Simulation of Business Systems
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Lecture - 06
Terminologies and Like Approaches

Good afternoon students. Here, we are I welcome all of you to yet another lecture of the course on Simulation of Business Systems.

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And, today we are going to cover the topic Terminologies and Like Approaches. With some additional terminologies and the stochastic simulation that we are talking about, what are the type of stochastic simulations that we are going to cover as part of the course also going to be looked into it. So, the today this sections major learning agenda are 4, the first one is we talk about the analytic actually the not the analytics it is analytic versus the numeric methods ok.

So, let us see how the analytic method and numeric method compares and most of the type of variables, these are the variables to be used in simulation.

And then some on the simulation like methods and we will look into Monte Carlo simulation. So, today's topics are mostly related to slightly advanced concepts and we

will also do an example of Monte Carlo simulation. So, that you can understand what is Monte Carlo simulation?

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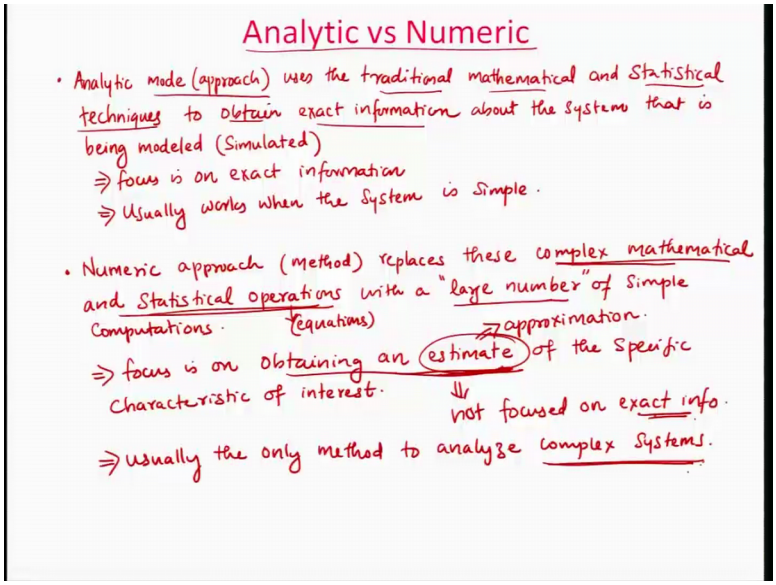

So, let us get through the first topic today called as Analytic versus Numeric simulation or analytic versus numeric approach. And by definition the analytic mode ok. So, let us write it this way, analytic mode or approach uses the traditional. So, the analytical mode or approach uses the traditional mathematical and statistical techniques; mathematical and statistical techniques ok.

They use the analytical and mathematical techniques to obtain exact information exact information about the system that is being modeled or what we call as that is being simulated ok. So, the most important thing is this approach this mode utilizes the traditional mathematical and statistical approaches ok. We use traditional mathematical and statistical approaches or techniques and why do we use that? So, that we can get exact information about the system, that is being modeled. So, the focus here is the focus is on exact information ok.

We want the exact value ok. And, usually works when the system is simple? Ok. So, this is the analytic approach. The numeric approach whereas, the numeric approach or we can call it as the method, the numeric method replaces these complex mathematical and statistical operations mathematical and statistical operations with a large number of large number of simple computations ok.

So, what do we do is we eliminate the complex mathematical and statistical operations or we eliminate those equations and then we replace them with a large number this is important point large number of simple computations ok. And, focus is on obtaining an estimate an estimate of the specific characteristic of interest ok.

So, the idea here is that, our main aim is to the focus is on obtaining an estimate ok, unlike so, not focused on exact information ok. So, we do not look for the exact info instead we want an estimate an estimate is in another way to think about it as an approximation ok. You want estimate the specific characteristic of an interest ok. Also, usually the only method to analyze complex system or systems so, when the system is complex, when the system is complicated, then you would rather like to use the numeric method or the estimation method, where the aim is to obtain an estimate and not an exact information unlike the mathematical and statistical technique. And, the numerical approach replaces the complex mathematical and statistical operations or the equations ok.

These operations can also be taught as the equations ok. So, here you replace the equations or the mathematical operations with a large number of simple computations, we use computers to do a set of simple computations to obtain this particular thing called as a estimate or an approximation, that is a numeric method. And, when the systems are complex when you dealing with large complex systems, then you use numeric approach instead of the analytical mode, which is more dependent on the traditional, mathematical and computational approach.

