Data Analysis and Decision Making - II
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Lecture - 29
ELECTRE

Welcome back my dear friends and dear students; a very good morning, good afternoon, good evening to all of you. And as you know this is the DADM II lecture series under NPTEL MOOC, which is DADM is basically means Data Analysis and Decision Making II.

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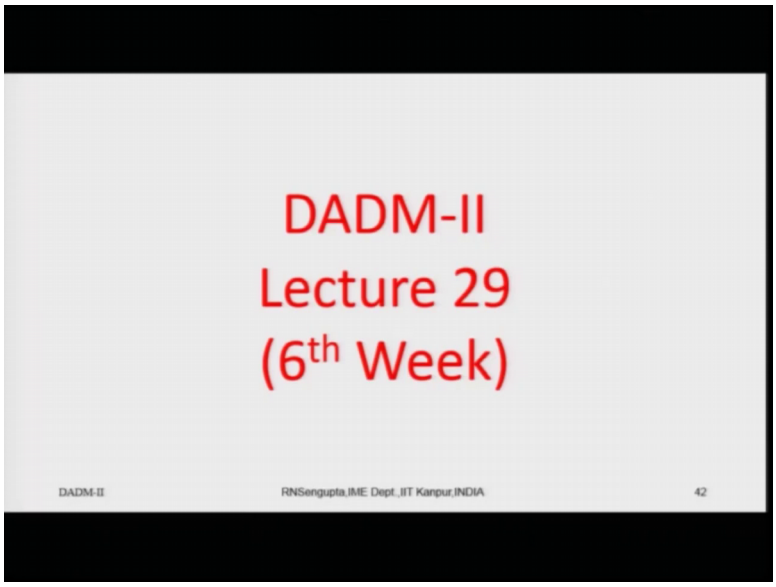

And this course whole duration is for 12 weeks which is 60 lectures; that is 30 hours and each week we have 5 lectures, each being for half an hour and if you can see in the slide this is the 29th lecture; that means, we are in the 6th week. So, with this 29th and 30th; we would wrap up the 6th week.

And if we remember we had just discussed in the first three lectures, we have in this 6th week we have been discussing about the ELECTRE method. And then after discussing the concept of ELECTRE we did the small problem in the ELECTRE method over the ranking concordance and discordance concept were considered. And then in the last few minutes or about last 5-6 minutes in the last lecture which uses the 28th one, I started the

epsilon ELECTRE where in the concept of epsilon; we are now stating that any decision basically would have three different sets.

One is the concordance which is the liking one, discordance is the disliking one and one is the indifference one where I am still not decided that what should be the decision between A k and A l alternatives considering the criteria.

Now, remember one thing I did mention there that in the utility functions we had considered the quadratic utility function which was equally penalized. Then I went in the concept of LINUX loss function, linear exponential loss function; where I showed and I where I discussed those three examples the civil engineering, electrical engineering and the marketing one.

And I also did mentioned that you can basically have an area where you are indifferent depending on overestimation, underestimation of or more liking, less liking. And based on that we have the epsilon ELECTRE method and remember in the epsilon ELECTRE; the epsilon value which you are going to take will depend on what is your risk averseness property.

Now, this has something to do with if you remember the AHP method; we had considered the principle diagonal to be 1 on the off diagonal element work asymmetric, but the scores were like 7, one seventh 9, 1 9 3 one third. So, the level of liking disliking was basically based on the fact that what is the score I am going to give to the positive one. And what is the inverse score I will give to the negative one negative one; that means, I am forced to take that decision under the AHP method. Similarly, we did that for the ELECTRE method.

And now, we will continue extending that that they may be a small bandwidth or a small set of values where I am indifferent between taking either A k or A l, whether I take A k and then when I comes considered with respect to A l; k nl are the suffix which I am utilizing for the alternatives. And j are the different type of criteria; so, j changes from 1 to n and k l which is i changes from 1 to m n is in Mangalore.

So, given the weights; weights is basically what is the priority I am going to put to the different scores; I already have the values of y which is coming from the scores which or the utilities values which are assigned to decisions or alternatives for the criteria's. And if

we remember, they were converted using the logarithmic utility function with base of 10; that was just a simplistic assumption it could have been done for base e also an opinion log. And then, we considered that the normalization would be done based on the fact that actual cell value divided by the sum of all either the rows or the columns.

