Data Analysis and Decision Making - II
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Lecture - 25
AHP

Welcome back my dear friends and dear students; a very good morning, good afternoon, good evening to all of you wherever you are. And as you know this is the DADM – II, which is decision, Data Analysis and Decision Making - II course under the NPTEL MOOC series. And this total course duration is for 30 hours, which is for 60 lectures spread over 12 weeks and each week we have 5 lectures, each lecture being for half an hour and we are in the 5th week and as you can see from the slide which is the 25th lecture which is in the last lecture for the 5th week. And my name is Raghu Nandan Sengupta from the IME Department, at IIT Kanpur in India.

So, we were discussing the AHP ranking of buying a car and when you are taking a decision, if you remember that we had considered first the subjective criteria style, fuel economy, cost and so on and so forth. And you then you first compare amongst themselves, make a priority, vector based on the priority matrix, then you take each and every criteria for all the alternatives come and together one at a time, then also you have separate priority vectors. Combine this priority vectors for both the levels at both the hierarchy multiply to get the score.

Now, if the cost factor comes as the fuel economy was there, kilo meters per litre that can also be normalized using the same concept on normalization which I have which I am repeating it time and again that can also be considered and that can be added on to find out the final ranking. Now, if the cost structure comes I said that adding or subtracting one of the criterias or one of the not all the alternatives, but the criterias may change the relative ranking of the criterias by bringing a fourth one in this case or say for example, adding, say for example deleting of the any arbitrary third one provided there are more than two the ranking may change, relative ranking.

So, here also you consider the cost structure, and they can be different flavours of the problem remember that and you can basically find out the relative scores.

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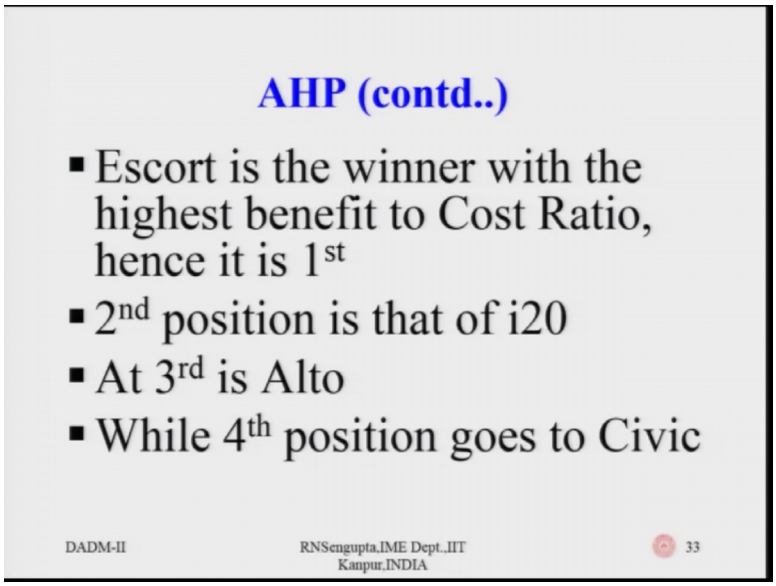

So, continuing with this once you have the combined scores, so Escort is the winner with a highest benefit to cost ratio hence it is in the first position. Combine all the scores, second position will go to i20, third to Alto, while the fourth position goes to Civic. So obviously, at this position depending on the alt the criterias which I have already mentioned in the cost structure you have this.

So, the problem concept is very simple. You have to find out the priority vector, priority matrix, priority vectors, rank them combine one level at a time. And also told that they can be primary level, tertiary level, then below the tertiary level one level, two below a levels below the tertiary level and can go on such that you add up this goes as you go up and keep multiplying and adding them such that you give a final scores to rank them.

Now obviously, there are some pros and cons for AHP. So, it allows multi criteria decision making which is a very good point. So, multi criterias with different types of con subjective, objective, concepts can be nationalized. And obviously, you have seen that consistency ratio, consistency values and other scores will give you how consistency in concept can be utilized in trying to analyse how rational your decisions are. So, that can be utilized.