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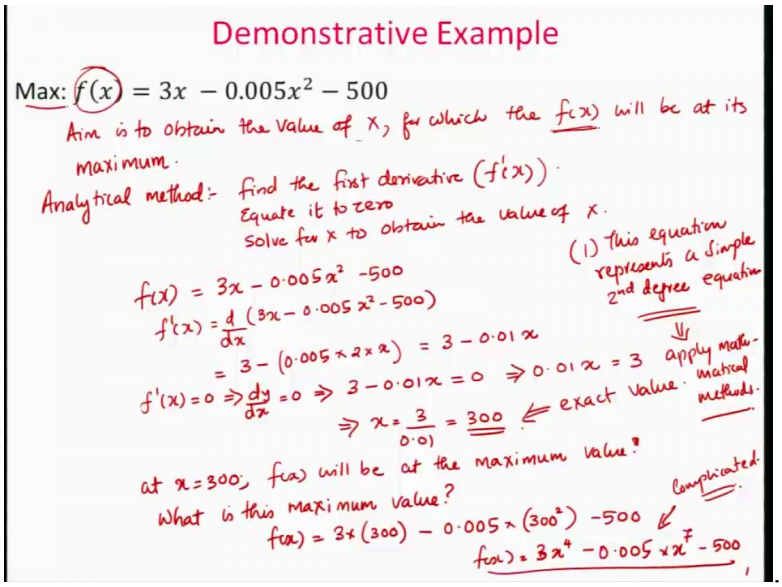

So, let us here look at an simple example ok. And, this demonstrative example we will first try to take a look into this example in the form of an analytic approach ok. So, the system is the aim is to obtain the value of X, for which the f of x will be at its maximum ok. That is what the max concepts is the maximum, but so, you are trying to find out that value of x particular value of x for which the f of x, the resulting function will be at it is maximum, what will be the value of this? So, the analytical method, the analytical approach method amounts to what? Amounts is find the first derivative or what we call as f dash x then equate it to 0 ok.

Then solve for x to obtain the value of x ok. So, let us see how it is done? So, your f of x is equal to 3 x minus 0.005 x square minus 500. So, the f dash x the first derivative equal to d by dx of 3 x this is your y 0.005 x square minus 500. So, the which will translate to this will become 3 minus this will be 0 0 5 times 2 times x and this will be 0, because the constant derivative will be 0 which will be equal to 3 minus 0.01 x ok.

Now, the thing is you say f dash x the first derivative you set to equal to 0 implies d by dx dy by dx is set to 0 implies 3 minus 0.01 x should be equated to 0, which gives you what we call as 0.01 x is equal to 3 right, which implies x is equal to 3 divided by 0.01 or equal to 300 ok. So, the analytical method tells that at x equal to 300, f of x will be at the maximum value and what is this is an exclamation sorry ok. So, what is this maximum value?.

What is this maximum value? That maximum value is obtained by f of x is equal to 3 times x will be 300 minus 0.005 times 300 square minus 500, if we solve this equation then you will get the value of f of x which will be the maximum. So, this is the what we call as the exact value that we were looking for. So, the main thing is that this equation represents a simple second degree equation, think about simple quadratic equation right.

So, now, because of this you can apply mathematical methods, you can do that and then you get the result. So, let us say instead of this if the question was this equation was complicated, if the equation instead of this was something like this f of x is equal to 3 times x to the power of 4 minus 0.005 times x to the power of 7 or something like this minus 500, then it would not have been that easy like what we are done now? Ok. This would have been a complicated case, then you might have resorted to something else.

So, the same question or same problem that we are talking about how can we actually solve this using the numerical method is what we are going to see in this slide?

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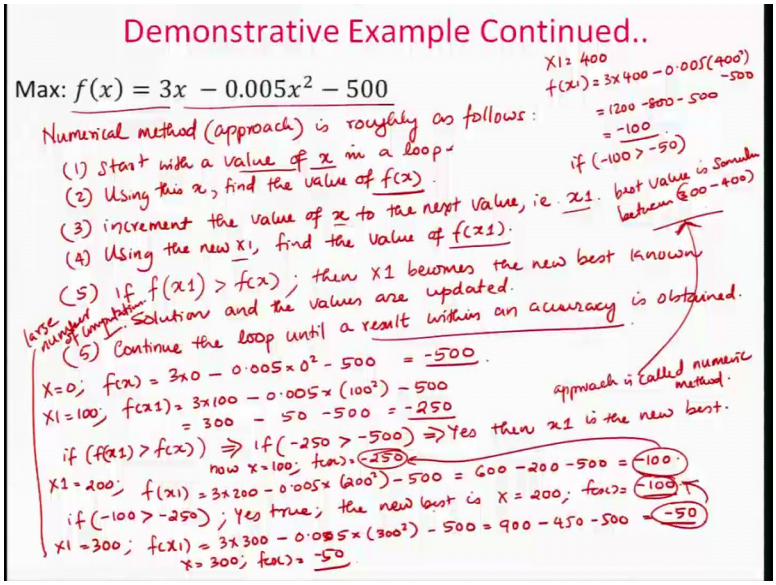

So, the same system ok, the same equation 3 x minus 0.005 x square minus 500 we are trying to solve this using the numerical method. So, the numerical approach, the numerical method or the numerical approach is roughly as follows ok. So, the rough approach is like this first start with a value start with a value of x in a loop ok.