So, this is normalization would give you what is the overall utility concept we are trying to use. But remember as I said once you decide on the utility stick to the utility function for that particular decision maker throughout the different stages of decision making point 1.

And point number 2 is that we will also consider the decision makers utility for any group decision. If you remember in the AHP method we considered that buying the either the car or trying to basically consider buying the house or say for example, Ram and Sham and their father and mother that the parents giving their feedback, it would be considered that the utility function would stay throughout the set of decision makers as same we even though it may not be true practically; obviously, that will not be true.

So, based on that I also mentioned whether you normalize along the row or on the column it would not change, but the answer; but the ranking respective ranking. But remember that once you follow normalizing along the row or along the column stick to that.

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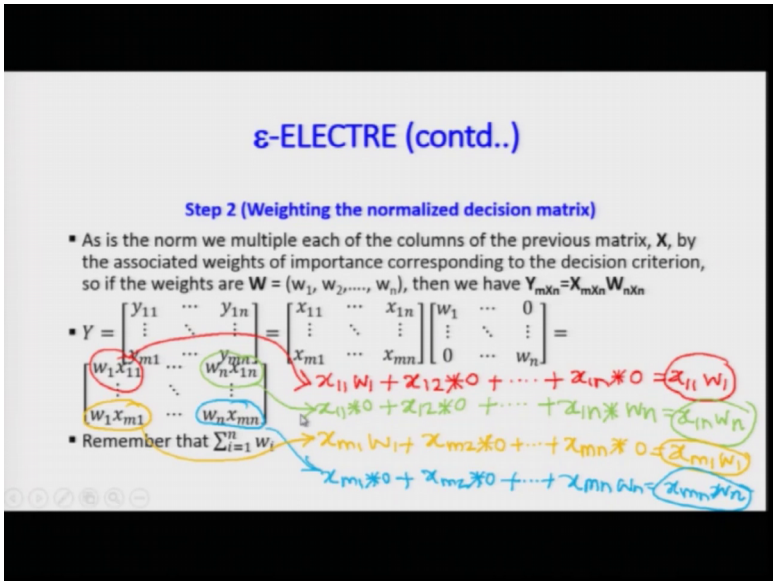

So, as is the norm we multiply each of the columns of the previous matrix X which has been normalized. If you remember the values were given then logarithmic utility function we utilized then we normalized; so that is X.

So, X is multiplied by the associate weights of the importance corresponding to decision criteria. So, if the weights are given as w 1 to w n and remember the sum or the weight should be 1; then we have, Y is equal to the matrix being the multiplication of X matrix, multiplied by Y and W. And W is a vector here we will convert it into a matrix.

So, Y will be given values the cell of the Y's would be given by x 1 1 multiplied by y 1 1 similarly the second values and so on and so forth. So, once we have that we will proceed and find out the values. Just one minute I think, I should have calculated accordingly. So, these values are not 0; my apologies I think these are. So, because if I am multiplying just give me 1 minute, if I am multiplying the first cell it will should within when I am adding the last values; it should be x 1 1 into w n.

So, I will w n here. So, it will be x 1 m my apologies there was a typo error here. So, this way; so all the cells have value they are not 0 as more turned out and the next last one would be w 1 because this is the last value ; so w 1 this multiplied this x m 1; so these are the values my mistake.

So, this will be the I will just mark it to make it more clear and remove you should consider this value; I am using different color. So, this would be x 1 1 multiplied by w 1; then the next value would be plus x 1 2 into 0 and all the values till the last one which is x 1 n would be multiplied by 0. So, the value becomes x 1 1 w 1 this is the value which I have; so this red color and red color match.

Now, I go to green color. So, if I multiply; so this is this multiplied by the last one. So, this would be x 1 1 multiplied by 0 plus x 1 2 multiplied by 0 plus dot dot dot dot plus x 1 n multiplied by w n is equal to x 1 n, w n. So, these value matches with the circled one; so it matches.

Similarly, if I take the color orange this one would match when I multiply x m 1 multiplied by; so I should remove this multiplied by w 1. Then it will be x m 2 multiplied by 0 plus dot dot dot dot plus x m n multiplied by 0. So, the final value becomes x m 1 multiplied by w 1; so this matches the colored yellow.

