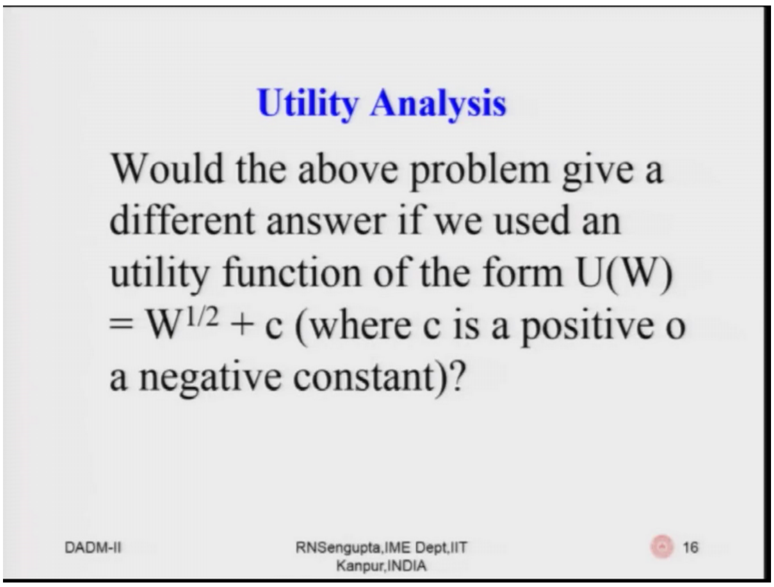

So, the question is that would the above problem give a different answer if we use a utility function of the form W to the power half plus c. Now c is basically is a positive or a negative quantity. So, if basically it is a positive or negative quantity. And any fair only considering the linear function only not comparing against each other. So, if we take the value of 0 to be the least value then; obviously, for higher negative values of c all of them would become 0 so and obviously, they would be at the same rank. But if we consider the value of c to be to be negative highly negative.

And then we considered that W to the power half plus c and the expected value is also considering the negative terms then obviously, we can rank them so the highest negative ones would be ranked the least and the lowest negative ranks with more towards 0 and positive numbers would be ranked the highest. And similarly if c is positive it will would not basically change the ranking system because they would basically in the values would come out in the positive range. So, the ranking relative 19 remains the same.

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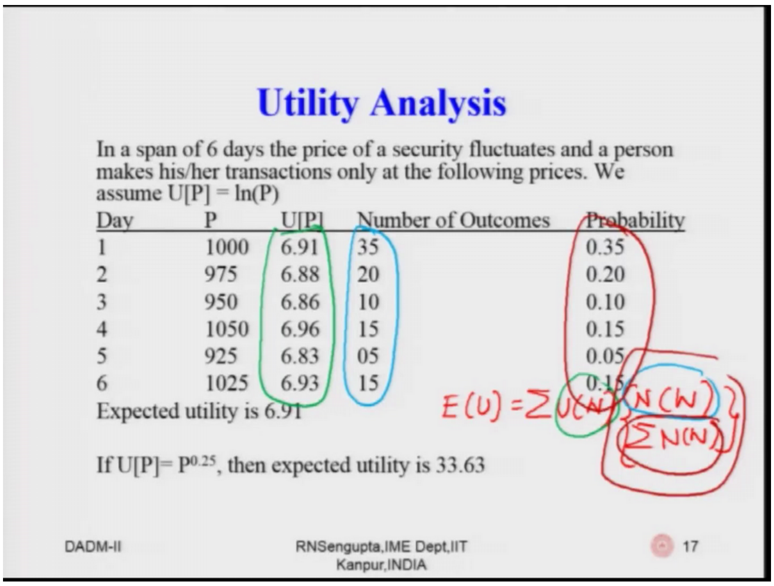

Consider the second example or third example. So, in a span of 6 days the price of a security fluctuates and a person makes his or her transaction only at the following prices. And we assume the utility function to be lager to make all utile function which is log of the price. And in the table which is shown we have the days which is given as 1, 2, 3, 4, 5, 6 the 1st column. The prices are given which is in the 2nd column; so there are different instances of the prices 1000 rupees 975, 950, 1050, 925 and 1025 so this is I am considering at random values. And then in the 3rd column we find out the utility of those prices.

So, we find out the log of 1000 log of 975 so and so forth and utilities are given. Now the outcomes are given so that outcomes here basically means the number which is NW; which means that if we take a snapshot these days. Now let us consider the disk concept not there if we consider the price of 35 it technically means that the number of outcomes which were favorable to the price of 1000 out of some instances of the values which I have noted they are 35 in number. Similarly if I consider the value of 925 rupees or dollars or yens whatever the price is; then the number of instances which are favorable to it that comes out to be 5.