Then, using this x, find the value of f of x see first start with a value of x in the loop, you take in a put it in a loop, computer loop, programming loop, and then which you put a value of x a particular value of x and for that you find the value of f of x ok. Find even once you find the value of f of x, then the third one is increment the value of x to the next value, that is x 1 you increment of the value of x to the next value that is x 1 right, then 4 using the new X 1, find the value of f of x 1 right. So, you first use x find the value of f of x then increment the value of x to x 1, then using that X 1 find the value of f of x 1.

Then, you compare if f of x 1 is greater than f of x ok, then X 1 becomes the new best known solution ok. And, the values are updated then the 5th one is you know continue the loop you repeat this process until a result within an accuracy is obtained here you are not looking for the exact result. So, with the reasonable accuracy we can actually get this. So, let us take a look into what we call as this let us start with the loop where x let us put it as X equal to 0. So, then at x equal to 0 f of x will be is equal to 3 times 0 minus 0.005 times 0 square minus 500 which will be equal to minus 500

So, at x equal to 0 f of x will be minus 500. Now, let us update X 1 to be 100 ok, for the time being. So, the f of x 1 is equal to 3 times 100 minus 0.005 times 100 square minus 500 ok. So, we look into this then that gives you what we call as that is 300, and this is 100 square is 1000 multiplied by this will give you 50 and this minus 500 which gives you minus 250 ok. So, now, if f of x 1 is greater than f of x implies if minus 250 is greater than minus 500 is this correct yes this is correct yes then x 1 is the new best ok.

Now, let us see what happen when we take the next x 1 ok. So, let us now increment the x 1 to so, now, the f of x is become, so, this is now x equal to 100 f of x is equal to minus 250. Now, X 1 equal to 200, if that is the case then what happens? Then f of x 1 will be 3 times 200 minus 0.005 times 200 square minus 500, which is equal to 3 times 200 will be 600 minus 200 square times this will be 2 minus 200, then minus 500 which will give you what we call as minus 100.

Now, if you check this if minus 100 is greater than minus 250. So, now, you are comparing with the previous value ok. This 100 is being compared with what we call as the minus 250, yes this is true, yes true. The new best the new best is X X equal to 200 and f x is equal to minus 100 is the new best. Now, let us take X 1 is equal to 300, then f of x 1 will be 3 times 300 minus 0.0 5 sorry 0 0 5 times 300 square minus 500.

So, that will be 900 minus 450 minus 500, which will be minus 50. So, now, if you look into this value minus 50 is larger than 100 minus 100. So, your new X will be equal to 300 and f of x will be minus 50. Now, do we stop here or do we continue. So, now, we will take the next one which we called it as X 1 is equal to 400 ok. And, at X 1 is equal to 400 we will get f of x 1 will be equal to 3 times 400 minus 0.005 times 400 square minus 500 which will give you 1200 minus 800 time 500.

Now, this will give you minus 100. Now you know that at x 1 equal to 400 the value has gone. So, now, if minus 100 greater than minus 50 is this true no this is not true. So, then what happens is the new value? So, now, we figure out that the best value of x, best value is somewhere between 300 and 400. You know, what you do is? How you do a loop between 300 and 400 and try to compute do this again. So, you can see that there is a this is what we called as the large number of computation ok. That is the reason why we end up doing large number computations to do this and then now, we repeat the same thing for 300 400 and we repeat this process.

So, this approach is what we call as this approach is called numeric approach or numeric method ok. You are not really trying to find out the exact value; instead you are trying to find the estimate of this. So, this will look silly for such a simple problem, but as a problem becomes complicated as soon as degrees of the equation becomes larger, than you have no other choice, but to basically go with this method and then find the solution to the problem ok.

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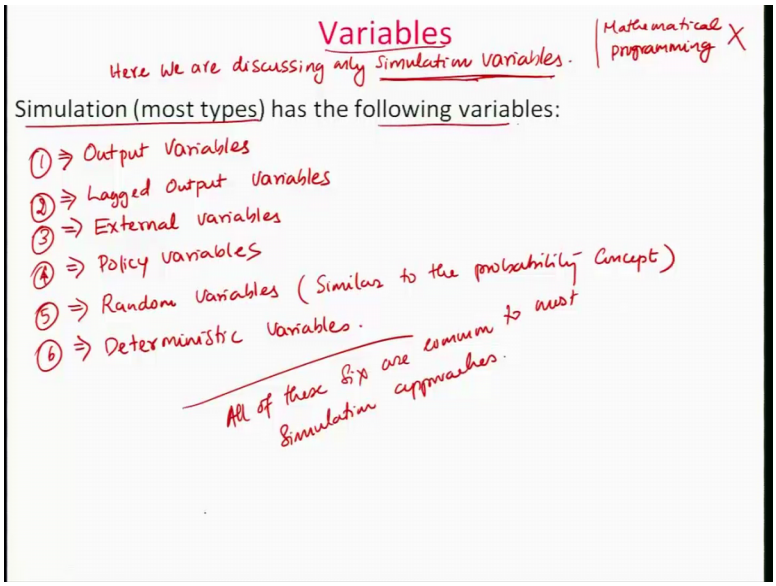