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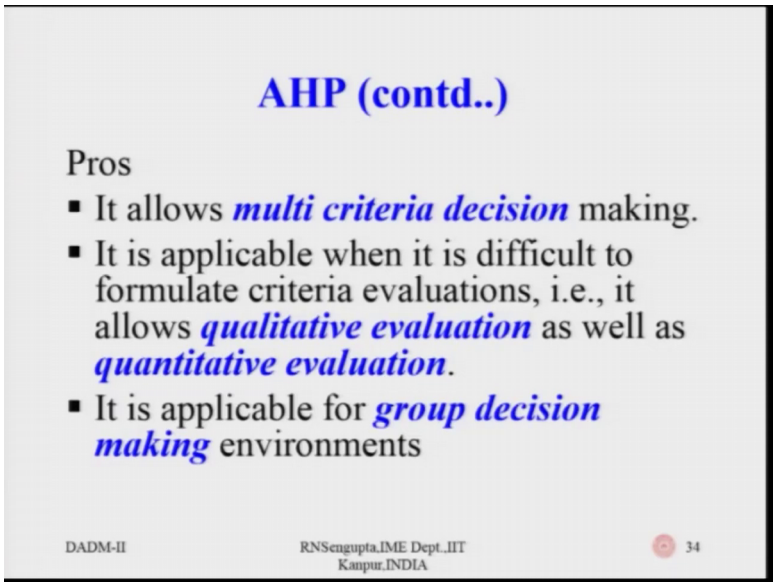

So, AHP is applicable where it is difficult to formulate criteria evaluation, that is trying to basically combine. Qualitative and quantitative is much more rational using AHP, if rationality definitely would be considered we can add them accordingly. It is applicable for group decisions. So, when there are more than one decision maker and the decision has different type of alternative, different type of criterias. Like if you remember very very nice example though not solved in details because it was very simple problem Ram and Shyam making a decision to go to the MBA schools and then their father come. So, at each level there are many decision makers and then you combine.

Say for example, you are trying to analyse the problem where you have say for example I have mentioned it very briefly. So, say for example, candidates has applied for the HR general manager or HR senior manager and on the other side of the table the MD, the chairman, the vice presidents like say for example, 5-6 persons us are sitting and they want to analyse how the good the or bad the candidate it is. So, you will try to basically analyse the candidates academic credential, what type of work experience he has, and what is his psychological profile, and what his demand of salary and all these things would be considered.

But, all the people who are trying to analyse the candidate will have different criterias different levels of scores for the same set of criterias. So, combining them would give you a much more better perspective.

Now, when you combining for the case of Ram and Shyam their parents were giving equal weightages to their sons Ram Shyam. It may be possible the chairman and the MD has a higher say, hence technically this course which you will assign for the overall ranking coming from the point of view of the chairman the MD would be hired while the general manager and the vice presidents who are trying to analyse the candidate their score would be low. So, which would be much more? Rational decision process when you combine all the alternatives, all the criterias and take subjectivity into consideration.

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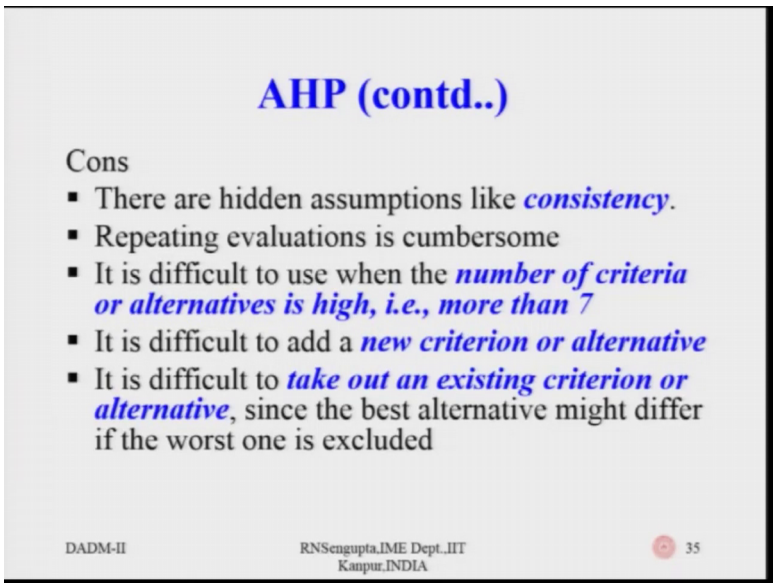

What are the disadvantages? So, there are hidden assumptions about consistency and consistency trying to find out level consistence to be equal for all the decisions is difficult. Because it becomes very difficult to form for me maybe it is very difficult when I am trying to analyse the decisions on a one to one basis; that means, i is to j and forgetting about all the other criterias or all other alternatives which are there. So, I only concentrate on i and j, but when I j come to j and k I only consider on j and k.