So, if I find out the numbers given in the second last column then the corresponding probabilities which we will find out using that outcomes would be NW divided by summation of MW there. So, that it would be 35 divided by 100 because 100 is the sum of that set of values which is there in this second last column. And when we add them up it as it is100 so, 35 divided by 100 comes out to be 0.35 correspondingly the other probabilities are 0.15 0.05 and 0.05 so if we add them it definitely comes out to be 1. Now, if you basically multiply so what we are going to do is we will multiply the you want we want to find out the expected value expected value would be the sum of utility into NW by summation of NW.

So, this is the equation we need to find out; so I use different colors repute the green one. So, the utilities are given here, the numbers NW s are given here, I am just marking the columns. And the value of summation and blues once we find it out it is 100 so the probabilities which you want to have is given here. So, we can find it out and do the calculations accordingly. Now in case if utility function is given as a function P to the power one fourth or P to the power 0.25 then the expected utility comes out to be 33.63 percent 33.63 in value terms. So, you can basically make a decision accordingly depending on different values are there find out the utilities multiplied there by the probabilities and find out the expected value and laying them accordingly.

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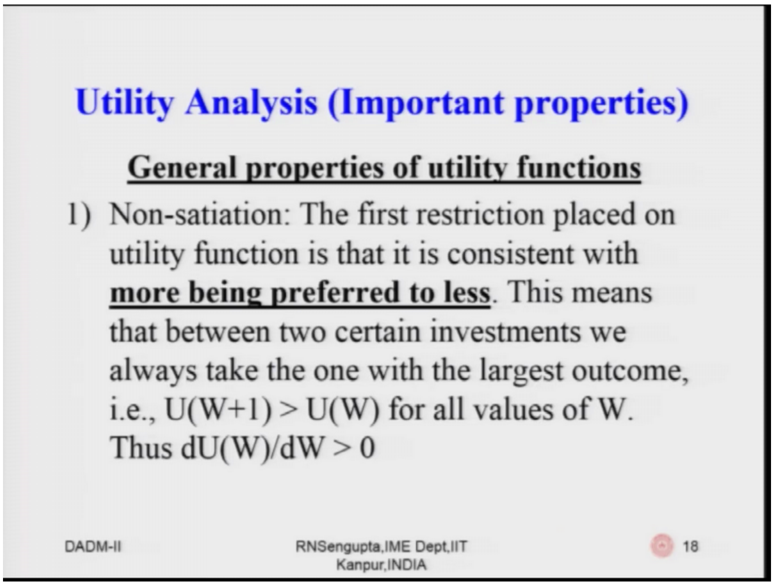

Now we will consider different properties or important properties of utilities. So, the general properties of utility functions we will go through two main one and come repeatedly come back to these in order to solve our problems. So, one is the problem the concept of non satiation. So, the first restriction pace not placed on the utility function is that it is consistent with the more being preferred to less. That means, more I give you more you want; that means, more it is better more the merrier.

So obviously, it would mean that in practical sense it may not be true because say for example, in many of the consumptions problem after a certain limit your marginal utility will start decreasing so. But still we will consider that point not to be true for the problems you may are trying to solve the optimization problems. So, this means that between any two certain investments or amount of value being W value means the wealth. So, if you take it the UW would always be less than UW plus 1.

That means, UW plus the utility based on W plus delta W would always be greater than UW; which means the first derivative the utility function with respect to W would always be positive. Now the question will come later on that this positive value which is there whether in the case when we take the second derivative whether it is increasing at an increasing rate increasing as a constant rate or increasing and decreasing it that would basically differentiate and give us the concept of the second property which you are going to come now.

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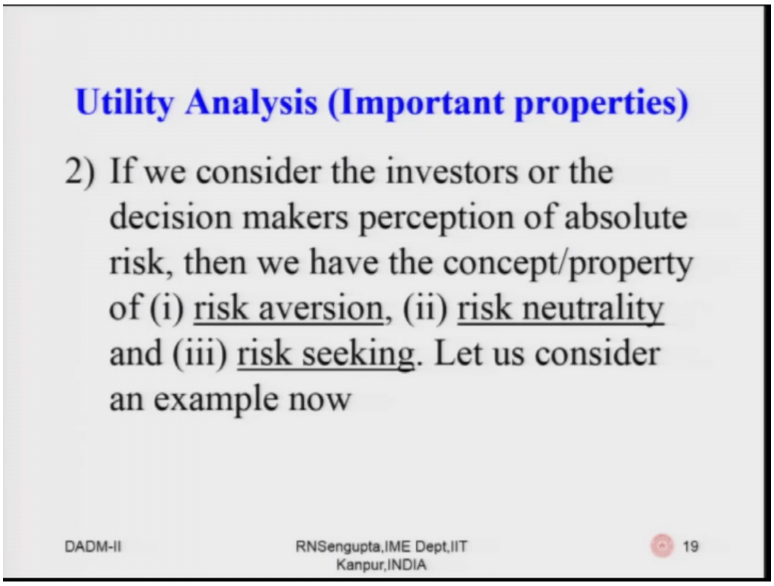

