Data Analysis and Decision Making - II
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Lecture - 14
DEA

So, welcome back, very good morning, good afternoon, good evening to all of you. And as you know this is the DADM - II course under the NPTEL, MOOC series and we are in the 3rd week and this is the 14th lecture; that means, with two lectures 14 and 15th we will wrap up on the 3rd week. And as you know this course is for 12 weeks, which is 60 hours each week we have half an hour such 5 lectures and after each week we have an assignment.

Till the 13th lecture we discussed about the concepts of utility and how you can consider a utility for different things, I will come to the decision trees later on, but first I would like to start off with the concepts of data and analysis; as a concept then come back to solving problems accordingly.

Now, so this is PDF files while scroll down, so, please bear with me. So, consider you have a system, so system is basically any unit with take some input and basically give some output.

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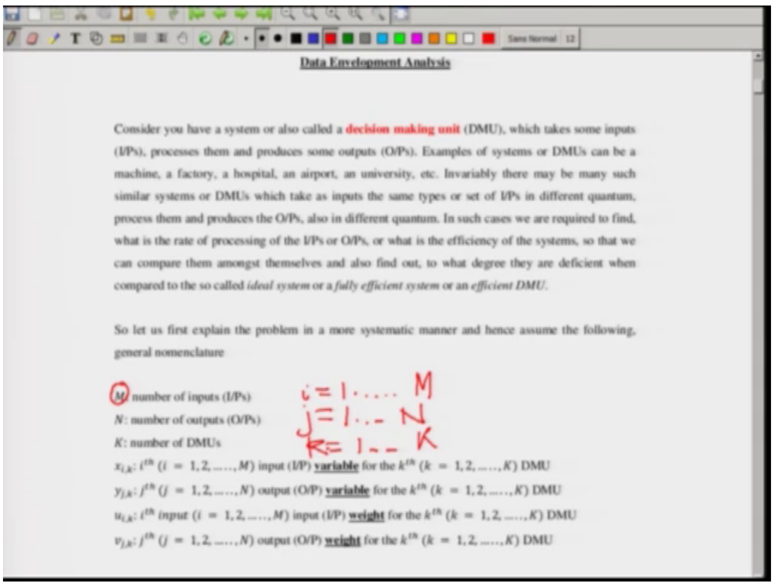

And consider those units are called the decision making units. Example can be hospitals, so Apollo hospitals if they are at different parts of the country, so, in Mumbai, in Kolkata, in New Delhi, in Chennai in Bengaluru. So, if you have different units of Apollo, so they will be considered a decision making units or DMU's as mentioned in the first line. Or it can me say for example, in the state government of different states Maharashtra, then you can be Chhattisgarh, Uttar Pradesh, West Bengal, Odisha. So, we will basically consider there as some decision making units. It can be in school also or we can be the IITs, also all like Kanpur, Delhi, Mumbai, Bombay, then IIT Madras, IIT Kharagpur or it can be limes also.

So, you have some decision making units and what are the inputs. So, say for example, for the hospital the inputs can be nurses, can be paramedic staff, can be doctors, can be number of beds, can be different type of services which I given OPDs, it can be either related to (Refer Time: 02:35), it can be related to some cardiac related treatment whatever and the output can be see for example, the occupancy rate or the bed.

So, lower the occupancy rate better because people have being, they are recovering and then going out of this nursing home. So, how rate the people move out from the system; it can be the number of successful operations, it can be number of success successful cases, critical cases handled, it can be see for example, also be the number of very sophisticated operations which are being done on. In the case of say for example, IITs and IIMs in the inputs can be students, can be teachers, can be staff, can be the library, can be the infrastructure, computer, classrooms, labs. And the output can be quality of students who were working in different types at sector of the country, it can be how well they are placed.

Later on, what is this average salary package, how many are going for higher studies, how many going for research, how many joining government services, how many are starting their own businesses or it can be how the IITs and I am sorry able to help, the people from the lowest rate of the society get education. So, it can be different things.

Say for example, for the government it can inputs can be government servants input can be say for example taxes, output can be say for example, the quality improvement as system, quality water then traffic problems are eased out, crime and law and order are minimize, so, they can be different outputs.