Similarly, finally, if I do it for the last one, it will match I am not writing it or let me write it. So, it will be easy for all of you want to understand. So, this will be x m 1 multiplied by 0 plus x m 2 multiplied by 0 plus dot dot dot plus x m n into w n. So, this is x m n w n this matches; so it balances. So, the blue one matches this, the orange one matches this, the red one matches this and the green one matches this; so all these things are solved.

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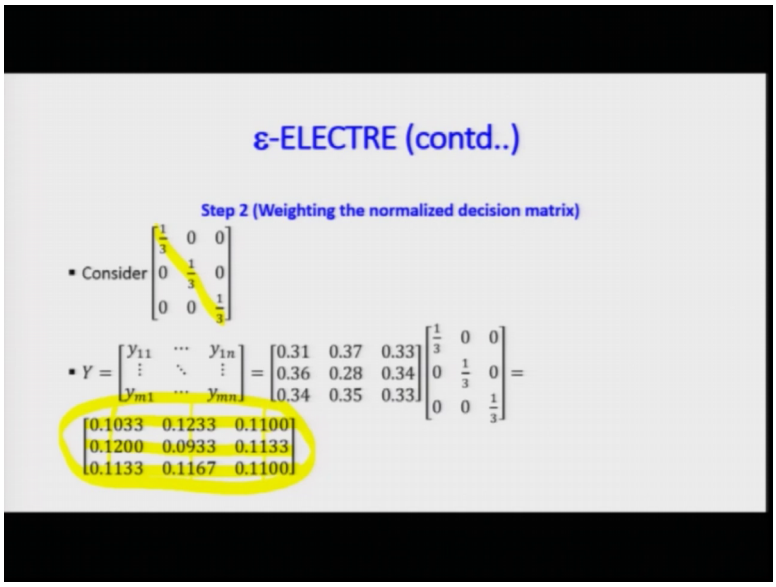

So, once we have this I consider a matrix of weights of one third, one third, one third. So, this is a very simplistic assumption let me highlight it. So, this values are one third one third one third I am taking in order to make our lives in nothing else.

So, once you have that you multiply. So, if you remember these are the normalized values either the row or the column for the logarithmic of the utility function or the values and then normalizing by the sum of the logs, multiplied by one third gives you these values. So, this is the values based on which will work now.

So, remember before I proceed, I will request all the students or all the participants who are hearing these lectures; please write these values. So, the first value is 0.1033; I am calling along the first row. Then 0.1233, then 0.1100, second value second rows are 0.1200, next value is 0.0933, third value is 0.1133, third row is first value is 0.1133, second value is 0.1167 and the last value is 0.1100.

So, please write these this matrix which is Y multiplication of X into W and based on this will do the calculation.

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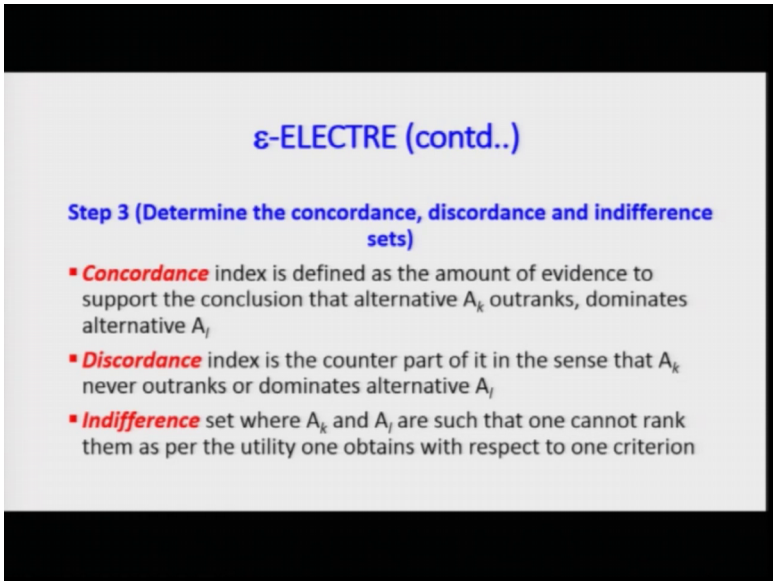

Now, in a epsilon ELECTRE method, I told you that we will basically have three different sets based on the indices; one will be the concordance indices which will give us the concordance set which is capital C, then we will have the indifference indices will give us the indifference set or the matrix with capital I.