The next property is basically considered the concept of risk something to do the risk. So, if you consider the investors or the decision makers perception or absolutist then we will have the concept of the property given categorize under three categorizes. One is the concept of risk aversion; that means, I am want to avoid risk one is risk neutrality; that means, I am indifferent to risk and the third one is basically this seeking; that means, I want to take risk. So, in this case what the point which I mentioned just few minutes back a few seconds backs that; the utility function is always increasing with respect to W and that is the property of non satiation which means the first derivative of U W with respect to W is always positive

Now, the second point which is there on the slide it will mean which I mentioned that the first derivative can increase at an increasing rate, can increase at a concentrate and increase that decreasing rate. And that will give us the properties I will come to that in to details within few minutes that will give us the properties of risk avoidance or risk aversion risk nearly neutrality or risk indifferent and risk seeking property of any decision maker.

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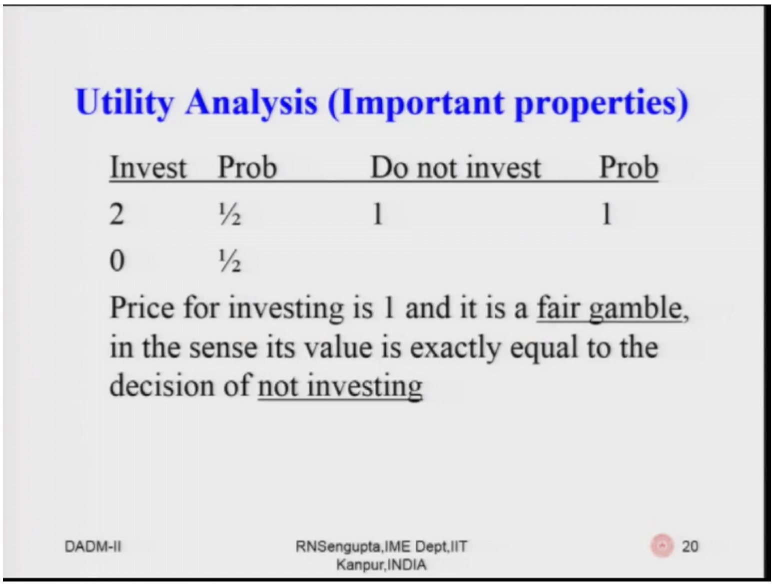

So, let us consider a very simple example. So, consider in on one table there is given and this values which is given which you see 2 and 1 they would not make a much sense. But let us consider those values as given as values only. So, consider you if you invest un amount to or a unit to and when you basically toss the coin it is actual coin unbiased coin probabilities are half enough. If a head comes that investment which you make would basically yield you the value of say for example 2. And in the other case whatever investment which you have made consider that as for the time being as 1.

So, one you invest you get two and another case one you invest if the tail comes you do not get any money. So, if I want to find out that were and this is the unbiased coin remember. So, if I tell you that if I keep doing that experiment tossing the coin finding out that in half probability I get to half I get nothing. So, in the long run you will say the expected value is 1, why? Because 2 into half; that means, 2 is basically the value which I am getting and remember this two is the value I am giving and I am not talking my utility no it is just a value and you consider the value as it is and that is the value which accrues to you.

So, the probability would behalf would be multiplied by 2 plus half another half would be multiplied by 0 the expected for a value is 1. So, leave aside this experiment come to the other experiment on table 2; there is a Sholay coin. So, all of you must have seen the film Sholay and those outside India may not if they have not seen that you should definitely see the film as Sholay. So, in Sholay coin the coin is biased in the sense both our heads consider. So, if their head whichever way I toss the probability of head coming is 1. That means when I when I toss the probability is 1 and the investment which I get is also 1.

So, if I tell you to find out the expected value you will immediately multiply the probability of 1 into the outcome which is 1 and the expected value is 1. So, if you consider the scenario and on table 1 and scenario in table 2 both give you the same expected value. Now where the differentiation starts? Now if I ask this values 2 see if I ask all of you who are taking this class that which of the decision would we take on table 1 where then unbiased coin or table where is the Sholay coin. Maximum the people are maybe all of them would definitely choose the first one which is in table 1.

Because you are even if the expected value is the same in the long run you are always thinking that your main focus is on probability half where you will win - 2 units. Now let me change the scenario of the game; the coin remains as unbiased in table 1 and coin remains as the Sholay coin and the biased coin with probability 1 on table 2. But the values which were here which you see now increases to say for example, 200 and the value remains 0 while the value 1 which is there on the other table is 1 becomes now 100. So, again I ask you the question. So, again I tell you that if I want to find out the expected value for these two decisions on table 1 and table 2.

You can find out that 200 into half plus 0 into half gives me 100 as a unit. And then 100 into 1 on the table 2 gives us 100 so the expected values are the same. Then again I ask you the question, now consider that out of all of you who are taking the class some may be a little bit doubtful of taking decision one which is on table 1. Because now you think that your focus of attention is slowly shifting to the case that what if a tail comes you lose everything; that means, all this 200 is gone. Let me again increase the value of 200 to say for example, 20000 I am just at increasing at very high jump. So, 2 becomes 20000, 0 remains 0, 1 becomes 10000.