With that now, let us get into the next major concept that we are going to see as part of today, which is called as the variables. And, these variables are pertaining to ok. Here, we are discussing only simulation variables ok. We are only focused on simulation variables; we are not talking about what we call as the mathematical or programming variables? We are not talking about mathematical and programming variable instead; we are talking about simulation variable or variables that are used as part of your simulation model ok. So, most of the simulation or most types of the simulation has the following variables ok. So, we list the number of variables now here and then we will see or the major variables here and we see what are the main one?

The first one is the what we call as the output variable and we will see each 1 of these variables in detail will be following after we list all these variable, then the one that we call it as the lagged output variables; I will explain to you what the lagged output variables are and they are also important in the case of simulation. Then, what we have is called the external variable. Ok. We used external variables, then after that what we have is the policy variables I will also explain to you what a policy variable is all about policy variables, then we have what we call as random variables ok, whereas this is similar to the probability concept probability concept, but we will see this in detail.

And, then the last type of variable is what we call as the deterministic variable. Ok. So, these are the major variables the first one is the output variable, second is the a lagged

output variable, third is the external variable, fourth is a policy variable, fifth is the random variable, and sixth is what we call as the deterministic variable. So, we will talk about these 6 variables which are quite common. All of these 6 are common to most simulation approaches in one way or other these 6 variables gets used in the simulation model of simulation approach that we do ok.

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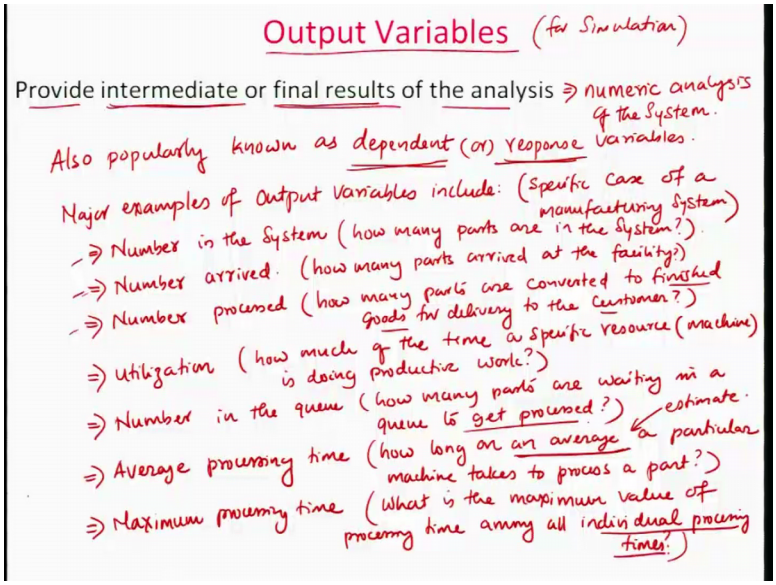

Now, then let us see what are the or the main thing called as the output variables ok? So, the output variables pertaining when in the in the case of for simulation? The case of simulation the output variables the most important thing is they provide intermediate or final results of the analysis ok. So, analysis that is the numeric analysis, numeric analysis of the system, what we are focusing here is the numerical analysis of the system. So, it gives you the intermediate ok, in between or final results of the analysis, the numeric analysis of the system also popularly known as we call these variables popularly known as dependent or response variable.

Say somebody say is I am talking about the dependent variable or response variable. What they are talking about it is there is a talking about the talking about the output variable. And, output variables provide you an intermediary or a final result as part of the numeric analysis of the system. So, the same term the dependent variable or the response variable that people use is also the also the same as that of an output variable. So, major examples major examples of output variables include ok.

Let us consider the special cases specific case of a manufacturing system, we are talking about a manufacturing system for that case the specific variable output variable some of the variables included this ok, number in the system ok. Which tells you, how many parts are in the system? Ok. You are answering that question number in the system ok, most commonly known as number in the system. So, it is an example of an output variable ok. Then number arrived in a manufacturing system it will be how many parts have arrived in the system? That will be the number arrived ok.

How many parts arrived at the facility or the manufacturing factory? Ok. So, that is the number arrived, but then number process ok. So, how many parts are converted to finished goods for delivery to the customer? So, that is the number of parts are processed. So, how many parts are converted to finished goods? Ok. That gives you the number processed.