Whenever you are considering this in DMUs, the inputs and outputs need not be in quantitative terms, need not be in terms where you take an basic quantify in rupees, in dollars, in yens. Say for example, I want the number of students to graduate from the universities. So, it will be only numbers or inputs can be say for example, the library or output can be the salary, on median salary average salary. So, for the salary we have some values for the library it may not be quantifiable it may be more qualitative.

So, with this let me read it consider you have a system or also called a decision making unit which takes some inputs, a bundle of inputs, processes them whatever the processes. Say for example, if is a CNC machine, it is taking the raw material, it is taking the cutting fluid, it is taking the electricity, it is utilizing the man over of the worker who is working on that, so they can be different things and output can be is different type of HP Motors which are being produced, can be the scrap material which is to be sold in the market. So, they can be different type of outputs.

So, let me continue reading it. It processes them and produces some output. So, we will denote inputs by IPs and outputs by Ops. Example of such systems on DMUs can be as I mentioned as machinery, a factory, hospital and airport and university. In a invariably there may be many such similar systems, like if I am talking for the example for the IITs or the universities can be all the IITs. So, they are each single DMUs. If you considering the government or each state is a DMU, if you considering the airport example it can be would be the different type of airports which you have in India.

So, invariably they may be many such similar systems or DMUs which takes as inputs which is the same types of inputs in different quantum. In some of and the and gives the outputs in different quantum. It may be possible that in some of the machineries which you utilize into produce. Say for example, different HP Motors. Some of the DMUs may not produce those DMUs, so obviously, we produce those type of motors we will consider them to be 0 in that case, but we will consider bundle of a in inputs and bundle of outputs have the output and inputs as IPs.

So, invariably they may many such similar systems or DMUs which take as the inputs and the same types of sets of inputs IPs in different quantum, process them and produce the outputs also in different quantum’s, in different numbers. So, in such a situation we are required to find that what is the rate of processing of this inputs and outputs or what is the efficiency of the DMUs. So, what is the efficiency? Would be considered relatively; that means, how good or bad is DMU 1 with respect to DMU 2 or how good or bad is DMU I with respect to DMU J. So, we will be basically compare the efficiencies of the DMUs and say there where the improvements can be done.

So, in such cases we are required to find what is the rate of processing of this inputs and outputs or what is the efficiency of the system I am just repeating this statement. So, that we can compare them against themselves that this is the DMUs and compare against themselves and find out to what degree their deficient when compared with the ideal system or in what respect their efficient even considered with different other DMUs which are there. So, obviously, they would be two ways of comparing, one would be the way of when you are trying to compare the absolute scale and the one of the reality scale.

Say for example, I will give an example which may not make immediate sense say for example, you are ok. Let us let me consider we are doing the marking of DADM - II. So obviously, many students up to in different marks, so, they we can consider or generally what is followed they can be two marking scheme; one is the relative marking scheme, one is the absolute marking scheme or after scale relative scale.

In the relative scale the highest marks is say for example, picked up to 100 and then basically the grading found out and based on 80 would be in; so say for example, 80 might 100, so proportionally in the person who get 70 is mark is made his or her marks is basically picked up to the relative scale. Similarly, of 40 marks student his or her marks would be picked up to that relative scale when where 80 has been made 100. So, this is the relative scale, none obviously, the relative ranking would be given.

In the other case and obviously, the grading would different depend because you have already decided on the brackets of the grace depending on the marks. Now, when you come to the absolute scale? So, we will consider the absolutely marks whatever it given a it is as it is. So, if the person has got 80 it will be considered 80 out of 100 and it will basically pick the mark, so, the grace of the other students accordingly to a absolute scale.

So, to first explain that let us explain the problem in a most systematic manner and let us assume the following. Number 1, m is the input numbers so, they would be 1 2 3 4 till capital M, capital N, N is Nagpur. Say for example, is the number of output; so 1 2 3 4 till capital N, if k number of DMU, so 1 2 3 4 till capital K. So, what time basically highlighting are these, so, this is basically and use a. So, this is M is the number of inputs. So, you will basically have the inputs as going from 1, i is equal to 1 till capital M j would basically be the output, so it will be going from 1 to capital J and k would basically be the number of DMUs it will go from 1 to capital K. So, M j sorry my mistake so, this would be, so, this would be till N.