So, trying to combine and then when I come back to i and k then j is removed. So, trying to combine i j, j k and i k, i j k are say for example, the decisions of the alternatives whatever it is on the criterias and then trying to believe basically combine them and find a related ranking between i j k may not give us a very good consistency ratio. So, this has to be thought about. Repeating evaluations is cumbersome because repeating the same thing time and again may not give us the same relative scores which we actually want to

have. So, if you remember the principal diagonals one there is no problem, but trying to basically combine for Civic and i20 on different fronts, that the differences of the scores may be very vague if you keep repeating that. So, hence the actual scoring system which should be between the cars or between the criteria may not come out when any person is analysing.

Or it may be possible they say for example, when buying a part car the first person analyses and gives the score that the second person when he or she wants to gives his or her score he or she may be influenced by the decision of person one. So, those influences we consider are not there, but it may be possible those decisions are there. Say for example, in the other example when trying to come back come find out how would you recruit senior manager for the HR. So, the chairman may have a higher say and his or her saying may have an influence on the vice president on the general manager he is also trying to analyse and they may be influenced.

It is difficult to use when the number of criteria alternatives is high its more than 7 because trying to compare 7 c 2 combinations, because taking two combinations from 7 set becomes very difficult and very confusing when you keep repeating this comparison. It is definitely difficult to add a new criteria in an alternative because the relative ranking of the alternate on the criterias would change; absolutely true. This is one of the most difficult points for AHP which does not give it a positive light. Obviously, it is on the negative side, but still AHP is used quite heavily.

And it is also difficult to take out an existing criteria on alternative, since the best alternative might differ when you compare them against the existing alternatives when one of them has already been taken out. So, the relative ranking would suddenly change whether you add that or you take it out; as I mentioned if you have say for example, 7 different alternatives. I am considering in a very arbitrary sense and if you take out the one of them, then the remaining 6 ranking may change relative ranking or if you basically have 7 one and you add eighth one, then I also the related ranking may change because the scoring pattern which you are doing for even the same person would obviously change quite drastically.

Now, I will just go through some brief very simple mathematics. This is just for information. So, this, I thought I should have covered in the initial part, but when I was

when I was making the slides I thought that best for AHP is to do discuss the concept, go directly in the problem and then come back to the simple concept of consistency which I had been repeating time and again today also; that means, in the 25th lecture and also in the 24th lecture.

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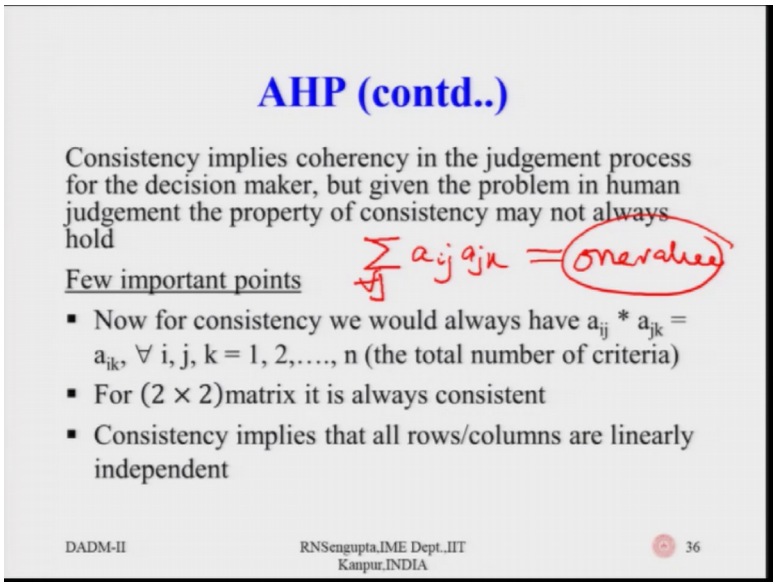

Consistency implies coherency in the judgment process. So, this is coherent concepts. So, coherency of logic logical sequences of steps are there when you make a decision, whoever it is making a decision buying a car, choosing a house or trying to basically recruit any person in your company. And basically, the coherency should be there in the judgment process as I just mentioned for the decision maker, but given the problem in human judgment the property of consistency may not always hold true. So, that may change. So, that is why we do the consistency check through the ratios.

Few of the important properties for consistency is (Refer Time: 10:52). Now, for consistency we would always have the, if you remember the i j k is this suffix which were there and the numbering was depending on the number of criteria number alternatives. So, if you have i j k running from 1 to n and if you find out the multiplication; so, if you remember I multiplied the matrix with the vectors to give the overall ranking. So, if you multiply this a ij cell with a jk cell the values gives us the total number of such scores addition of this of the score. So, you what you are doing is you

are adding up the values i jk a jk for all values of j. So, that will give you one value which will be in the vector of of the final score, if you remember we have done that.

