**Simulation of Business Systems**
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**Lecture – 05**
**Example of Simulation Model**

Good morning guys. And, welcome to the yet another lecture of Simulation of Business System. So, now, having seen the basic assumptions and fundamentals that we in required to start the simulation model. We are now going to do the simple Example of the Simulation Model. Before, we doing that there is also one more thing that I need you guys to understand which is called as the table of random digits.

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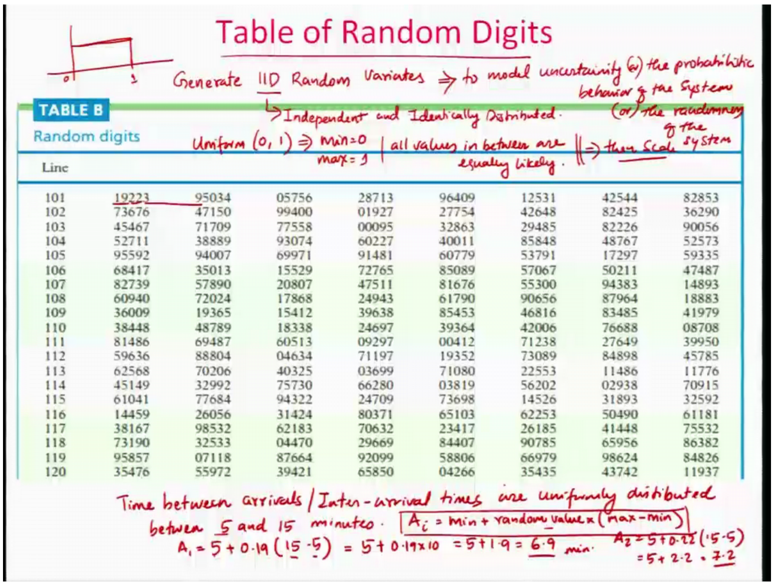


TABLE B

Random digits

| Line | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 101 | 19223 | 95034 | 05756 | 28713 | 96409 | 12531 | 42544 | 82853 |
| 102 | 73676 | 47150 | 99400 | 01927 | 27754 | 42648 | 82425 | 36290 |
| 103 | 45467 | 71709 | 77558 | 00095 | 32863 | 29485 | 82226 | 90056 |
| 104 | 52711 | 38889 | 93074 | 60227 | 40011 | 85848 | 48767 | 52573 |
| 105 | 95592 | 94007 | 69971 | 91481 | 60779 | 53791 | 17297 | 59335 |
| 106 | 68417 | 35013 | 15529 | 72765 | 85089 | 57067 | 50211 | 47487 |
| 107 | 82739 | 57890 | 20807 | 47511 | 81676 | 55300 | 94383 | 14893 |
| 108 | 60940 | 72024 | 17868 | 24943 | 61790 | 90656 | 87964 | 18883 |
| 109 | 36009 | 19365 | 15412 | 39638 | 85453 | 46816 | 83485 | 41979 |
| 110 | 38448 | 48789 | 18338 | 24697 | 39364 | 42006 | 76688 | 08708 |
| 111 | 81486 | 69487 | 60513 | 09297 | 00412 | 71238 | 27649 | 39950 |
| 112 | 59636 | 88804 | 04634 | 71197 | 19352 | 73089 | 84898 | 45785 |
| 113 | 62568 | 70206 | 40325 | 03699 | 71080 | 22553 | 11486 | 11776 |
| 114 | 45149 | 32992 | 75730 | 66280 | 03819 | 56202 | 02938 | 70915 |
| 115 | 61041 | 77684 | 94322 | 24709 | 73698 | 14526 | 31893 | 32592 |
| 116 | 14459 | 26056 | 31424 | 80371 | 65103 | 62253 | 50490 | 61181 |
| 117 | 38167 | 98532 | 62183 | 70632 | 23417 | 26185 | 41448 | 75532 |
| 118 | 73190 | 32533 | 04470 | 29669 | 84407 | 90785 | 65956 | 86382 |
| 119 | 95857 | 07118 | 87664 | 92099 | 58806 | 66979 | 98624 | 84826 |
| 120 | 35476 | 55972 | 39421 | 65850 | 04266 | 35435 | 43742 | 11937 |

And, as I said earlier that we use we generate I I D random variates and as mentioned I I D stands for Independent and Identically Distributed random variates. And, why do we generate random variates to model uncertainty or the probabilistic behavior of the system or the randomness of the system ok.