So, capital M, capital N and capital K need not be equal; obviously, M and N need not be equal also. So, you will denote x suffix i k as the ith input variable for the kth DMU. So, basically you have the DMUs 1 2 capital K. So, for each one they would be that inputs, so, we will consider i number of inputs are there, i can many number. So, you will basically have x 11, x 12, x 13, x 14 so on and so forth, where the first number will denote what is the number of the input and the next number that is 11 or 12, that 1 2 3 4 which is the second number would denote what is the DMU size.

So, basically x 11, x 12, x 13, would basically denote the first input for all the corresponding DMUs 1, 2, 3, 4, 5, 6 so on and so forth. So, we can basically do it in the another way also by in doing the nomenclature as x 11, x 12, x 13 where the first number basically may denote the DMU also. Similarly, you will basically have y j k where it will denote the jth output corresponding to the kth DMU.

So, again similarly y 11, y 12, y 13, y 14 will denote as for this nomenclature; the first output with respect to the first, second, third, fourth, fifth and so on and so forth DMUs or if you follow the policy where the first subscript denotes the DMU, then basically we will denote as y 11, y 12, y 13, y 14 so on and so forth where the first number 1 would denote the DMU and the next number 1 2 3 4 5 6 would basically denote the output. And obviously, the outputs and in inputs and the DMUs would be denoted by the maximum number N, M and K.

Will also denote u i ks and v j ks as the rest as the ith input for the kth DMU and the jth output weight these are the weights, so, u and vs are the weight. So, v is v j k is the jth output weight for the kth DMU. Again, you will basically have u 11, u 12, u 13 so on and so forth, where the first number would denote the in input number and the second number 1, 2, 3, 4, 5 as per this nomenclature denotes DMU. It can also be reversed in the sense you can consider x 11, x 12, x 13 so on and so forth where the first number one denotes the DMU and the second number 1, 2, 3, 4 so on and so forth denotes the input. So obviously, you can basically make the differentiation accordingly.

Similarly, when you have v 11, v 12 so on and so forth as per this nomenclature the first number denotes the output number and the second number 1, 2, 3, 4 denotes the DMU. You can also reverse it where the first number would denote the DMU and the second number would denote the output.

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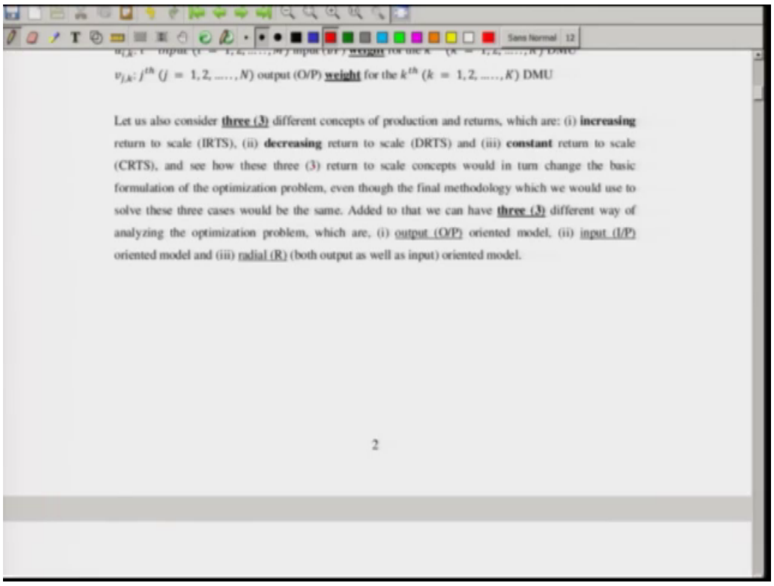

Now, let us consider a let us highlight here. So, let us consider 3 different concepts of production and scales of returns. So, that will give us the picture why we are trying to do the formula of form formalization of the optimization problems accordingly. We will come to the solution later on. So, let us also consider 3 different concepts of production returns which are number 1 increasing return to scale IRTS, decreasing return to scale DRTS, constant return to scale CRTS.

And we will see that how these 3 returns to concept would a would in turn change the basic formulation of the optimization problem, even though the final methodology based on which we will try to solve the problem the simple optimization problem concept would remain the same will all the 3 cases.

Added to that what we said is basically this increasing, decreasing and constant return to scale we will have 3 different type of models; one is the output oriented model, one is the input oriented model and one is the radial model we will consider the combinations on the output and the input coming into the picture. So, output would have some implications, input would have some implications. So, the output word means some implication with respect to the output, input means some implication with respect to the input and radial (Refer Time: 15:14) would basically combination of them. So, let us consider this graph.