So, those doing that number of times obviously, will give you a different levels of consistency. For a two by two matrix it is always consistent, we would not go into the proof, but I will state it. So, consistency implies that all rows and columns are linearly independent. So, it is a something to do with matrix a linear set of equations. If you remember the Gauss Jordan method of elimination, we are considering that there are n number of equations and n number of variables and each of them are independent all equations independent of each other hence you will get unique solutions.

So, if you basically have the matrix concept. So, here all the rows or all the columns are independent and none of them can be expressed as a linear combination all the other hence the concept of rank will come out of the matrix such that the rank would be n depending on the number of rows or number of columns which you have.

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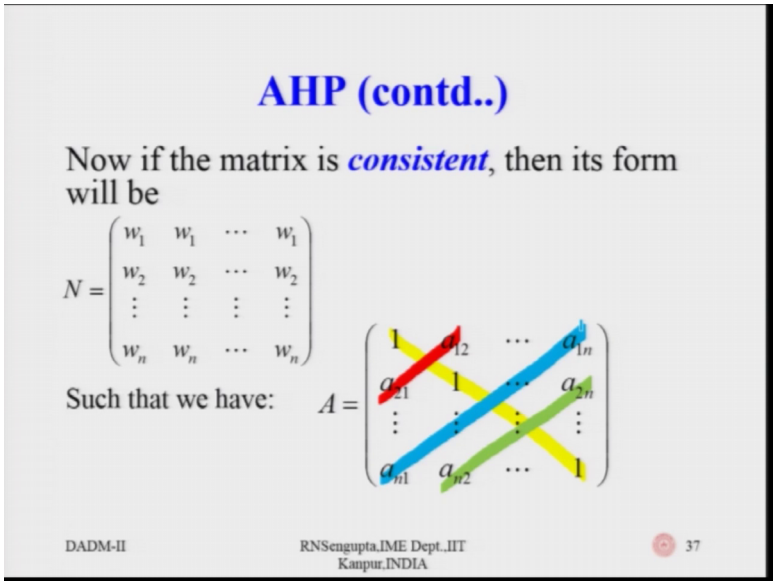

Now, the matrix is consistent. So, consider it is of the form that you have the weights given as w 1, w 2, w 3. So, these are the weights for the first one is w 1 second one is w 2, similarly for the last one is w n. And the values which you have for the priority matrix; obviously, as usual the principal diagonal is 1 and the of the diagonal element which you have these are not symmetric. So, a 21 is not equal to a 12 because if a 12 is 9 a 21 will be 1 by 9 or if a 21 is 7, a 12 will be 1 by 7. Similarly, if I consider a n 2, a 2 n again they

are not symmetric because the scoring pattern which you are doing for the criterion and the alternatives are such that you follow the seams seam the scheme of scores of 1, 3, 5, 7, 9 and if this ambiguity you been bring the even numbers.

So, in this case it is an asymmetric matrix the principal diagonal being one. So, it is the priority vector.

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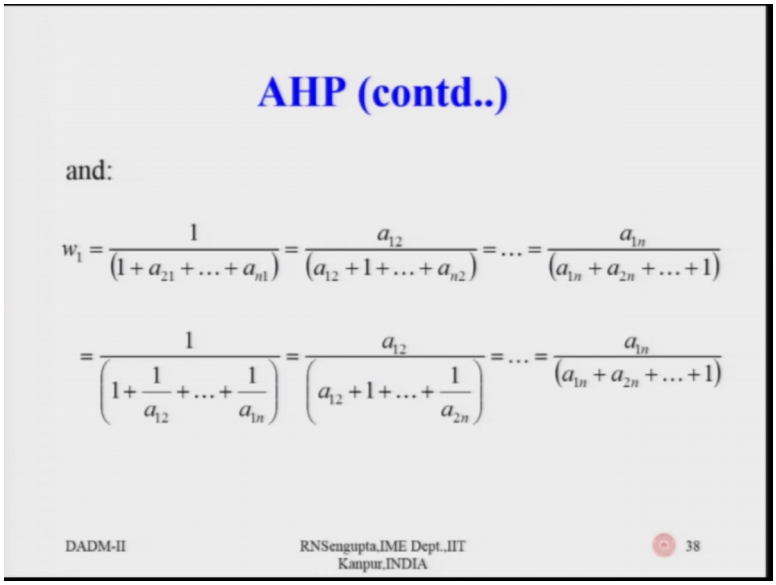

So, if I find out the weights. So, weights would basically be the ratio of the sum of all the corresponding rows of the columns and if I have them the corresponding d in the weights w 1 or w 2, w 3 would be found out accordingly and we can find divide it. So, this is just a end result. I am not going to go into the actual mathematical derivation and the use of that. But remember one thing consistency property is important for AHP, point one. The ranking system would change if you add or delete a particular criteria or alternative that is also important.