And the finally, we will have the discordance indices which will give us the discordance matrix which is capital D. So, our main task would be to find formulate, find out capital C, capital I and capital D. Now, let us go one by one what we mean by concordance indices values in or index values indifference index values and discordance index values; that means, liking, indifferent and disliking.

Concordance index is defined as the amount of evidence to support the conclusion than the alternative A k; outranks or dominates alternative A l with respect to say for example, any of the criteria which is basically j; j changing from 1 to n. So, we will compare A k with respect to A l for each of these j's. Similarly discordance index is the counterpart of it opposite; if you remember in general ELECTRE process we had a concordance indices and a discordance index.

So, a discordance index is the counter part of it in the sense that A k never outranks or dominates alternative A l on the contrary it is negative utility or negative value accrues to taking A k with respect to A l. So, that is the discordance index based on which will have the discordance matrix capital D.

Similarly the last one the in different set or indifference index would be when we compare A k and A l and the comparison is such that one cannot rank them as per the utility one obtains with respect to one criteria which you have. So, for criteria j if we compare A k and A l we are indifferent. So, that will basically have an indifferent index value and that value would be utilized to formulate the capital I which is the indifference matrix.

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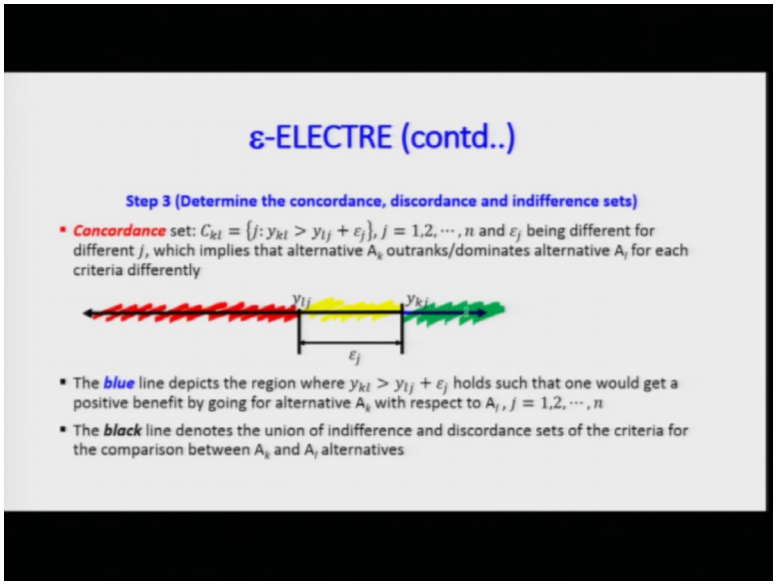

Now, here is basically where it things becomes more interesting. So, when you are trying to formulate the concordance set, discordance set in the indifference set. If you consider very simply; you will basically have the straight line or the real line where we will have the scoring.

Now, consider that if it was a general ELECTRE method either you will take a value anything value more; that means, if I am considering the real line from my side anything positive on to the right more towards positive would basically go into the concordance set. And anything basically on to the left would basically go into the discordance set; that

means, we are dividing the whole set into two disjoint sets such that union of that is the universal set. So, they are mutually exclusive and exhaustive two sets.

Now in this epsilon ELECTRE we basically proceed further and divide the whole real line into three different zones or three different sets. So, say for example, the middle portion will be the indifference set or indifference indices; indifference values what is that zone, length of the zone we do not know that will be basically dictated by the utility function on the human being or the decision maker has.

On to the right so of that zone if I am looking from my side on to the right more towards positive with the concordance set, concordance indices, concordance matrix, any value on to the left on indifference zone would basically the discordance set or discordance index and the discordance matrix.

So, now we will need to formulate the rule that how do we get when we compare A k with A l based on each and every criteria what decision rule should we apply such that we can put the comparison of A k and A l in either the concordance set or the discordance set or in the indifference set that's all. Now here we will add a second level of intuitive field or actually what is true.

So, the indifference set would basically be made accordingly which I will mention, but I would not go into solving that only mentioned as we solve the problem, then general problem and then basically talk about is variance. So, the concordance set or the concordance values; C kl when we are comparing k and l; it will basically for any j, j is basically the criteria, where we will basically have the difference of y k that is that cell value which is therefore, y kl would be greater than y lj plus some epsilon value depending on the liking disliking proposition which we have between k and l.