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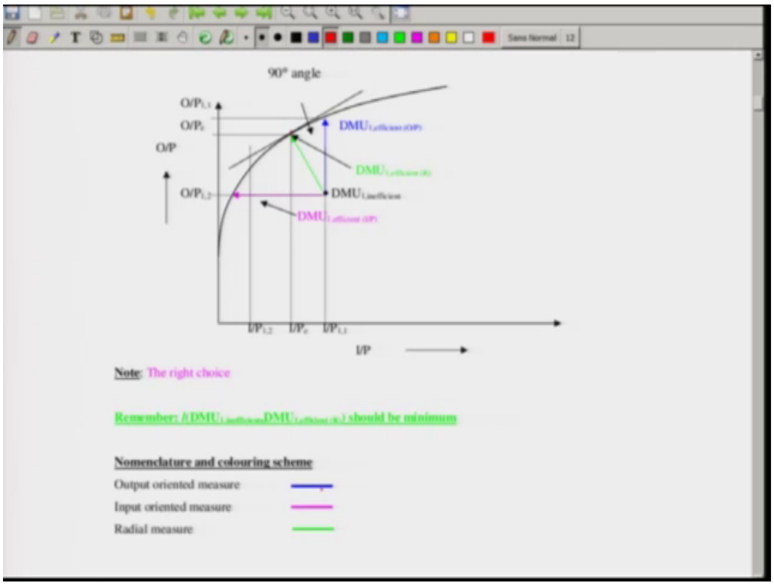

So, this is the decreasing returns to scale, so which is a I will write it here is me, ok, I think I should use the. So, this is the decreasing return to scale let me decreasing return to scale. So, this is the decreasing return to scale which I have just highlighted. Now, let us look at the graph.

So, first note down the y axis and the x axis. So, when the y axis you have the output, on the x axis you have the input. And thus the graph which gives you the combination on the output and the input a's ,which I will again highlight. Now, using say for example, the light pink color, let me use another color, let me continue using the yellow one. So, it will be is because yellow one is not here. So, the graph is this one, this one which you highlighted.

Now, notice one thing. Notice the black one DMU 1 which is mentioned as inefficient because the set of all the DMUs which are combination, so why I am saying that let us consider. Consider arbitrarily which I will erase, but a consider arbitrarily a point is here, a point is somewhere here and this is the DMU. Now, if you consider the input, the input is here, the output is here. Now, if I am looking at the bundle of input and output what I would basically think, they can be two ways; one is at the same level of input or output I will basically try to minimize or maximize is in this way.

Consider there are other sets of DMUs which has scattered. So, let me use this as the set up points. So, I am using a, these are set up points which are there all the points which are there a DMUs. Now, we has taking as a example I will erase this later on. So, consider the point which has is here which is just vertically up, it means at the same level of input the DMU can get an higher output which is here I would I just very lightly mark it because I want to erase it later.

So, rather than this point I would rather go to this point where my output would be more, consider other way around. I want to basically keep the, so here I keep the input fixed increase the output. Now, on the other hand consider the output is fixed, but the input is very high. So, what I would do? I will try to go, so in this case sorry in this case I will try to go up and in the later case when I am go when it is horizontal I will go more on to the left such that at the same level of output I will try to basically utilize less and less input. So that means, I am always trying to move up vertically up and on to the left horizontal.

So, if you consider this, so now, I can erase this set which is here which I have drawn. So, if you consider the point which is the black one, this is the inefficient which means it can go vertically up which means we have following the blue line and it touches this point which I have not drawn and it is marked as DMU efficient output bracket output what I am going to come to that. So, this is in blue colour and or else I can go to on to the left horizontally which is the pink line such that I reach the DMU 1 which is efficient, but with respect to IP. So, IP is the input and OP is the output. So, what is actually means is this. So, actually it means that this DMU 1 which is inefficient can be made efficient with respect to the input also or with respect to the output also.

Another way, it this inefficient can be made efficient is to move in such a way diagonally such that it reaches that curve which was there. So, the curve, why it is a decreasing in the sense this decreasing return to scale concept is basically it means that as you keep increasing the input. The output increases, but the rate of change of the output starts decreasing; that means, the first derivative is definitely. If you remember the utility concept the first derivative is positive, but the rate of change of the second derivative is basically less than 0; that means, per unit each unit addition of the input you are getting a little bit less and less amount of output correspondingly.