Trying to find out the priority matrix, you have to have the level of consistency judgment falling up in the rational fronts such that consistency ratios or consistency concepts are not violated in the final decision, given the consistency matrix you will try to find out the consistency vector. So, sorry the priority matrix you will find out the priority vectors and go from the lowest bottle level from the tertiary to the primary and so on and so forth to give the cumulative scores where you find out the sum of the multiplicative factors of the scores

And another thing which I have repeated time and again so, remember that the utility function which you follow on normalizing whether through the rows or the columns such that the sum is equal to 1 after normalization. The normalization concept of whether is quadratic or exponential or power function or logarithmic function whatever you are utilizing would be used consistently for the same person for all the alternatives and decisions. And try to use the same normalization concept of the utility function for all the different persons when you are trying to combine the decision makers like decision maker 1 or 2 or 3 or 4 when trying to arrive as a collective decision. Like say for example, for the car or say for example, trying to recruit a person the example which had given.

So, with this I will end this AHP part and then start of a new concept of the Electra process. So, this technically I thought as we are going flow in the flow it would have ended in the 25th lecture and end of that, but I have a finish a little bit ahead of time. So, whatever time is left for this 25th lecture which is the last lecture for the 5th week I will consider and a new important topic to be added. So, just bear with me I will open the slides and discuss that accordingly. So, this is the AHP part and just one thing. So, finally, to wrap it up for AHP; so, then once we find out sorry for that, I just missed this slide.

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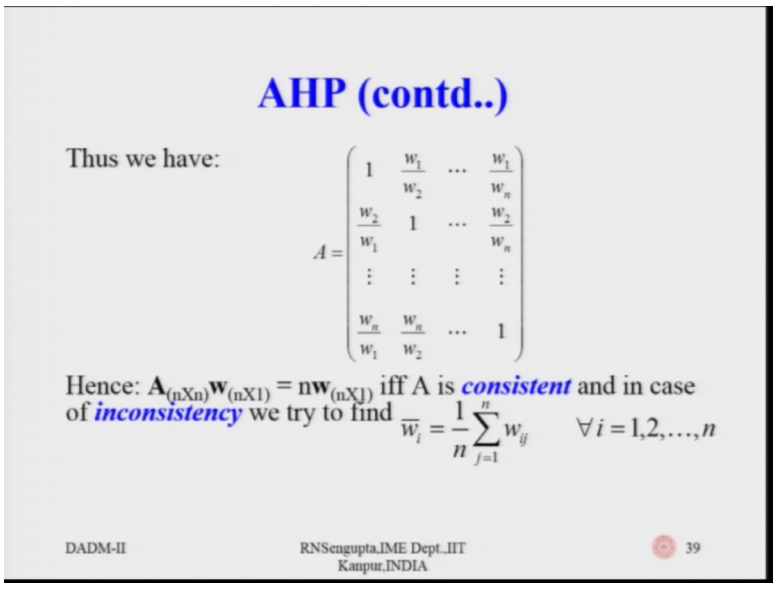

So, when we find out the actual a matrix. So, they would be normalized, so and finding the ratio. So, once you find on the ratios. Hence, the matrix a which is the normalized concept you multiply the weights is it will be equal to n into w and if and only if a is consistent and in this case if the are inconsistent you can find it out, but for the consistent scores you will use the average. If you remember in the first example when we are doing the consistence ratio we found out the averages and proceeded accordingly. So, these are just for interest you can pick up the Sathy's book the references which are already given when you started that class. So, you can check that and find out the actual mathematical formulation of consistency how it can be utilize.

Here in this course we remember we were just discussing the application of different above MCDM processes. So, with this I will end I will again repeat m and AHP and then I will try to start. So, we are continuing the 25th lecture, so just for when if at all I upload the slides, so I thought they should not be any confusion, ok. So, we are going to start a new method which is known as ELECTRE, which is Elimination and Choice Translation Reality.