Again the expected values would be same 10000 10000. But again if I ask you the question which decision will take the table 1 and table 2. Slowly you will see that as I keep increasing the value at some point of time all people will now be slowly being different that they do not know which one decision to take and as this value 2000, 20000 becomes 200000s continuously 10000 becomes 100000 and it is 2 and 1 this values basically are multiplied by higher and higher values you see that all of you wait basically be tempted to take a decision which is the certainty when which is on the table 2.

So, this gives us the con concept that at some point of time any human decision we are willing to take can basically be of any three categories depending on the wealth, depending on my experience, depending on what my risk attitude is depending on my age all these things would basically dictate that how I take that decision. So, let me come back to what is mentioned in slide. So, price for investing is one which I have already mentioned and it is a fair gamble fair in the sense because the probabilities and a half and half. In the sense that is values exactly the decision of not investing which is there on the right hand side in table 2 and we will use this concept of fair gamble later on.

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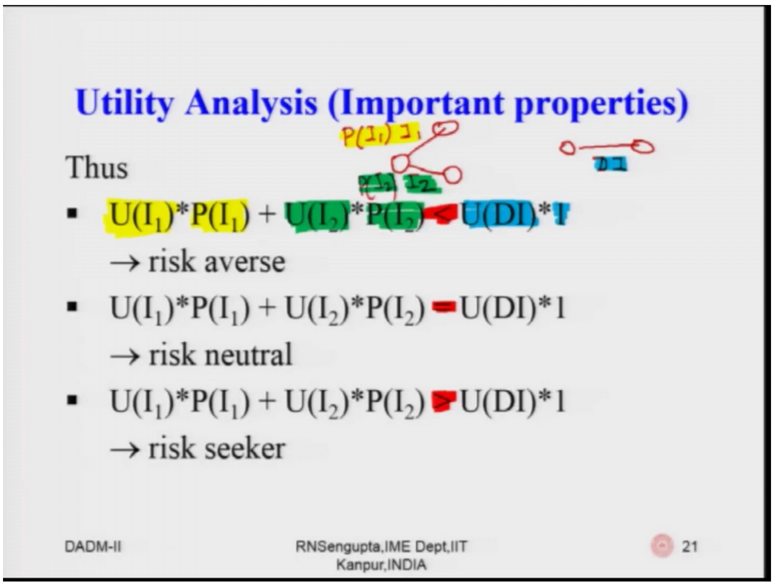

Now, the equations which are given are just simple equation and notation of whatever I have done. So, thus if I consider the investments or decisions as I 1 and I 2 so, this means my decisions are like this; this is I 1 with probability P I 1, this is I 2 with probability I 2. So, the utility and on the other hand obviously, you will basically have the deterministic event DI. So, the probability is here it would be half probability here it can be half. Technically if you have in a fair gamble, but now I will consider the probabilities as P I 1 and P I 2. So, now, let me go to the highlighter.

So, this part so obviously, probability P I 1 comes here and that I 1 gets replaced by the utility because I 1 is the value. So, once I basically assume the utility function to be some functional form. So, that utility becomes U I 1 as noted down. Now if I go to I 2 this value is basically I 2 and the probabilities of PI 2. So, you see PI 2 is here and the corresponding you reduce UI 1 UI 2. Finally, consider the deterministic 1 D UD 1; that means, utility of the term is to even and obviously, the probability being 1 it is 2 so that means, you have this two decisions. Now, let us consider and I will highlight it with the red color the less than sign the equal to sign and the greater than sign.

So, if you see the equations on the left hand side are the same equations the right hands are the same for the three bullet points only the inequality equality sign is changing. If less than sign which means that I am more inclined to take the deterministic event because I want to avoid risk; so you can understand from the situation. If both are equal I mean different and in the left hand side a is more as in the last bullet point which means I am willing to take the risk. Even if the expected value for all the three cases may be the same is exactly the problem which I have discussed.

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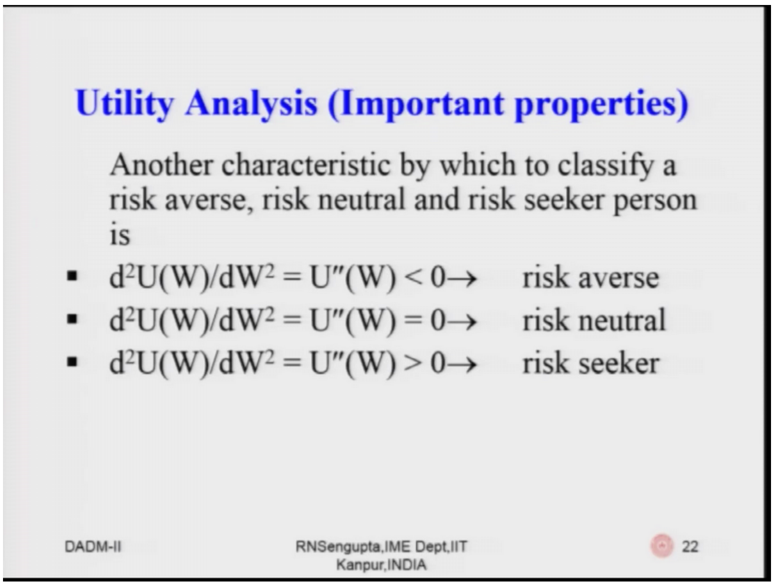