So, the value will basically check as j changes from 1 to n; we will find out that how the large the difference of y k is there with respect to; y kl with respect to y lj based on the fact that is epsilon j which will give you give us some nonzero difference between the values of y kl and y l; larger it is better smaller it is not that good.

So, this epsilon j is being can be different for different j's. So, say for example, for criteria 1; epsilon j is 0.01, for criteria 2 the epsilon value epsilon 2 can be say for example, 0.002; similarly for epsilon 3 for j is equal to 3 the epsilon value can be 10 to

the power minus 4; depending on; however, you basically formulate the problem liking and disliking.

So, this values are epsilon will imply that alternative A k outranks or dominates alternative j for each of the criteria's differently as I mentioned. So, if you remember the number line; so this is what is going to come. So, the whole number line I have not colored it, I will come to that coloring later on, the whole number line or the ranking set is such that we as I mentioned we divided into three zones; I am drawing it here, but technically is more much for an explanation, actually we would not formulate it may be solve the problem.

So, generally we should have a region which is not liking. If I consider red as say for example, definitely look like, if I consider orange as indifferent, like in the streetlights what we have? The red, the yellow and the green and the green one being for the case when we basically have a liking. So, actually this should be red, yellow and green, but I am using the blue color in order to mention that. So, again note discordance red, indifference been yellow and definitely I like concordance when green. But I am using the blue color to give the notation that the concordance set is there.

So, the blue line depicts the region where y kl is greater than y lj. So, which is plus some epsilon j depending on j the criteria and it will hold such that one would get a positive benefit of some amount plus something given for alternative; given that alternative A k is taken with respect to A l for A j is equal to 1 to n when n are the criteria. The black line now on to the left is basically combination of the discordance and indifference set.

So, it basically says the black line denotes the union this is what was important; it basically gives the union and indifference and discordance sets of the criteria for the comparison between A k and A l for any of the j criteria.

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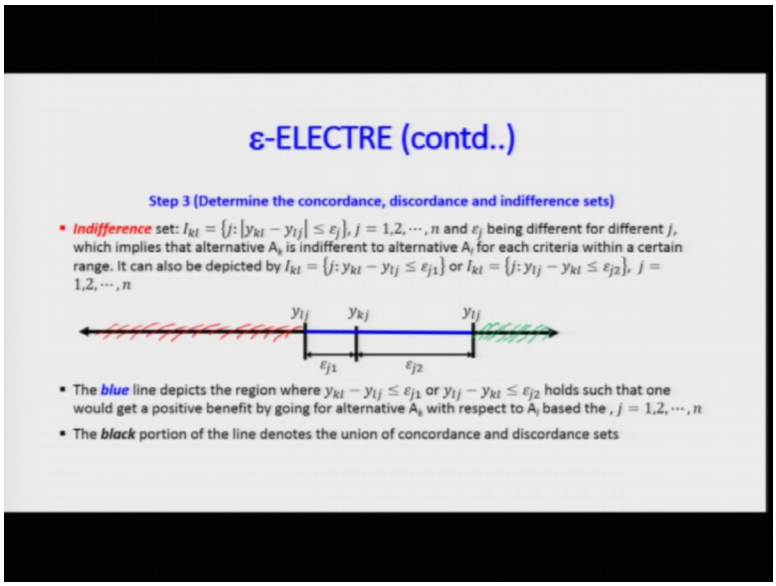

Now, let us come to the indifference set. So, invent set if you remember I have marked it at the yellow one in between; now that difference can be asymmetric, depending on what level of asymmetricity you want: that mean; like indifference can also be out of different values.

So, in the indifference set I kl will when we compare A k with A l for those j's values, if the mod of the difference are less than some epsilon value. So, epsilon again j; j means 1 to n would keep changing depending on the criteria and epsilon j as I said. So, it will imply that A k is indifferent with respect to A l within a bandwidth of A kl minus A jl or; that means, y; I would not be using that A, y kl minus y lj is that mod value is less than equal to epsilon in different alternatives A l for each criteria within a certain range and that range would be dictated by the criteria.