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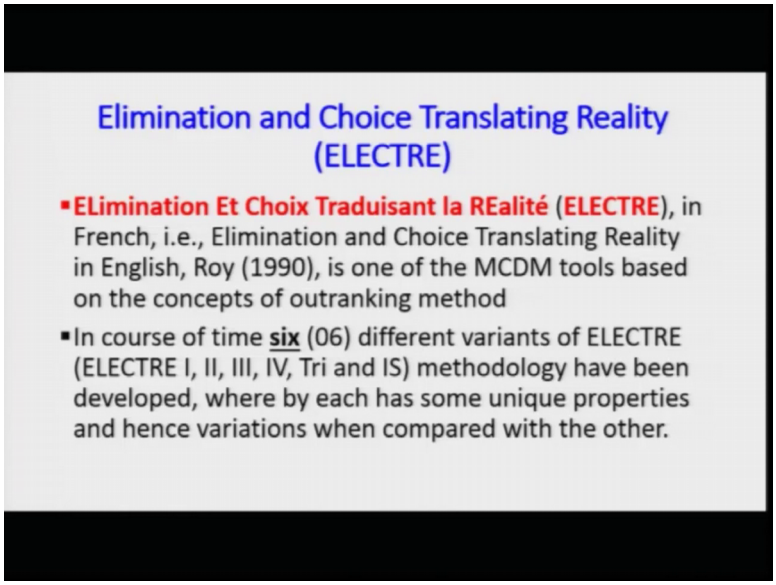

Now, I will try to for to finish all this standalone topics in such a way. So, this continuity without this break which happened in this (Refer Time: 18:56) end of the 25th lecture. Obviously, there is no confusion, but to make a smooth flow it will be much more easier for you all to understand and easy for me to continue without any break in the flow.

So, what does ELECTRE means? So, ELECTRE is basically the word is a French word which basically means elimination in charge translation reality. So, you are doing the concept in such a way that you have a set of criterias and decision. And you try to find out the best possible decision which is a theoretical concept, such that you will try to find out the positive distance of liking from that the so called theoretical set a theoretical value for all the decision making this is an alternatives which are there, and also try to find out the negative weights; that means, the negative distance from that so called theoretical value for all the decisions which you are forced to take due to some circumstances.

If you remember I did mention one of the example, that you are a bank and you are forced to or due to government regulations you are told to open it in the rural district. So, if you consider that profit motives a important factor; obviously, that is not the best choice, but still you have to do that. So, how do you make a decisions accordingly? So, you will consider some distinct matrix coming out such that you will divide the overall decisions into two sets, number one this is a positive set which gives us positive values and a number two is the negative set which gives us negative values. And then try to compare and combine this positive set and the negative set and try to basically take a collective decision.

Now, the point of positive set and negative set is in this way. Set means a set of scores which you are giving. Now, the point of positive and negatives sets in this way. Say for example, there are two alternatives, i and j. Now, we will consider as we have done in the AHP that you give a score of 9 is to 1 9, 5 is to 1 5. So, the point of 9 or 5 or 7 or 4 whatever it is you are giving to the case of the decision of the alternative which is positive to you and you give a inverse of the score to a one which is definitely not positive to you, but you are being forced to take the decision due to some circumstances. In the similar way we exactly do this.

We basically compare two decisions two alternatives i and j and if i is better we give a positive score of positive point to i and when if you are forced to take j we also give a score. But the point is that you try to rationalize the now the score in the normalized scale that i is to j and j is to i if the points are equal which means that I am equally disposed whether I take decision i or decisions j and if I am not equally disposed between i and j. Say for example, I like i more than j or i like j more than I in that case

the preponderance of the scores between i and j and j and I; if I like i and if I like j more than i then the preponderance of the score of j to i and i to j will be such that the points of j would be higher. This would be coming up correspondingly using some very simple concept of liking set and disliking set.

Here remember in the initial case will consider the liking and the disliking set to be such to be made. In such a way that the distant matrix are of equal proportions, in the sense if I like something my propensity of distance would be of same magnitude at as to the level when I dislike something, which may not be true. Because in the actual sense if you remember I have discussed the concept of asymmetric loss functions where I am assigning a highest score or lower score or a higher loss or a loss depending on whether my estimation is high or low.

If you remember I have considered three very simple examples one was for basically for building the damn from the civil engineering concept, one was basically from the electrical engineering where you have a set of a machine is there and you have vacuum circuit breakers to understand that when you should basically do the is general maintenance of the very costly machine. And another example was that when you are trying to float in a product in the market from the market perspective the warranty life it is high or low would basically help you to gain the market in the initial case, but you lose the market as you as your product fails earlier.