So, another characteristic by which to classify risk aversion proper the person and this neutral person in this seeker person we will consider this. That is the second derivative would have a sign; in one case it will be less than 0 in another case it will be equal to 0 and third case it would be greater than 0. The second derivative which means in the second a root is less than 0 it means that utility function is increasing because; obviously, the first derivative is 0 is increasing, but the increasing is happening as a decreasing rate; that means, I am risk averse I want to run away from risk.

If it is basically the second derivative 0 which means the utility is increasing that is U W by W that is d UW by dW that is the first derivatives positive and it is increasing as a constant right; that means, I mean different. And in the third case if it so and this means I am a risk neutral person and in the third case if the second derivative is greater than 0; that means, the first derivative obviously, 0 if it is and the second derivative is increasing it means that the utility function is increasing at an increasing rate; that means, I am a risk seeker.

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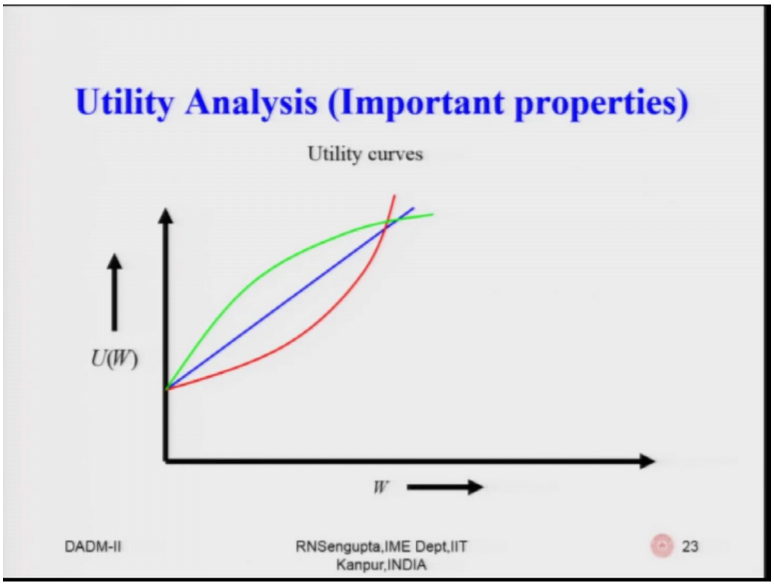

So, these are the graphs if we see the green one basically has a utility function which is increasing money increasing and decreasing rate. The blue one is basically utility function is increasing what increasing at as constant rate. And the red one is basically a utility function which is increasing, but increasing in increasing rate. So, these are very obvious from these graphs.

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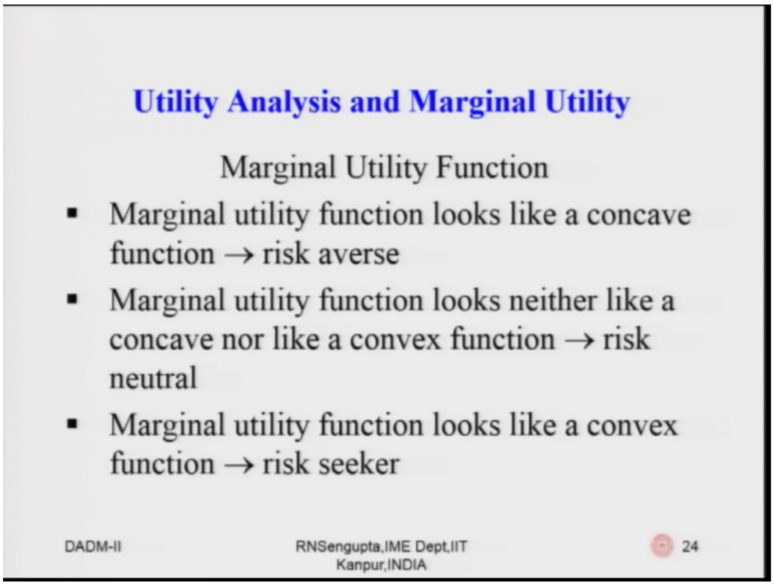

Now, let us come to the concept of marginal utility functions. So, margin you to marginal utility basically would be corresponding in the first derivative. Marginal functions looks like a concave curve is a risk aversion person; that means, I am avoiding this. So obviously, the same if the second derivative would be negative. The and I am risk averse marginally to second bullet point says the noisy root functions looks neither concave nor convex which is a straight line.