So, here either we are trying to model uncertainty or the probabilistic behavior of the system or the randomness of the system all of the means predimising ok. So, one way to do it is we basically on the most common option to take it is you take uniform 0 1 random variates, which means the values will be always between 0 and 1 and all values are equally likely; that means, minimum equal to 0 maximum equal to 1 all values in between are equally likely or in a way to draw this will be if you say this is 0 and this is one. So, then the graph of this will be somewhere like this. So, every value in between this will have the same probability of occurrence ok.

And, then scale to whatever you want to do ok. Other option is that for anyway for doing this you require some form of random numbers or at some place. So, there is a table of random digits and in which you can take random digits and then use that as a way to do it. So, most of the time what I do is I take this table of random digits and I say if I say that the time between arrivals say for example, time between arrivals which is also the inter arrival time arrival times are uniformly distributed between 5 and 15 minutes so, multiplication becomes easy minutes. If I say that then what do I do is I use these digits are sampling the uniform 0 1.

So, let us say I am going to you take 2 digit accuracy. So, I will just take the first 2 digit value and the first arrival time A 1 will be equal to the value will be 0.19 it will be 5 plus 0.19 times 15 minus 5 or the equation is arrival time A i is equal to min plus a random value multiplied by maximum minus minimum value. So, 5 is the minimum value and then the random value is 0.19 that I got from here 2 digit accuracy, if I take 3 digit will be 0.19 2 and this is the max and this is the min. So, this will be 5 plus 0.19 multiplied by 10.

So, it will be 5 plus 1.9 which will equal to 6.9. The first arrival time will be 6.9 minutes seconds whatever you want to call it let me put it as the minutes. Then you have to calculate A 2; A 2 will be these next value next 2 digits, 5 plus 0.22 15 minus 5. So, this will be 5 plus 2 pint 2 this will be 7.2 like this. Then the digit will be 3 and 9 39 ok. So, 0.39 so, we keep on doing this. So, this is a process in which your random digits can be used to generate your arrival times same is true for the service times.

So, in the next model that I am going to demonstrate to you guys we will be following this approach ok. This approach to calculate the arrival time service times and other things, but I will not be taking this is random value for the time being we will be just taking numbers randomly picking up number. So, that you can see how the mechanism works, but these numbers can be derived using this process which will actually model randomness. The reason why we are doing this is instead of taking all values exactly the same, we are basically saying is in this particular case is that all values between 5 and 15 minutes are possible for each consecutive arrivals. If the one person is arriving at after the sixth minute the next person can arrive anytime between 5 to 15 minutes after that.

So, then if the next person arrives in 10 minutes then; that means, 6 plus 10 which would be 16th minute is the next person will be arriving. And, that is how the simulation clock will advance? I hope this makes sense.