It can also be dictated by a different levels of epsilon like say for example, for the value of criteria j 3; we can have a positive and a negative one. So, one epsilon value for the indifference being in the positive side and in the negative side; that means, we have the bandwidth of indifference that indifference can also be divided into 2 unequal zones, that unequal zones would basically be dictated by the epsilon 3 1 and epsilon 3 2; number 3 is basically for the criteria and 1 and 2 are the different levels of epsilon we assign.

So, that yellow region now is that this one which we have divided unequally; it need not be unequally it can be equal also. So, we basically formula the rule epsilon j 1 and j 2

which I said epsilon 3 1 and 3 2. So, the blue line which is in between; so if I remove it will be you will be able to see.

The blue line depicts the region where the difference is either less than equal to epsilon j 1 or less than equal to epsilon j 2 depending on which side you are looking for. And you will hold true the such that one would get a positive benefit by taking A k with respect to A l depending on epsilon values and the j criterion number.

The black portion of the line denotes the union, now on the left and the right. So, if I go this is the discordance value and this is the concordance value. So, the black line is basically the union of red and green.

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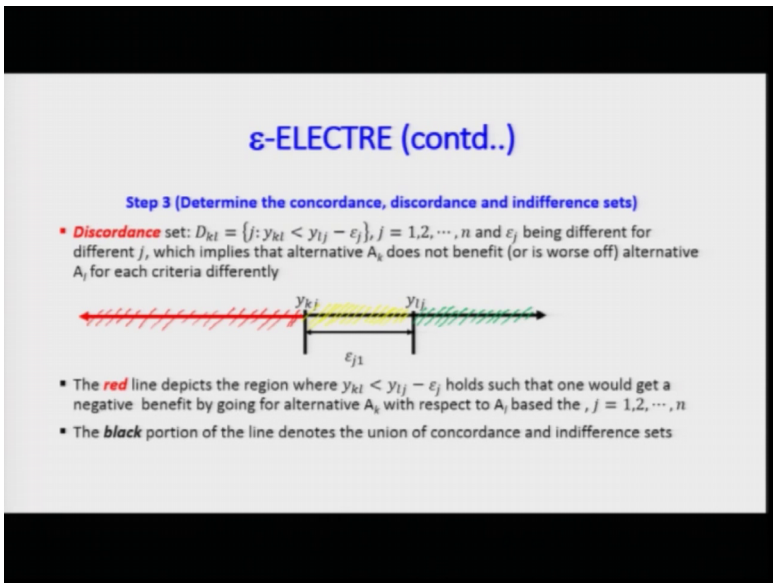

Now, let us come into the discordance set. So, the discordance set would be for those j values where y kl is less than equal to y lj minus epsilon value. So, higher or lower the epsilon value corresponding to that j would dictate; what is the level of discordance and to what level.

So, here also we have this I have marked red actually I should be. So, this is the discordance region which we have and technically this is basically the indifference region. So, and this is basically the concordance and liking region. So, again the universal set of the whole points is divided into three zones.

So, you will basically have the discordance set D; now concordance will be denoted by C with the corresponding suffix indifference will be denoted with I with the corresponding suffix and in discordance sets or indices will be denoted by D with the corresponding suffix. So, D values for each and every j would be calculated where epsilon j being different for different j's which implies the alternative A k does not benefit or is words off by taking that alternative A k and not A l.

So, here the red line depicts the region where y kl is less than y lj minus that epsilon j depending on the value of epsilon j. So, such that one would get a negative benefit or a dis benefit by taking A l with respect to A k with respect to A l, my apologies based on the criteria number because epsilon j would dictate the level of in discordance concept.

The black portion again would basically denote the union of the concordance and the discordance set. So, concordance this is the values in indifferent and this is the value for the concordance. So, we have this black line, I should have drawn, but I have omitted it.