So, is it is neutral person. And in the last case when the marginal function looks like a convex curve it is if; that means, it is increasing; that means, the second derivative is greater than 0, which I just showed in the last graph. So, it is a risk seeking person because the second a data UW prime is positive.

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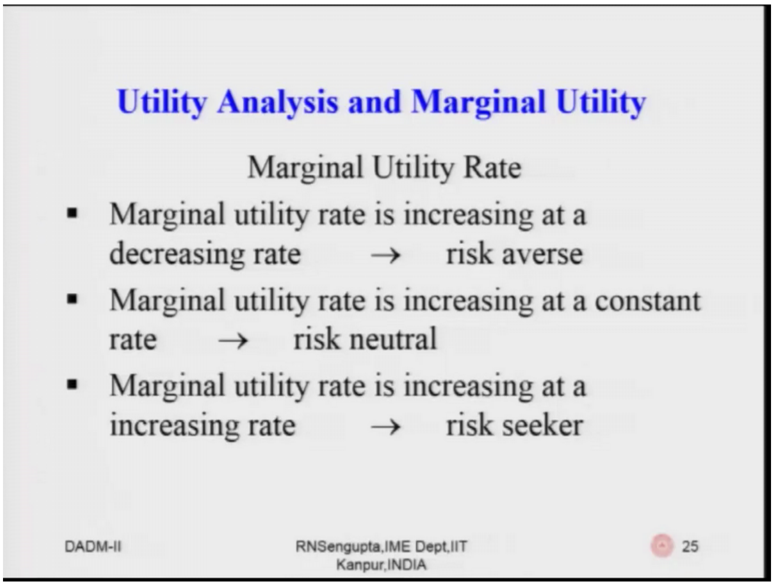

So, which means that the marginal rate is increasing and I mean decreasing rate for the risk averse person. The second bullet point is marginal utility rate is increasing at a constant rate which is a risk neutral person. And the third bullet point is the basically the marginal utility is increasing at an increasing rate which is the risk seeking person.

So, increasing decreasing being risk averse increasing constant being risk neutral and increasing increasing being risk seeker the first part of; obviously, would be increasing, but as per the non cessation point it will always be positive.

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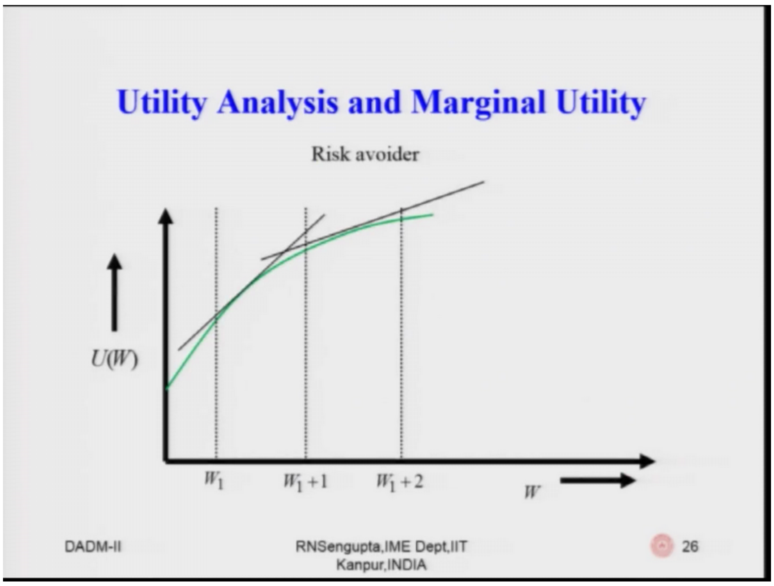

So, here are the graphs which have tried to draw it as neatly as possible for you to make understand. So, on the y axis you are measuring the utility functions on the x axis case you are measuring the wealth some amount of wealth. And the green curve which you have if you see the dy dx of that or d UW de W it basically slowly tan of that angle slowly starts decreasing so hence it is a risk a version.

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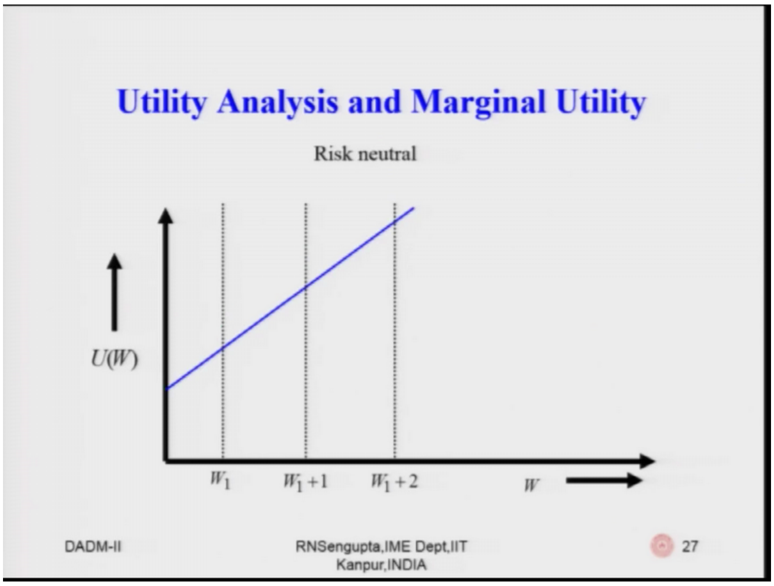