Then, utilization ok, how much of the time or specific resource, let us call it as a machine ok, specific resource a machine is doing productive work? Ok. That is, what is the utilization aspect of it? Then number in the queue ok, it measures how many parts are waiting in a queue, queue to get processed? So, how many parts are waiting in a queue before a machine in front of a machine. So, that it can get processed that will be the number in the queue. Then average processing time ok. It is how long on an average a particular machine takes to part? Ok. So, here is an average or what we call as an estimate? Ok. They are all estimate on the on that regard right.

So, how long does a particular machine takes to process a particular part on an average that is what he called as the average processing time? Ok. Similarly, also maximum processing time; so, this is the what is the maximum value, maximum value of processing time among all individual processing times? Ok. So, what we are doing here is that here measuring you have individual if you are producing 100 parts, each one of the part will have an individual processing time. And, from that individual processing time 100 find the maximum value and that will be the maximum processing time.

So, these kind of variables that I mentioned to you the number and system number arrived number processed all those kind of things, are quietly common when you are studying any of the systems in the for simulation. And, these gives you approximate idea

of what is the final performance of system, when the simulation is completed or when the simulation is running as time progresses? Ok.

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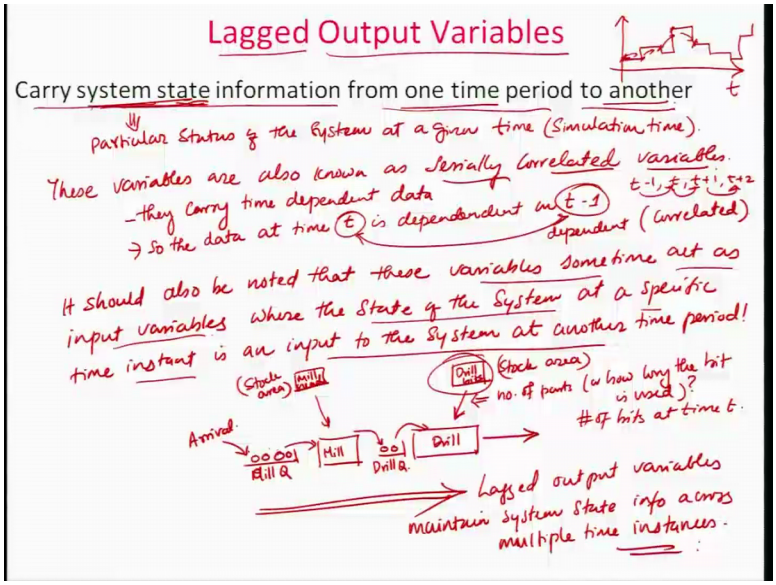

The next variable, we are going to talk about is the lagged output variables ok. The lagged output variables are unique category of variables in the case of simulation, because is lagged output variables can carry, these variables can carry the system state information from one time period to another ok.

Remember that we draw the graph of the behavior of the system and on x axis we have time and y axis there were different factors that we are measuring. And, we were seeing that the behavior the system was doing something like this and this kind of stuff ok. So, the time dependent behavior of the system ok, when you carry that information of the time dependent behavior from one time instance to another from this particular time instance to this to this to this. So, this time instances the way you transfer this information to this time instances, is what we call as the lagged output variables.

So, they carry system state information remember state means the particular status of the system at a given time. This time is a simulation time not the real time ok.

Then, that is that system state that gets transferred from one time period; one a particular time period to another is what is the job of the lagged output variable? Ok. These variables these variables are also known as serially correlated variable ok. These

variables are correlated purely, because of one reason, because they carry time dependent data ok. So, the data at time t is dependent on t minus 1 the previous time period or t minus whatever you want call as.

The t and t minus 1, they are related or they are dependent on each other. Hence, it becomes correlated. So, it is called serially correlate, because of time progresses you will see t minus 1, t plus 1 and t plus 2 etcetera. So, they are you know serially these values are correlated. So, hence it is called as a serially correlated variables right. Also, it should also be noted that these variables sometime act as input variables, where the state of the system state of the system at a specific time instant is an input to the system at another time period ok. So, the variable sometimes these variables they act as an input variable, because the state of the system at a specific time instant is an input to the system at the other time period.

So, let me give you a small example. Let us say that you have a milling machine and a drilling machine ok. So, the each one of the machines have it is own queue and parts arrive in the milling machine and there at the drilling machine. So, they come to the mill from there they come to this queue from this queue they come to the drill and then they leave ok. So, here is the arrival ok. So, the parts arrive from there they is a machine. So, here the machine is free, then they go directly to the mill if not the machine is it will join the queue this is mill Q this is the drill Q ok. And, then once the milling is over it will look, whether the drill is free and if it is free when release free and if it is free it will go directly otherwise it will join the queue ok.