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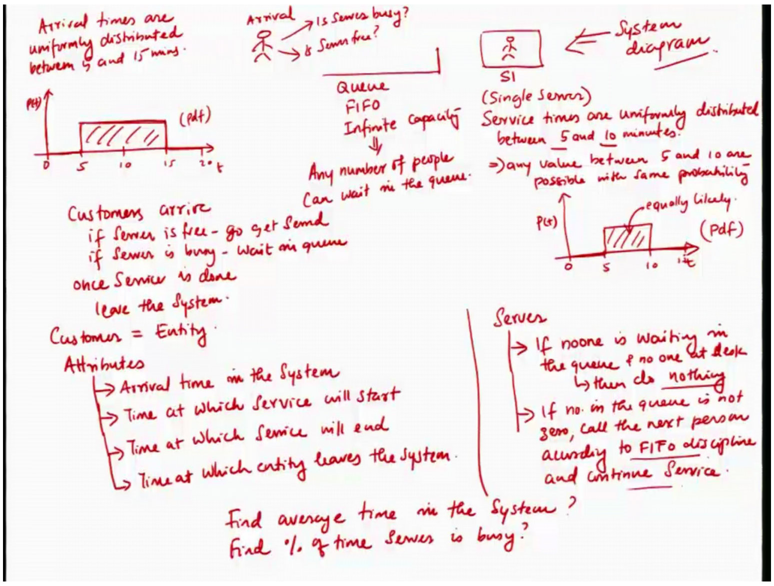


So, let us try to do a simple hand simulation of our single server model. So, the first I will do is a diagram which is typically called as a system diagram, we call this as a S 1 or the single server and here is a person who is waiting there to get serviced. And, let us assume that the service times are uniformly distributed between 5 and 10 minutes. So; that means, if anybody comes in the service time can take any value this implies any value between 5 and 10 are possible with same probability.

So, if I draw the graph of this it will be like this here will be your time this will be your probability ok. And 0 then you have 5 then you have 10 then you have 15 like this. So, up to 5 there is no probability at 5 it jumps up to 1 value and then up to 10 remains the same and it is comes down. So, all values between 5 and 10 everything here is equally likely same probability of occurrence ok.

Then people come in and there is a queue also in the system when an arrival happens a person arriving into the system ok, an arrival this is an arrival let us say arrival has 2 things it can do one is; is server busy, is server free, these 2 questions the arrival has to answer.

If the server is free it will go directly gets served, if the server is busy it will come and stay in the queue. So, let us and let us say that the arrival times are uniformly distributed. We will take uniformly distributed for the time being to make life easy and then we will see complicated models later the arrival times are uniformly distributed between 5 and 15 minutes ok. So if I draw the graph of this arrival times will be something like this 0 5 10 15 20 the time goes like this and here you have the probability associated with it. So, up to 5 the probability is 0 and 5 it goes up and then it comes down like this up to with all values here are equally likely ok.

So, this is what the so, this is called as the pdf probability density function the pdf of the arrival and service time distribution. And, this is a queue of the server it is FIFO first in first out and has infinite capacity. Any, number of people can this means infinite capacity means any number of people can wait in the queue ok. That is what we talked about the infinite capacity queue. With this now what we will try to do is we will try to simulate people arriving into the system. So, logic is customers arrive, then if server is free go get served, if server is busy wait in queue then once service is done leave the system and so, the customer is the is your entity ok.

So, the attributes of the entity includes the attributes of the entity includes arrival time in the system ok, time at which service will start, time at which service will end, time at which entity leaves the system ok. So, these are the attributes of the model that we are talking about. So, we take at this point that is a customer as the entity. If you look at the server logic a server logic if no one is waiting in the queue, then do nothing queue and no one at desk. There is nobody waiting in the queue this no one at the desk then do nothing or remain idle. Other option is if number in the queue is not zero, call the next person according to FIFO discipline and continue service.

So, the logic of the server who is doing the service is pretty much like this. No one is waiting in the queue and no one is left at the desk then do nothing remain idle. If someone is waiting in the queue the queue length is not zero, then call the next person according to the FIFO discipline and then continue the service. And, other end of the day we are trying to find average time in the system and, find percentage of time server is busy ok. So, these are the 2 questions that we are interested in answering at this particular simulation.

I hope all of you understand the example setup that we have and also remember when we draw a diagram like this with a queue then the server and other things. This diagram is also called as the system diagram. It is different from the system diagram that is drawn in the systems engineering, but in this is more of a simulation system diagram. So, I hope everybody understands this example setup and using this example setup we are going to do the simulation. So, as I said earlier.