In a risk neutral person the dy dx or d U W dW is straight line so tan on the triangle does not change. So, it is basically risk neutral person and again imagine measuring UW along the y axis and W values for the wealth values on the x axis.

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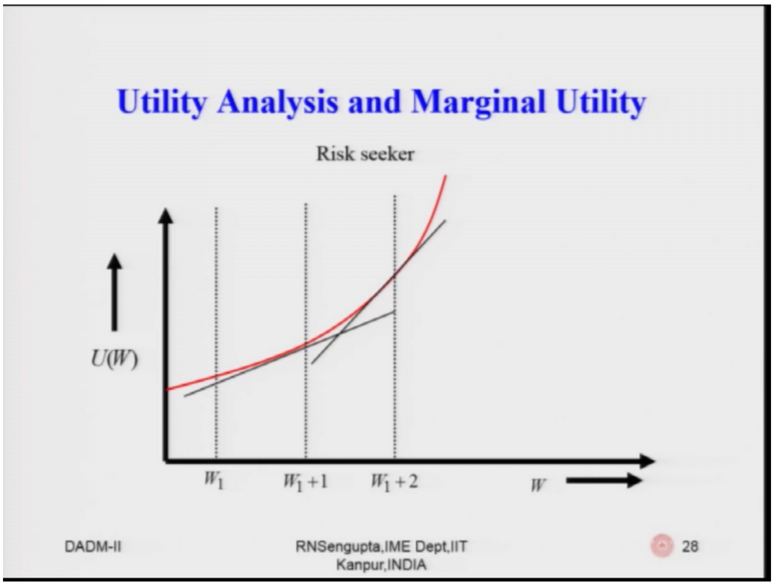

Similarly, if I take the utility analysis and the marginal rate for the curve which was the red one. And if I plot again y axis being UW x axis being W and if I find out the dy dx which is d UW dW is increasing; that means, the tan of the angle is increasing hence it is basically risk secret person.

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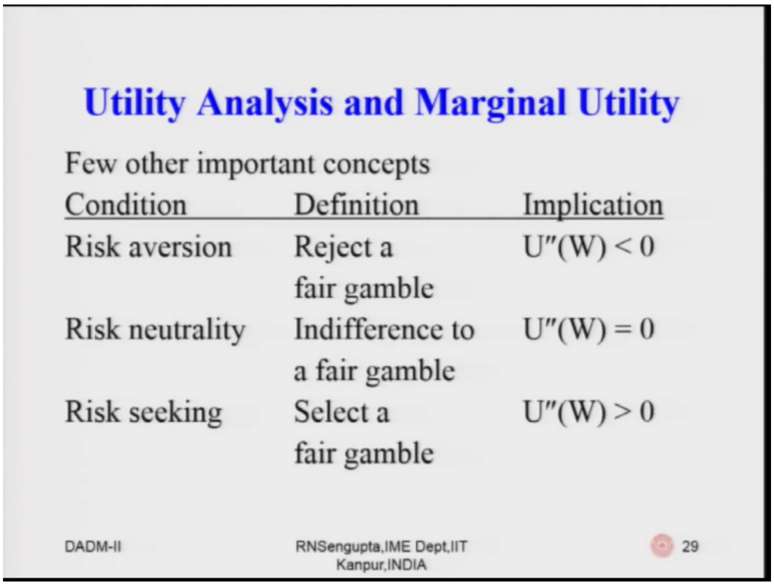

So, few other important concepts which we are kind of considering the utile analysis. So, consider the risk aversion person would basically as per the definition would be he would reject a fair gamble. Because his implication would be the second derivative is negative because that is why his reacting a fair gamble and going for the certainty event. This neutral person would be indifferent between the fair gamble and indifferent between certainty events. So, because in that case his or her second derivative would basically be 0.

And a risk seeking person would be when the person who will select the gamble because technically they expect a value for him or her remember that is very important. So, the decision which is being taken by the human being him or her is based on his or her perception of risk only. So, coming to the third point which is mentioned I am again re rereading it. So, select the person whose risk seeking would select the fair gamble because the second derivative is basically positive.