And, let us say let us assume that here is a drill bit or drill bits ok. So, here is like a stock area, you are stocking drill bits here and here you have what you call as a mill head? Ok. So, this is another stock area, where the milling heads are stocked here. So, depending upon the number of parts that are processed the drill and mill bits will come here, which will result in changing the milling and the drilling head depending upon the tool wear. So, then this is dependent on number of parts or how long the bit is used? How long the drill bit has been used is one important question. And, if the particular time reaches, then the drilling machine will stop and it will say send me a new drill bit ok. And, then only the process will remove.

So, the state of the number of drill bit, how many drill bit are there at a particular time? Ok. Number of bits at time t is an input to this particular system. So, hence we can say that the lagged output variable maintain system state information info across multiple time instance. So, they keep so, they keep track of what is happening in the system over a good period of time. So, that we can we can actually get the right information that is necessary.

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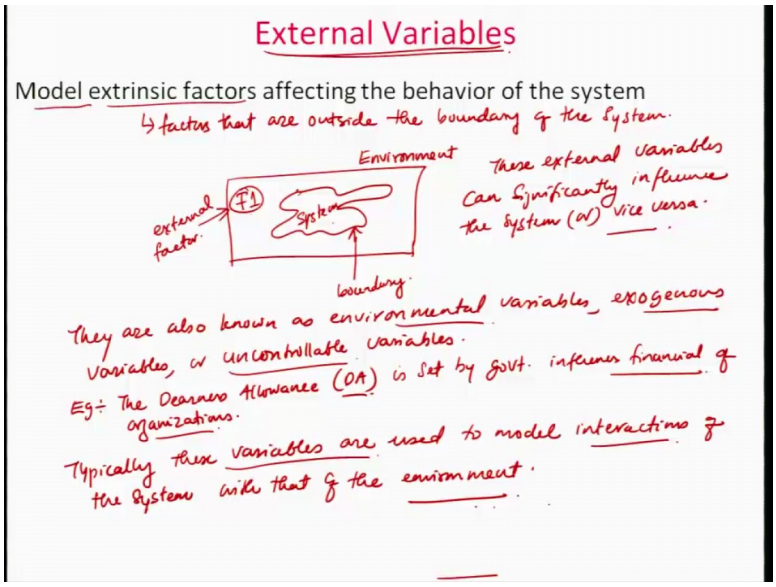

Then, we talk about what we called as the external variables. These variables, they are used to model the extrinsic factors ok. Extrinsic factors means factors that are outside the boundary of the system. So, as I have drawn earlier is that if you think about this as a system, it's not a system this as a, what we call as environment and let us assume that this is a system we are studying ok, this is a system of interest, then this is the this party is the boundary ok. So, factors that are outside, so, if there is a particular factor f 1 write here, then that factor is this is a external factor is a factorial representation of the say ok.

So, the influence of the external factor how this external factor will influence the behavior of the system is being studied using external variables. So, these external variables, variables can significantly influence the system or vice versa.

So, what we are saying is that it can either significantly influence the system or it can may not influence system at all both of them is possible ok. There also known as they are also known as environmental variables, environmental variables or exogenous variables

or uncontrollable variable ok. All these say environmental variables exogenous variables and uncontrollable variables there all one and the same they all referred to what we called as external variable. So, an example of this is the dearness allowance popularly known as DA dearness allowance is set by government ok, influences financial of organization.

You have no control over what is the DA that the government is going to set the government will set the DA according to many conditions and it is on terms and fancies. And, once the DA is set then you have to use that DA and give the salary to the people who are working in your organization. So, hence such a factor that you are absolutely no control over set environmental factor, but it have a significant impact on the way your business operates, whereas the other time the government might give the minimum support price of rice. And, the minimum support price of rice might not be that important to the automotive industry who for which rice is not a raw material. So, then the impact of the minimum support price of rice on the automotive industry will that be quite low compared to the dearness allowance rates that is said by the government, which directly influence the employee salary alright.

Also, typically these variables these external variables are used to model interactions of the system of the system with that of the environment, with that of the environment ok. So, if you want to model the interaction of the system with respect to the environment, then what you need to do is you need to use the external variables to model that interaction.

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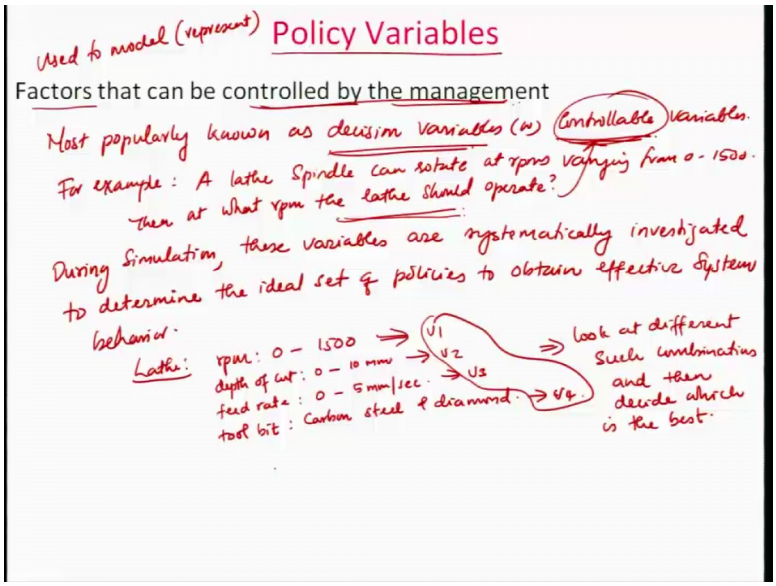