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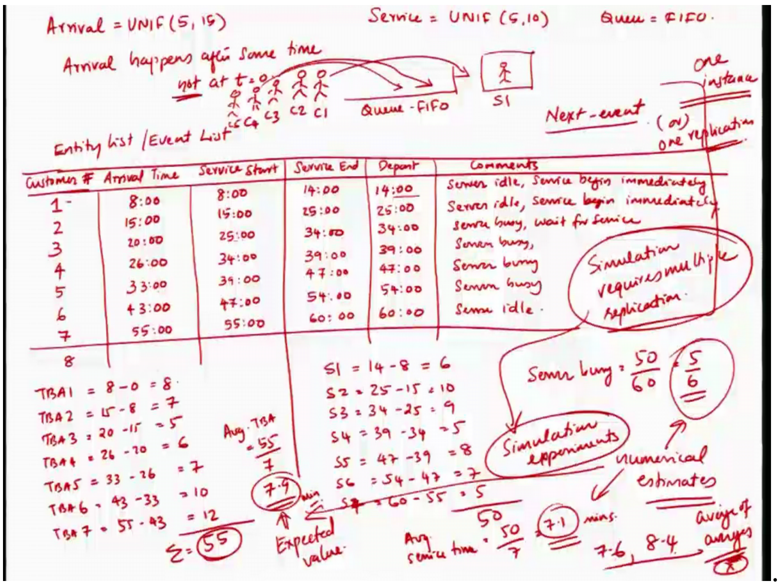


Arrival is uniform 5 and 15 service is uniform 5 and 10 and queue is FIFO, First In First Out ok.

So, I am doing a system diagram once again. So, that you have an idea server waiting here as one and here is the queue is a FIFO queue and arrivals is going to happen here. So, we first make a table or a tabular format remember and this is called as the entity list or event list. Let us all think about it as an entity list for the time being and then we can say it as customer number, then you will call it as arrival time ok, then service start, service end, depart alright. So, now, we also told that arrival happens after some time or not at t equal to 0. So, the system began a time is sticking time is going on. So, first the arrival time is between 5 and 15. So, I am giving a taking a random value I take the first random value as 8 ok. So, the customer one arrives at 8th minute ok.

So, the customer one arrives C 1 ok. He comes and he checks what happens is the server idle yes server is idle. So, he goes straight to the server ok. So, the service can start at 8 (Refer Time: 18:12), I will write my comments here. So, it is easy for you to follow, server idle service begin immediately ok. And, now the service time when will the service end you have to sample a time between the 5 and 10 uniform between 5 and 10. So, let us say I take it as 6 time units ok. So, the service starts at 8, then 6 (Refer Time: 18:47) it will take 6 minutes to complete the service. So, it will be over at the 14t minute and thus the idea.

So, up to from 8 to 14 the service will at 14 time interval the service will end and at the entity will also depart exactly at 14 ok. And, one arrival has already happened you have to schedule the arrival of the next because as I said it is a next event. So, every time you do one event you have to also schedule the next event. So, in this case our entity is the customer so, we schedule the next event which is the next arrival. And, again for that you sample a time between this and let us say you got the value as 7 units ok. So, that 7 means the next arrival happens at 15 a plus 2 15 at entity will arrive, and at 15 minute will arrive this is your C 2. When it arrives it actually sees that is a server idle at 14 the time the entity has finish the service and it has left C 1 has left.

So, the server is idle, service begin immediately ok. So, the service will start at 15 and service I am taking the service time to be let us say as 10 time units. So, it will get over at 25, I am putting it as 10 because it any value between 5 and 10 is equally possible and it will leave at 25. So, now, I am doing the next event of this and I sampled a value ok. And, I got the time as I took the arrival time as 5 ok.

So, then 15 plus 5 the arrival will happen at 20, when it comes at 20 this is your C 3 ok, when it comes at 20 it sees that the server is not free it is busy. So, what it does is the C 3 will come and join the queue ok. So, it will wait and when can the service begin, it can begin only at 25 time units, because as soon as the server is finished server will look and see whether there is anybody in the queue sees there is one person in the queue it will immediately pull that person and starts the service.