Or in the second case that you are not able to win the market in the initial case, but you will slowly gain the market as your products being popular considering the warranty time is actually different with respect to the competitors. So, these three examples gives a, a gave us the concept that whether overestimation important for the case when building the damn, underestimation important for the case of the electrical circuit and where case overestimation or under estimation can whatever is important for the marketing example. So, this concept initially would we would not use in the ELECTRE, and, but will be try to bring that using the distance concept.

In course of time 6 different, let I have not finished the first bullet point. So, this is the elimination and choice translation reality which is in English it was developed by a Frenchman by Roy in 1990s, when trying to basically analyse different huge products projects, where decisions were and were both subjective an objective to arrive at a

rational decision in trying to rank projects where different criterias different alternatives were utilized.

It is one of the MCDM tools based on the concept of out ranking method; that means, how good or bad your ranking system is, how you are able to outrank other decisions or other alternatives. In course of time six different variants of ELECTRE came into the picture, they are ELECTRE I, II, III, IV, Tri and IS methodology. And they have been developed where each by has some unique properties and hence variations when compared to each other would be there, but will only consider the simple concept of ELECTRE how it is utilized.

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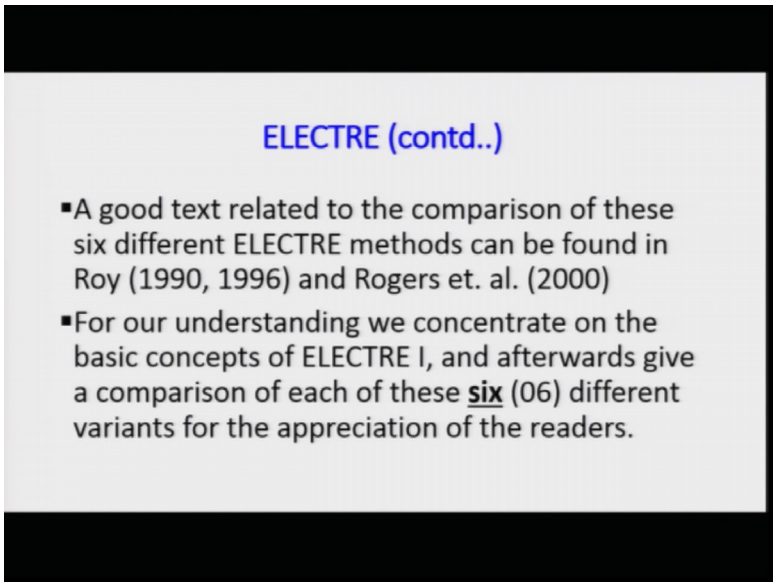

A good text related to comparison of this 6 different ELECTRE method can be found by the in who is was basically the inventor Roy in 1990 and one paper 1996, and by Rogers the paper was out in 2000. For our understanding we concentrated on the basic concepts of ELECTRE I process only in a very simplistic sense, but as I said I will add the concept of outranking in asymmetric sense also like whether I have liked or disliked the weights would be different depending on the level of loss function or the asymmetricity we which you want to bring into the picture.

And afterwards we will give comparison of each of the 6 different variants for the appreciation of the readers or appreciation of the students who are doing this course. But I will main concentrate on the ELECTRE method with outranking concert being equally

disposed for whether you like or not like, and then being unequally in disposed using the asymmetric concept of loss; the concept wise, we will bring it very simply.

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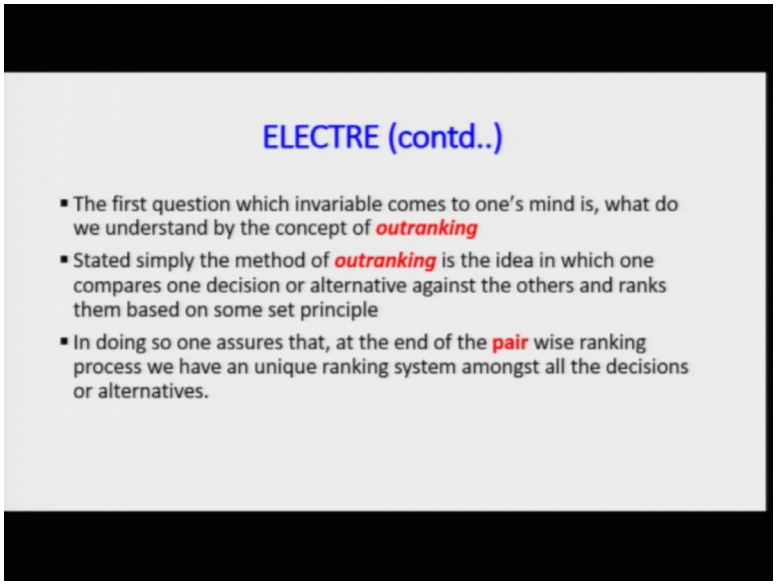