So, if you move, so the if the curve is there where I am hovering in the pointer and if I move diagonally and such that I move the least distance diagonally such that I am tangent to the point where I reach which is basically here, which is marked by IOPE and IPE which I am just marking with the my pointer. So, that point would basically would be the point where both the input decreases and it may also mean that the output can also be increase.

So, then I am trying to basically make a compromise where the input decreases in some proportion output also degrees in some proportion, but the overall efficiency is the higher such that in some cases it may be true that I have to decrease the input, but at the same time I have to increase the output or decrease the output proportionally in order to maintain the maximum efficiency.

Now, let me consider very simply hypothetically examples. Consider the vertical movement up, you have employed 10 workers and 3 machines. Now, it so happens that this workers you cannot be fired because they are permanent workers, but on the one hand and you have already you have purchase 3 machines very sophistic machines is be costing say for example, 20 lakhs. So, in that case what on an average price and in that case it will mean that you will basically have to increase the production in order to increase the efficiency because you cannot decrease your input.

Because decreasing input means what, you have to basically lay off one of the workers which obviously, mean a huge amount will litigation cost or you may basically not try to increase the production, but tell them to (Refer Time: 22:29) idle. But if this is idle obviously, as they being permanent workers you have to pay the salary is the medical bill, the insurances, the provident fund, electricity cost which you are incurring and so on and so forth.

On the other hand, so obviously, they are rational decision has to be taken whether you will go vertical up. Consider the case where you want to basically move horizontal left and in this case consider that you are taking temporary workers; that means, you can hire and fire them and as per the norm.

So, if you have taken temporary worker say for example, on 1 day unit 20, so, you will higher 20 keep them for 1 month. Say for example, it suddenly see the production it falling down and you want to maintain the same amount of say for example, output you may basically give each some person whose working a little bit more, but basically reduce the number of workers we are coming such that are or you we are employing. So, out of the 20 number of temporary workers you may be in a position two say for example, tell 7 of them that they need not come for the coming days and you will basically pay the salary to the 13 one and do the work. So obviously, it can be done accordingly.

Now, the right choice which you do for the decreasing for the decreasing return to scale when you have the this choice to make. So, the best one would be that either you can go up or you can go on to the left of horizontally or you can go diagonally. So, diagonally means that you are trying to compromise on both the concentration of their efficiency psi.

So, the nomenclature is what you can do is that output will introduce is given as I mentioned as the blue colour, input as the pink colour, and the radial one of the green colour. So, remember the distance of the DMU inefficient which is the green, the red black one which I am just trying to how are not mark, till the radial one which is the green diagonal point. So, this distance would basically be the minimum which would is the shortest distance which you going to consider.

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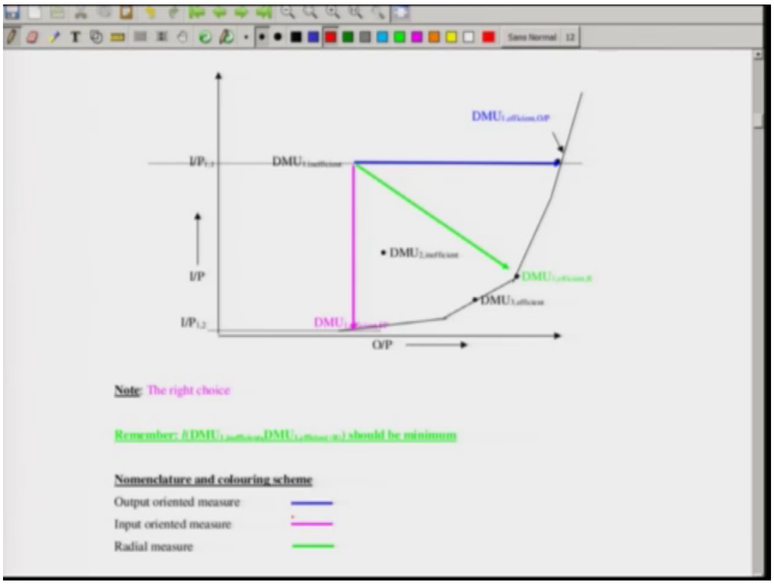

Now, consider the decreasing return to scale, but the axis is are inverted, in the sense you basically consider the input one along the y axis and the output one along the x axis. So obviously in that case the graph would be as input decrease. So, in that case input oriented in this decreasing return to scale the case was there as you increase more and more on the input the comments would what increase in the output is not happening.