So, service will start at 25 and let us take sample another value of the service time and let us take it as you know 9 units. So, excuse me. So, the 9 units the service will end at 34 a minute ok. And, this guy will depart exactly at 34. And, then this arrival that happened at

20, what will happen is you are to also the schedule the next arrival which is the fourth entity. Let us say we sample a time of 6, let us say randomly we sample time of 6. So, this entity arrives at 26th time unit. And, this is yours C 4 customer 4 and when he comes in he finds that no the entity is being served by the system ok.

So, what it will do is it will wait and the service can start only at 34. And, we will how long it will be we will sample little later. Now, that it is waiting in the queue this arrival will also trigger the next arrival the 26th ok. So, now, let us say we take the next time I got it as 7 time units. So, with the 7 time unit the 5th entity has arrived in the system. So, here is your 5th entity C 5. And so, 26 plus 7 that will be 33, it will arrive at 33 time units because so, 7 time in between. So, then when it comes at 33 it is not because it already sees is one other person C 4 has also joined the queue, and then C 3 will also join the queue and its service can only start little later down (Refer Time: 23:04) we do not know when it will be.

But, now that it reaches 33, now what happens is the system clock if you look into it this was 20 then it was 25, then it this was 26, this is 34 is still 33. So, then the next event that will come out of this is the 33 because we go we the simulation clocks jump from. So, the first the clock will start at 8 it will jump to 8 and 14, then after that 14 will jump to 15, from 15 it will it is not suppose to jump to 25 it will jump to 20. And, once it jumps to 20 then it will jump to 25, 25 it will get service started.

Then it is supposed to go to not instead of the 34 it will go to 26 and it is scheduled the service at 34, then instead of this it is sampled to the next one and it got the time as 33, 33 it will schedule the next arrival. Let us say we took the time of 10 so, it will become 43 time units ok. Then the next time that will come out of it will be 34 and 34 is a service beginning of the 4th entity. And, you sample a time and let us say we get the time value as 5. So, we get it as 39. And the entity will leave at 39 time units, then we look at the time period because 33 is over, then before 43 that is 39, it comes in and the 33 entity will start it is service at 39, because it is already waiting in the queue. And, then we sample a value because this is the next biggest time, we take the time as let us say a time units now a time units. So, that will become (Refer Time: 24:41).

So, that is 46 47 it will be over at 47 and the entity will leave at 47, then we look at 43 at 43, what happens is this entity has arrived; but it is still waiting in the queue. So, it will trigger the next arrival, we take a time of another 15 minutes. So, or let us say 12 minutes let us say so, 43 plus 10 53 so, at 55 the next entity will arrive. Then the time will go to 47, because at 47 this can begin the service. And, let us say we took the time of for the time of let us say 6 time units or 7 time units minutes. So, it will become 54 like this. And, the entity there comes and 54 55 will immediately start the service at 55 like this. And then the time will continue let us say we sample a time of 5 and it becomes 60 and this you know and then this will trigger the next arrival like this.

So, let us say we cut the simulation exactly right here at the 7 there is a stopping condition as soon as 7 entities are over we are done with this ok. So, in this case of the entity you can say that server busy wait for service. Here also we can see the server busy, here also we can say that server busy, here also server busy, the last case it is server idle.

So, if you want to calculate the time between arrivals ok. I am going to see whether I can erase this without making a mess. So, let me do this properly only up to this much. This also so, the time between arrivals TBA 1, that is equal to 8 minus 0 that is 8 TBA 2 time between arrivals that is 15 minus 8. So, that will be to 7 TBA 3 time between arrival 3, that will be 20 minus 15 that is 5.