The first question which invariably comes to into one mind is what do we understand by the concept of outranking. So, what does outranking mean? Does it is does it mean a relative score? Does it give us a absolute score? What does it mean? Stated very simply the method outranking is the idea in which one compares, one decision of alternatives against all others and ranks them based on some set principle and as you do that you are trying to basically find out the relative score where the decision on the alternative stands with respect to the all the other sets of decisions and alternatives.

In doing so, one issues that at the end of the pair wise ranking; here also it is a pair wise ranking. So, if there are n such alternative decisions you take the jth one and compare the j 21, j 22, j 23, so on and so forth and then you go to compare j to j minus 1, j to j plus 1 and go till j 2 and n, and put j alternative of decision in the, right perspective where it stands. So, let me continue reading it. In doing so, one I one assures that at the end of the pair wise ranking process we have a unique ranking system amongst all the decisions or the alternatives such that we are able to take the decision correspondingly.

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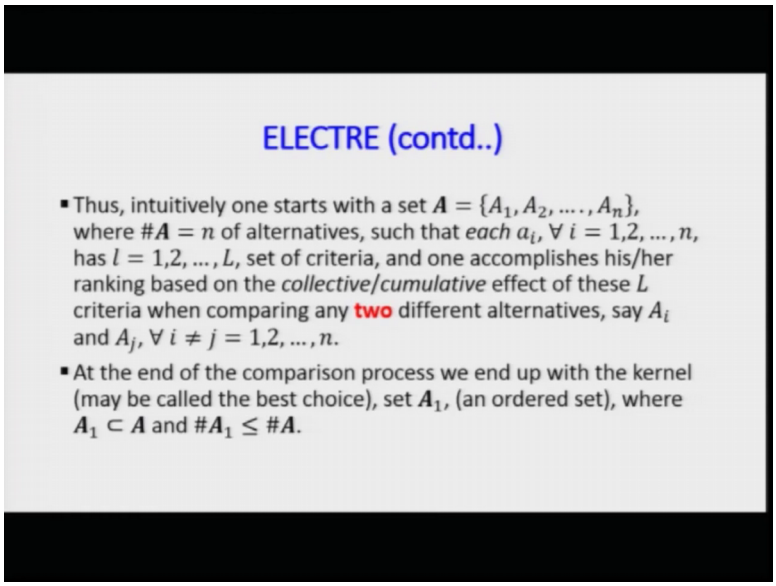

Now, nomenclature would be important, so I will go a little bit slow. Thus, intuitively considered you have the alternatives a given as A 1 to A n. So, each alternative will have criterias. So, it can be c 1 to c m for A 1, then again c 1 to c m for A 2 and so on and so forth. Now, the number of m which will have for A 1, A 2, A 3, need not be same. So, if they are not same so obviously, we will consider some of the cells to be blank or replace them accordingly in order to have a consistency concept of ranking. This word of consistency I am using not from the AHP point of view just from the English language point of view.

So, where the number of alternatives is given as n such that for each alternative, they would be not m, but we are considering L number of criterias. So, one when we are comparing A i, A i 1 to A i 2; A i 1 means the suffix is i 1 is basically the i 1 is one element of n and i 2 is also an element of n. When I am considering alternatives 1 to 2 or 2 to 3 or 1 to 4, like I said i 1 to i 2 then I will consider each and every criteria to be considered such that I am able to compare using criteria 1, I am able to compare A i 1 and A i 2. Then considering criteria 2 and again I am able to compare A i 1 and A i 2 and continue doing that such that I assign scores at each level of the distance and then do the collective combination of the scores.

So, it says thus these 1 to L are the set of criterias and one accomplishes his or her ranking based on the collective the cumulative effect of all these L criterias which I just

mentioned, when comparing two different alternatives one at a time and or say for example, considering the other things accordingly. At the end of the comparison process we end up with a with a best choice set, where we say see that A 1 or A 2 or A 3 are are ranked in such a way based on the on the criterias we can rank them accordingly.

So, with this I will end the last lecture of the 5th week and continue the discussion of ELECTRE in more details and solve a problem in the same concept, solve a problem. And, then come to the actual mathematical algorithm or decision process how you make that.

So, have a nice day, and thank you very much for your attention.