Now from there you will get into what we called as the policy variables ok. Policy variables are interesting, because ideally speaking these are the factors this model ok. So, policy variables are they are used to model or represent factors that can be controlled by the management. Whatever the factors that are controllable by the management whatever the management can control, that factors are basically modeled by policy variables. Most popularly known as decision variables or controllable variables remember that external variables were uncontrollable, where as in this particular case we are talking about controllable variables or decision variables.

There is we talk it as a decision variable purely, because this can be controlled by you your decision is dependent on controlling it. So, let us say that for example, lathe spindle can rotate at rpm varying from 0 to lest say 1500, then at what rpm the lathe should operate? That is the decision variable for you. Should it operate at 200 rpm should it operate at 5000 500 rpm should it operator at 1000 rpm, that question that decision, you have to take because the speed of the lathe is controllable it is this speed can be controlled by you. Hence, that is the controllable variable ok. And, during simulation, during simulation these variables during simulation these variables are systematically simulation.

These variables are systematically investigated to determine to determine, the ideal set of set of policies to obtain effective system behavior. So, assume that let us take the

example of the lathe again, in the lathe you can decide what the things that you can decide is rpm is varying between 0 to 1500 ok. Then, you have the depth of cut ok. It can vary from let us say 0 to 10 mm and then the feed rate let us say it varies from 0 to let us say 5 mm per second something like this ok. I am just putting random values. And, then tool bit you can say carbon steel and let us say diameter for the time being. So, then you can decide a particular value ok. So, this will be 1 particular value a v 1 this will be v 2, v 3 and then you put another value v 4.

And, you can have many set combination of this ok, you have this is one particular combination. Then look at different such combinations and then decide which is the best, ok. So, by doing this, what you are going to do? If you are going to basically find out the, ideal setup policies that you can actually use to decide the behavior of the system ok. So, now, we are back to the again looking forward into the variables.

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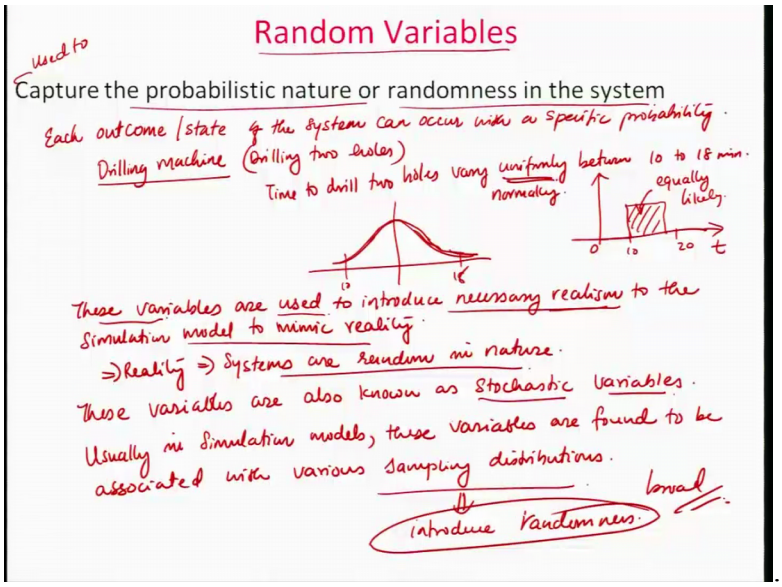

The next type of variable that we are going to talk about is the random variables ok. What is this use? These used to capture the probabilistic nature or randomness in the system ok. So, or what you are trying to do it is each outcome or state of the system can occur with a specific probability ok. So, if I say that so, for example, if you take a drilling machine and your part has been your manufacture, you are drilling 2 holes, if that is what you are trying to do.

And, then let us say if I say that the drilling time that time to drill 2 holes vary uniformly between 10 to 18 minutes, if I say that then what I am saying is I can represent a system something like this 0 this is 10 this is 20 like this is your time and here is the probabilistic nature. So, then up to 18 you will say all values between this are equally likely. Because, it is uniform uniformly distributed, if I say instead of uniform, if I say normally distributed, then it will be something like this, you will have you know the variation like this and you will probably say this is between 10 and 18 or something like this you know; however, you want to draw it ok. So, this will give the bell shaped curve.