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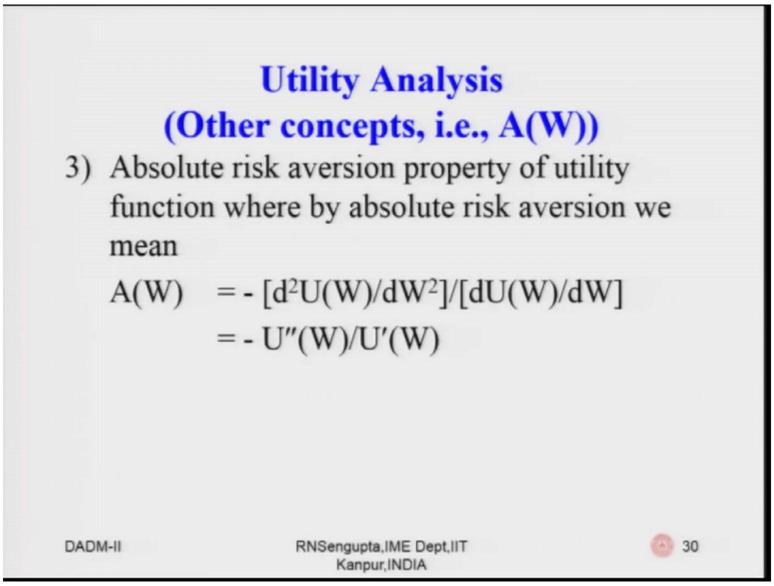

Now, we will consider some of the properties of utility analysis. So, absolute risk aversion is the property which would be important for our discussion. So, absolute risk aversion property of a utility function where we consider the absolute functions is even by this property where we consider the negative of the ratio of the second derivative divided by the first derivative. So, mathematically we are not going to deal with that because they are mouthing will proofs for that, but that is not necessary. Why they are necessary we will come to the practical implication of AW later on AW and A prime.

And similarly we will consider another concept also which is the relative risk aversion so this is the absolute risk aversion. So, if it is basically minus value of U double point by

UW U prime. So, U prime is always positive and there is a minus sign there. So, U double prime being negative being 0 and being positive with a negative sign would basically dictate that how the value of A is changing. That means, A changing means A prime has to be found out.

So, that would be important for us to find out the properties of the human being or decision maker who is taking. So, the if you can if you notice that that when you take the derivative of A prime A which when you find out the prime there the actual the information will be coming out for the utility function then you will see that in the later problems.

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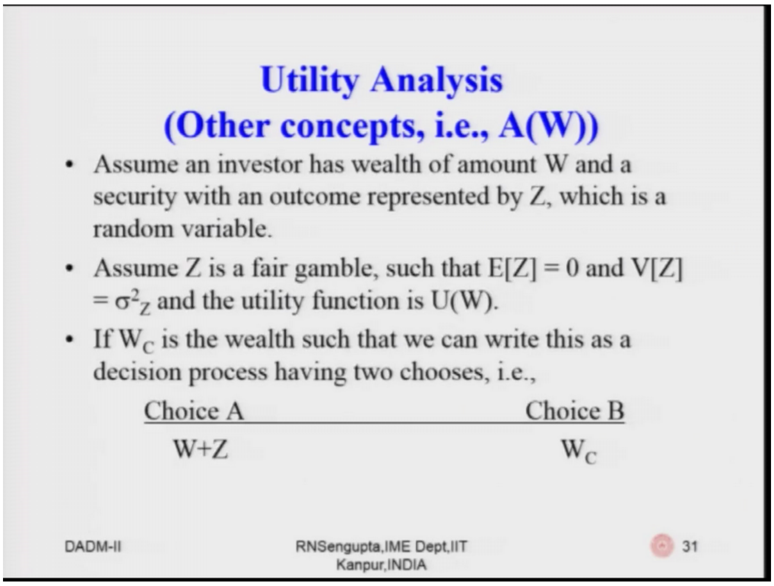

So, I will just briefly discuss how we generally doing it we will assume basically an investor this is just for the concept is nothing to do with the problem solving it is just a overall feel that how you can basically approach the problem. We will assume an investor has wealth of amount W, whatever that W is and a security is basically a financial asset with an outcome which is represent by Z which is basically random because security would have base in random returns.

And we will consider Z to be a fair gamble whatever the amounts are not their fair gamble will consider the probabilities have such that there is an equivalent deterministic event which can replace that. And we will consider the expected value of that fair gamble is 0 and the variance is given by some sigma square suffix Z and the utility is basically given by UW.

So, we will basically choose of wealth such that we can write this as to decisions in one decisions there is the we will WC and another decision is the well the blue plus the security. And based on that we will try to balance it and basically try to expand using Taylor series expansion and then trying to find out the Taylor series from the Taylor series expansion. We will try to find out the concept of absolute risk aversion which we just mentioned of the last slide. And then similarly find out the second derivative will come into the actual implications again I am mentioning as we solve the problems later on. So, with this I will end this third class and have a nice day.

And thank you very much for your attention. [music]