TBA 4 time between arrival 4 will be 26 minus 20, that is 6 time between arrival 5 TBA 5 will be 33 minus 26 so, that is 4 7. So, TBA 6 will be 43 minus 33, which will be 10. TBA 7 will be 55 minus 43 so, that will be 12. So, like this. So, this is the time between arrivals, individual time between arrivals so, here 7 entities that are come in. So, if you sum this up that will be 7 plus 29 plus 6 15; 15 plus 5 20 28 30 35 so, 3 1 2 3 plus 2 5 55. So, 55 was the sigma sum total of the time. So, average TBA will be 55 by the 7 entities that has arrived.

So, it is about 7 point something right 7.9 or something like that. So, on an average in this entity you can see that in the simulation every 7.9th minute you will end up seeing one entity coming in the system. Though the individual times will vary, but this is the average or what we call it as the expected value ok, kind of a thing. Now, we see whether I can clean up little bit more ok. Now, similarly now let us look at the service times so, the service time S 1 is equal to 14 minus 8 ok, S 2 is equal to service start and service end 25 minus 15 that is 10.

So, 14 minus 8 that is 6 ok. S 3 is 34 minus 25 that will be 9 right, then S 4 is equal to 39 minus 34 that is 5, S 5 is equal to 47 minus 39 I believe that is right 47 minus 39 right. So, 8 S 6 is equal to 54 minus 47, that is 7 S 8 sorry S 7 is equal to 60 minus 55 that is 5. So, now, if you sum this up the 7 values then you will get actually 5 plus 5 10 plus 8 18 plus 20 25, 25 plus 10 35 9 44 plus 650 ok.

So, your average service time is equal to 50 divided by 7. So, that is 7.1 so, 7.1 minutes is what you get it as your average service time. So, if you want to find out what percentage of the; what percent of the time the server is busy? Ok. So, the total time duration is 60 of which 50 time units the servers server was actually providing service. So, the server busy is equal to 50 out of 60 or is 5 by 6. So, that much is the time the server was busy serving the people the rest of the time the server was idle ok.

So, like this we end up doing things like this and we calculate. So, all of these estimates what we have they are all numerical estimates. And, we cannot say that this is the behavior of the system with one single replication. Now, or one single instance like this, instead we will keep on repeating the same experiment multiple times. And, with that we actually would end up doing, what we call as estimating the behavior of the system. So, this one particular example ok, this whole thing is one instance or one replication ok. When you say one replication; that means, it is one complete run of the system. Now, what you do is in simulation requires multiple replications.

So, you run the same model with a different set of random numbers many times ok. And, then each case you will get an average like this ok. Different averages the next average could be 7.6 third average could be 8.4 like this you will get different values like this also you will get different values. Then using these values these individual values these individual averages, you will calculate the average of averages or what we call as x bar we calculate that then using that we make the a predictions of different numbers. So, that is why since we keep on repeating this experiment multiple time, this is why this is called as simulation experiments. The experiments is the numerical analysis using the simulation model, where we create random variates and from there we study the long time behavior of the system.

So, what I would recommend all of you guys to do it is?

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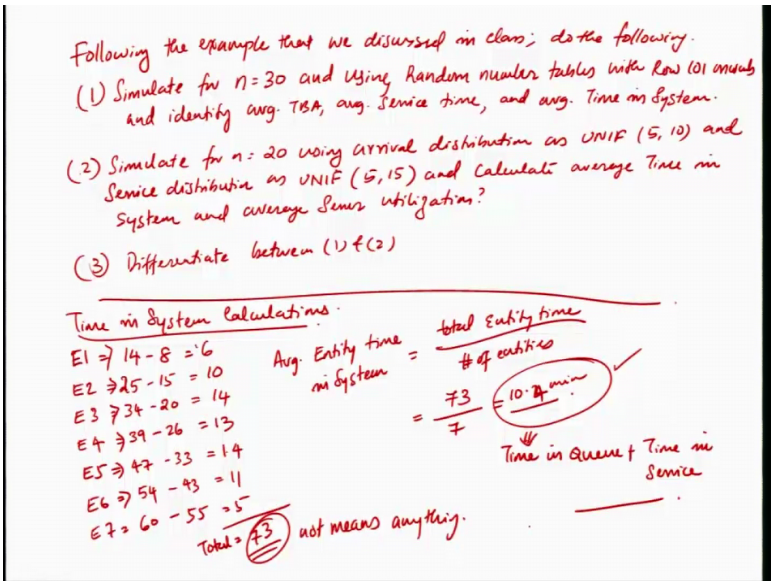