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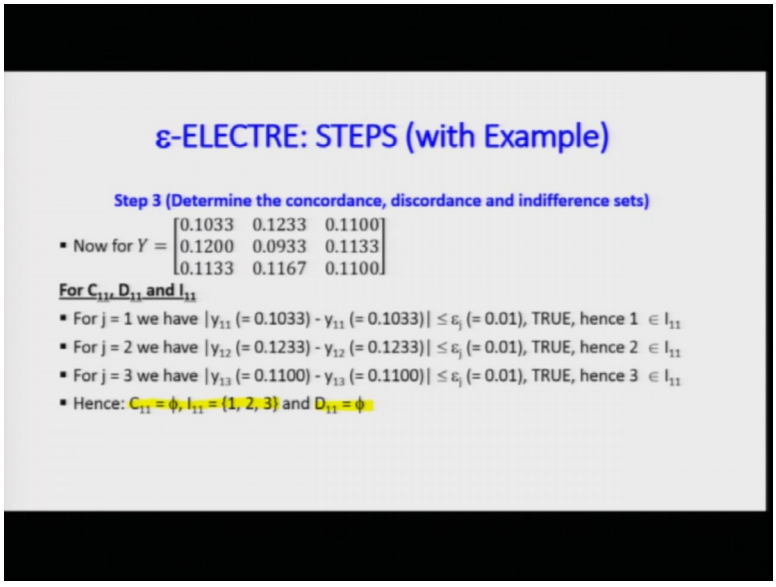

Now, these values, the y values were already calculated using the normalized utility values which mean there multiplied by the W values which is the weights and this matrix Y was given.

Now we will just follow the rule given and calculate the values and I will only talk about the values end results rather than going into the detailed calculations. So, we will first

find out C 11, D 11 and I 11 with concordance, discordance and indifference, initially for the ELECTRE method; it was only C and D. For j is equal to 1, we formulate and find out the difference and if we are considering epsilon equal to 0.01; it can be anything else also.

And I am considering remember this epsilon j, I am considering same for whatever j it is there; j 1 2 3 does not change the values of epsilon; even though I had mentioned that in the last few minutes, but I am considering that 0.01. Based on j is equal to 1; we find out that for the first criteria, it belongs to indifferent one. For the second one, it belongs to indifferent one, third one it belongs to an indifferent one that should basically be.

So, because we are comparing the values of 1 to 1 so; that means, I am comparing alternative 1 to 1. So, once we have this I will just highlight C 11 is a null set, D 11 is a null set, indifference are 1, 2, 3. Initially in the epsilon method these 1, 2, 3 would have gone into the C set.

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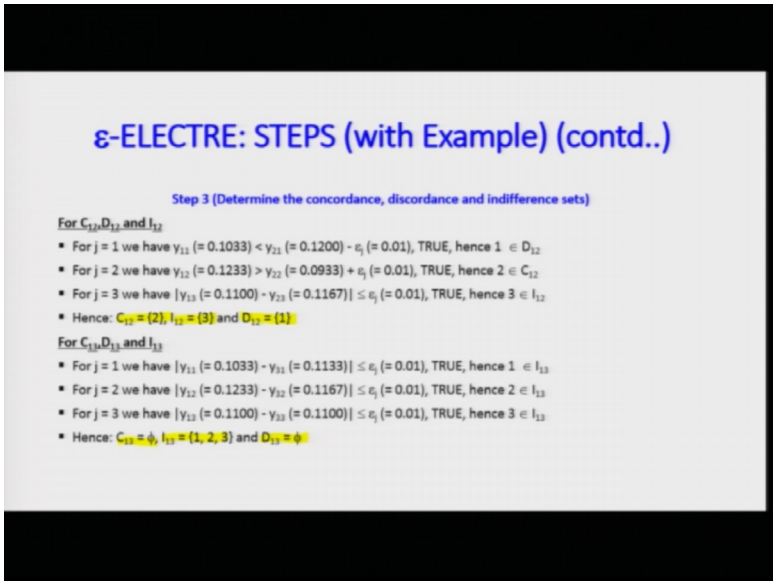

When I consider C 1 2, D 1 2 and I 1 2 and then I will consider C 1 3, D 13 and I 1 3. So, the values for j is equal to 1; 2 and 3 for C 1 2, D 1 2 and I 1 2 please calculate it considering the values of epsilon as fixed as 0.01. We have 1 belongs to the discordance set, 2 belongs to the concordance set and 3 belongs to the indifference set.

Similarly, for C 1 3, D 1 3 and I 1 3 to find out for j is equal to 1, 2, 3; again the same value of epsilon of 0.01; we find out; do this calculations if you this very simple. Then the concordance set element is null indifferent is basically 1, 2, 3 and discordance is null. So, we will proceed accordingly and then calculate and find out the values. With this I will end the 29th lecture for this DADM II and try to wrap up the epsilon ELECTRE in the last lecture for the 6 week.

Have a nice day and thank you very much for your attention.