So, this probabilistic nature or what we call as the probability associated with the behavior of the system is typically captured using with the help of random variable ok.

These variables are used to introduce necessary realism to the simulation model to mimic reality. So, these variables these variables means these random variables they are typically used or people use these random variables to introduce necessary realism ok, or necessary what you call as the randomness into the system, so that you can model reality ok. And, a reality is reality systems are random in nature or the behavior of the system has a probability associated with it this is not a completely true statement, but you can say that the behavior of the system has a probability associated with it. That to mimic that probability, that probabilistic behavior or that uncertainty in the expected output is created using such a system ok.

These variables are also known as also known as stochastic variables ok. So, when I say we are talking about stochastic variable, then the main aspect is this that we are talking about random variable ok. And, usually in simulation models these variables are found to be associated with various sampling distributions.

So, what we are saying here is that various sampling distributions are used to introduce. So, these sampling distributions introduce randomness. This is like a let us make it as a broad statement we will make it more except as we process progress later down the role in this case ok. So, as I said earlier the random variables capture the randomness in the system and this is done to mimic the reality or realistic behavior of the system. And, in reality that these systems have some stochasticity or the behavior of the system is dependent upon some probably, some probability and these stochastic variables are used

to actually introduce the randomness or the variation in the system, which is associated with that of the sampling distribution.

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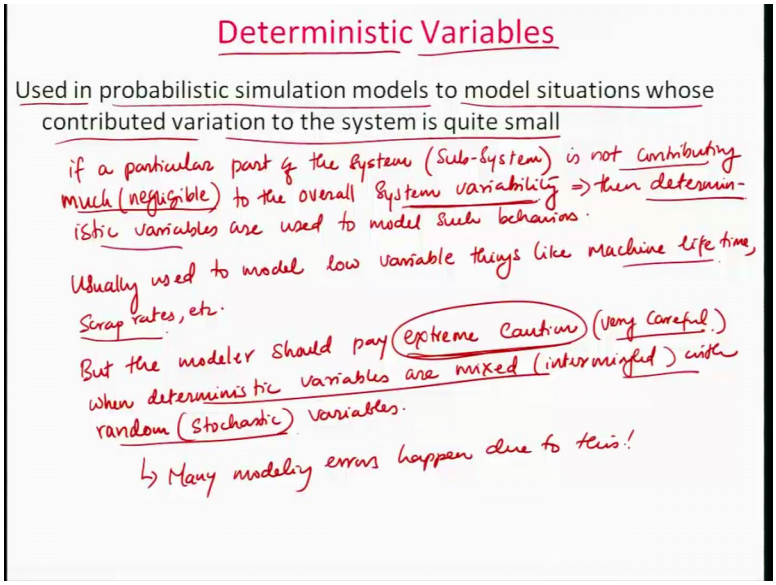

Now, let us get into what we call as the last set of variables, which we call as the deterministic variables. So, the deterministic variables they are used in probabilistic simulation models ok. So, deterministic variable in probability I will explain that to you use them to model situations whose contributed variation to the system is quite small ok.

So, if a particular part of the system or let us call as a subsystem, if a particular part of the system or a subsystem is not contributing much or let us call it as negligible much to the overall system variability, I am not talking about uncertainty now, I am talking about variability ok. We will study what is this system variability. In the following classes to the overall system variability, if a particular part or a subsystem does not contribute much to the system variability, then deterministic variables are used to model such behavior.

So, the deterministic variables are used in situations, where the subsystem does not contribute much or it is a negligible contribution to the overall system variability. So, these are usually used in used to model low variable things like machine life time, scrap rate etcetera. So, these are the things that does not contribute much to the variability the machine life time scrap rates all this kind of things are much less in the case of a

manufacturing system. So, hence you will use a deterministic value to pretty much you know model such behavior of system ok.

But, the modeler should be should pay extreme caution or care should be taken very careful, when deterministic variables are mixed or intermingled with random or stochastic variables ok. So, if you are a modeler and you are trying to mix deterministic variables are getting mixed or intermingled with stochastic variable, then use extreme caution or be very careful, because you know many modeling errors happen due to this ok.

So, my suggestion in this regard is that, when you are modeling deterministic variables you have to mixing them with the stochastic variables, then you know please ensure that you pay extreme caution alright. With this we come to the conclusion of this particular topic about the basic type of the approaches then difference between analytic and numerical approach and as well as different type of variables, that are used in the modeling or used in the simulation studies. And from based on this now we will get into simulation like methods and the next lecture we will look into Monte Carlo and see how Monte Carlo works.

And from there then we will get into an overview of probability in statistics that is necessary for our simulation course and then we will move towards studying various simulation software and other stuff. So, for this particular class, I recommend you guys to go through the lecture notes and start studying getting more information about this, additional reading materials will be provided and please read your text book at the meantime.

Thank you very much.