So, if you keep increasing the inputs then the outputs then the curve would just be the mirror image, inverse one with respect to the case where you had the outputs along the y axis in the inputs along the x axis. So, in this case again there set of DMUs would be on the upper side of the curve, you can either move vertically down or we can move horizontally on to the right. So, they are basically being shown vertically down being the pink one same thing.

So, it is you will move down such that you are decreasing or input, but trying to maintain the output and then other case you would basically move on to the right; that means, you are maintaining the same amount of input, but trying to increase the output. And the radial case you will try to move the least distance radially such that your tangent to the curves in order that you are able to decrease input and always make a compromise on the output also. So, this increase and decrease would depend on the type of problem you has doing such that you are trying to increase your efficiency.

Again, the nomenclature is the same thing. In the green line is basically radial one the pink one is basically the is input oriented where you only concentrate on the input the blue one is basically which you only concentrated on the output that is why it is known on the output oriented. And the again the same nomenclature of the distance from DMU which is inefficient till the tangent point is the least one which is a straight line.

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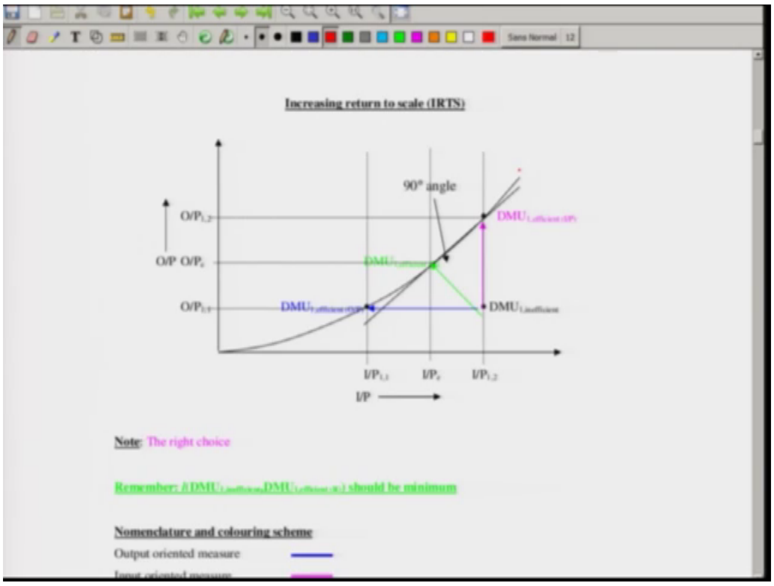

Now, consider the increasing return to scale. So, in the increasing return to scale you will consider the graphs again in the same way. In first case you will consider the output along the y axis and the input along the x axis. The graphs are such that as you keep increasing the input the output would increase, but in a higher proportion such that the rate of change of the functional form of output input is positive, but at the secondary would be higher. That means, per unit as you keep increasing the addition of the output is happening at a faster rate.

So, all the set of in DMUs if you consider the inefficient one, if you go vertically up which is the which becomes there is the pink line there you basically maintain the same amount of input, but increase the output. If you go horizontally left there you basically maintain the same amount of output, but decrease the input and the radial case you basically try to increase the output and decrease the input in such proportion that you get the best combination. Again, the same type of examples which I kept would be applicable.

And here the green line which is the radial one would be such that it is tangent which is the straight line. The nomenclatures again the same, the and for the output oriented model it is the blue one, the input oriented model is the pink one, and the green one is the radial one. And if you see the distance the right choice would be the pink one because you want to basically do the case where the increasing return to scale is there.

So, because increasing return to scale means the as you keep increasing the input the output also increase, but in a higher proportion. In the other case when it is decreasing return to scale it will be if you keep increasing the input it is not in it is going to increase, but it is going to increase as the lower scale.

So, in the next class I will discuss the increasing return to scale, but with that axis is inverted in the sense the input comes into the y axis and the output comes into the x axis. With this I will end the 14 lecture and continue with discussion of DA in more details at least the discussion and then come to the formulation once we complete few simple optimization problems.

Have a nice day and thank you very much.