The question for you to do at is following the example that we discussed in class do the following ok. Simulate for n equal to 30 and using random tables and a number tables with row 1 0 1 onwards. And, identify average TBA, average service time, and average time in system ok. I forgot to tell you how to calculate time in system, but I will show you in a quickly ok.

Second thing is simulate for n equal to 20 using arrival distribution as uniform 5, 10 and service distribution as uniform 5, 15 ok. And calculate average time in system and average server utilization ok. Then third will be differentiate between 1 and 2 ok. So, that is what I would expect you guys to do as homework by yourself and do the analysis.

Now, let me also teach you something called as what is a time in the system ok. When an entity time in system I forgot to mention to you so, this literally means that the time that entity spends, when it is arrived at the system. So, the first entity if you think about it departs the system at 14 it arrives at 8 so, time in system calculations ok. How are you calculating the time in the system ok; so, the entity 1 the E 1 the time in system is 14 minus 8 ok. So, left at 14 it arrived at 8. So, 14 minus 8 will be 6 equal to 6, entity 2 time in the system ok, that will be it arrived at 15 it left at 25 ok. So, it left at 25 arrived at 15 that is 10.

For entity 3 it is it left at 34 it arrived at 20 ok. So, it will be 34 minus 20 that is 14 ok, entity 4 we will look at into as it left at 39 it arrived at 26 ok. So, it arrived at left at 39 arrived at 26. So, that is 26 plus 10 36 plus 3. So, 13 time units. E 5 all these times are E 5 is it left at 47 arrived at 33 ok. So, it is 47 and it arrived at 33 so, that will be 14 right.

Then entity 6 it will be it left at 54 arrived at 43 ok. Left at 54 arrived at 43. So, it is 11 entity 7 it left at 60 arrived at 55, left at 60 arrived at 55 so, that is equal to 5. So, this is the entity time in the system individual entity time in the system. So, total will be 5 plus 1 6 6 plus 4 10 10 plus 6 16 16 plus 4 20 423 then that is 2 2 plus 1 3 4 5 6 7. So, 73 time units ok.

So, total time in the system you calculated this as 73, but average entity this not means anything, but the average entity time in system is equal to total entity time divided by number of entities ok. That will be 73 divided by there are 7 entities right. So, on an average the entity spends 10.2 minutes in the system, 10.2 no 10.4 (Refer Time: 39:43) something 10.4 minutes in the system. So, this is the average entity time in system ok. So, we can see that it is more than the service time, why because at some point of time, this time includes the time includes time in queue plus time in service ok. So, that is a reason why this time is larger?

So, my suggestion to you is that once it changes in these we can see that these values in 1 and 2 has changed and then you will actually see these changes in the entity time in the system, and similarly the utilization of the server also ok. So, I hope that now you have a clear idea how to calculate certain statistics? And all the statistics are basically dependent on discrete event. So, that the discrete set at which you are calculating the time point at a discrete level. So, kindly do this and I hope that you will understand the behavior of the system and the solutions will also be posted for this questions later.

So, thank you very much for your patient listening and continue to read the topics that we are already discussed in the class. And please also do the work by yourself, because this is a new topic to every one of you and unless you put your time and effort into this it will be very hard for you to realize, because once we start going into the probability stats and push Monte Carlo simulation. If you are not up to date on this one, you will it will be very hard for you to follow it up not that of an easy course. So, please spend your time and effort in this case and I will see you in the next lecture.

Thank you.